Towards Coverage Closure: Using GoldMine Assertions for Generating Design Validation Stimulus

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ABSTRACT

We present a methodology to generate input stimulus for design validation using GoldMine, an automatic assertion generation engine that uses data mining and formal verification. GoldMine mines the simulation traces of a behavioral Register Transfer Level (RTL) design using a decision tree based learning algorithm to produce candidate assertions. These candidate assertions are passed to a formal verification engine. If a candidate assertion is false, a counterexample trace is generated. In this work, we feed these counterexample traces to iteratively refine the original simulation trace data. We introduce an incremental decision tree to mine the new traces in each iteration. The algorithm converges when all the candidate assertions are true. We prove that our algorithm will always converge and capture the complete functionality of an output on convergence. We show that our method always results in a monotonic increase in simulation coverage. We also present an output-centric notion of coverage, and argue that we can attain coverage closure with respect to this notion of coverage. Experimental results to validate our arguments are presented on Rigel, a 1000+ core processor design as well as several other benchmarks.

1. INTRODUCTION

Design verification is a primary source of bottlenecks in the system design cycle. Although directed tests capture much of the desired system behavior, they don’t suffice in checking for unintentional erroneous behavior. A phase of random input vector generation is employed with an intention to capture infrequent or unexpected design behavior. Due to the practical infeasibility of exhaustive simulation, the termination point of random simulation is very nebulous. Contemporary industries often use a numeric value like a few million simulation cycles before concluding the random simulation phase. Evidently, such a methodology is unsystematic and inconclusive. Despite multiple coverage metrics, there is no assurance that there are no gaping holes in the design behavior.

Coverage closure, or the process of determining the completeness of functional coverage of input vectors is then, one of the most daunting challenges of the present day validation environment.

We propose a methodology for attaining coverage closure of design validation. The methodology is based on GoldMine, an automatic assertion generation tool that was introduced in [16]. GoldMine uses data mining and formal verification to generate assertions. A Register Transfer Level (RTL) design is simulated and the simulation traces are passed as data to GoldMine. GoldMine uses decision tree based supervised learning algorithms to mine rules from the simulated test data. A decision tree is used to statistically infer rules from the data, such that the leaves of the decision tree correspond to candidate assertions for a given output. These candidate assertions are true of the simulated design inputs, but may not be true over all possible inputs. In order to determine if a candidate assertion is true for all inputs or not, the candidate assertions are passed with the design to a formal verification engine. If the formal verification passes a candidate assertion, it is a system invariant. If not, it generates a counterexample. In [16], we showed the generation of complex and useful propositional (combinational) as well as sequential (temporal) assertions.

In this work, we incorporate feedback from the counterexamples generated by GoldMine to refine the simulation data that was used to generate assertions. The test data refined by counterexamples is now used to run another pass of GoldMine. The counterexamples from assertions that fail formal verification are again fed back into the input test suite. This iterative refinement continues until a pass of GoldMine where all the assertions pass the formal check. The test suite that remains at that point, along with the passing assertions, are the output of our method. We introduce a variation on the original decision tree data structure that is built incrementally with every iteration. An incremental decision tree per output adds information from a counterexample for every failed assertion on its leaf nodes.

Let us now look at how this counterexample guided automatic assertion/test generation process attains coverage closure. Firstly, in GoldMine, we stipulate that only 100 per cent confidence candidate assertions need to be considered for formal verification. Even a single contradicting example in the simulation data is enough to discard a candidate assertion. This ensures that a failing assertion that produced a counterexample is never reproduced by GoldMine in successive iterations. Since every counterexample provides a trace through the system and the addition of new variables, the corresponding input vector tests for as yet uncovered behavior. Every iteration, therefore, increases coverage of the test suite. This results in a monotonic decrease in the design space uncovered by the tests with successive iterations. In stark contrast to random or directed testing, where arbitrary long phases of coverage stagnation can occur, our method always makes forward progress with respect to test coverage. Secondly, the limiting condition for this algorithm to converge is when there are no failing assertions. At this point, for every decision tree corresponding to a design output, all the assertions in the leaf nodes are true. This provides a deterministic metric of progress for test development. Until all the assertions for a given output pass, the test suite can be improved upon. Thirdly, our method also provides an alternative notion of coverage— one that is output-directed. If all the leaves of a decision tree have true assertions, it implies that the (incremental) decision tree now captures the complete functionality of that output. The decision tree that was predicting design behavior by observing dynamic data, has completely captured the output logic function at the convergence point. The design functionality with respect to
that output is completely covered in our method for both combina-
tional and sequential designs. We provide a proof intuition for the
correctness and convergence of our algorithm. Since the decision
tree extracts information from dynamic, simulation data, it gener-
ates only the reachable state of an output. Unlike static analysis, it
is not possible to reach illegal or unreachable states in our method.
At the point of convergence, the input test patterns along with the
GoldMine assertions represent the validation artifacts required for
achieving coverage closure. We consider the validation task com-
plete when the entire functionality of all outputs in the design have
been captured, i.e. all the assertions for the outputs are true.
Our contributions in this paper are as follows.

- We present a counterexample guided iterative refinement ap-
  proach to mine tests using GoldMine, an assertion generation
tool.
- Our method provides a deterministic metric of progress in the
test development process through incremental decision
trees.
- Our method ensures a monotonic increase in the coverage of the
test suite with every iteration. We do not plateau or
  stagnate with respect to test coverage.
- We prove that for a given output, the incremental decision
tree will always converge. We also prove that the complete
  functionality of an output will be captured at the point of
  convergence.
- We also introduce an output-directed notion of coverage. The
tests from our technique will always stimulate only the reach-
able states of the design.

An outline of the paper is as follows. In Section 2, we present the
background information regarding the tool flow of GoldMine. In
Section 3, we describe our counterexample guided iterative refine-
ment approach to mine tests. We detail the creation of incremental
decision trees, the principal artifact in mining the tests. Since the
treatment of combinational tests is different from sequential tests,
we present these discussions separately. We outline a proof that
shows the convergence and completeness of our algorithm in Sec-
tion 4. In Section 5, we argue for coverage closure and forward
progress of coverage using our technique. Section 6 details an
example that runs through the entire algorithm. In Section 7, we
demonstrate experimental evidence of the merit of our approach,
using RTL designs from Rigel, a 1000+ core processor, as well as
standard ITC benchmarks.

2. INTRODUCTION TO GOLDMINE

We provide the background on GoldMine required to explain our
counterexample guided input pattern generation algorithm. Gold-
Mine combines two diverse technologies- data mining and static
analysis, to generate assertions automatically. Data mining com-
prises dynamic, statistical techniques that are computationally effi-
cient, but depend heavily on domain information for deriving rele-
vant knowledge from a system. Static analysis of designs (includ-
ing formal verification) captures domain (design) information, but
suffers from computational complexity issues. Together, these two
technologies offset each other’s disadvantages. The static analysis
when used to guide the data mining, gives rise to useful domain
knowledge, i.e. assertions. 1 shows the architecture of GoldMine.
It is composed of a Data Generator, Static analyzer, A-Miner, For-
mal verifier and A-Val components.

2.1 Data Generator

The data generator phase of GoldMine is used to provide the
data for the data mining algorithm. For a given RTL design, the
data is obtained through dynamic simulation traces. The design is
simulated for a fixed number of cycles (10000) using random input
patterns. Regression tests, if available can also be applied to obtain
traces.

The sequential behavior of a design is usually expressed in the
form of temporal assertions. GoldMine is capable of generating
combinational as well as sequential assertions. We need to provide
a mining window length, or the duration of time cycles for which we
want to capture temporal behavior. For instance, if we want to
consider the following behavior: once a is valid, it will be valid
two cycles later, the mining window length can be set to 2. All
generated assertions will span up to 2 cycles, including a \( \implies \)
XXb, which is the assertion of interest. 1

2.2 Static analyzer

The static analyzer extracts domain-specific information about
the design and passes it to the data mining algorithm. We extract
the logic cone of influence for every output that we are interested in
assertions for. The data mining phase (A-miner) is restricted to
analyzing only the logic cone of any output. This limits the search
space of the mining algorithm from all the inputs, to the relevant
variables in the design.

2.3 Decision tree based learning and A-Miner

The data mining phase of GoldMine is called A-miner, for asser-
tion mining. A-Miner uses a decision tree based supervised learn-
ing algorithm to map the simulation trace data into conclusions or
inferences about the design.

In the decision tree, the data space is locally divided into a se-
quence of recursive splits on the input variables. A decision tree
is composed of internal nodes and terminal leaves. Each decision
node implements a "splitting function" with discrete outcomes la-
beling the branches. This hierarchical decision process that divides
input data space into local regions continues recursively until it
reaches a leaf. We require only Boolean splits at every decision
node, since our domain of interest is digital hardware. The exam-
ple in Figure 2 for an output z shows the simulation trace data for
inputs a, b and c.

An error function picks the best splitting variable by computing
the variance between target output values and the values predicted
by decision variables. The predicted value on each node is the mean
of output values, denoted by \( M \) while the error at a node is denoted
by \( E \) in the example. When the error value become zero, it means
all output values are identical to the predicted value and the deci-
sion tree exits after reaching such a leaf node. When the error value
is not zero, the variable with minimum error value is chosen to form
the next level of decision tree. A candidate assertion is a Boolean

1We use LTL [14] notation for expressing GoldMine assertions.
We can produce SVA [1] as well as PSL assertions.
propositional logic statement computed by following the path from the root to the leaf of the tree. In the example, the splitting of input space into two groups after decision on variable \( a \) leads to \( E = 0 \), corresponding to assertion \( A_1 \). Along the \( a = 1 \) branch, another split occurs on \( b \). Assertions \( A_2 \) and \( A_3 \) are obtained at the leaf nodes.

\[
\text{Design function: } z = (a(\neg c)) \& b
\]

\[
\begin{align*}
\text{Simulation trace: } & a \ b \ c \ z \\
& 0 \ 0 \ 0 \ 0 \\
& 0 \ 1 \ 1 \ 0 \\
& 1 \ 0 \ 1 \ 0 \\
& 1 \ 1 \ 0 \ 1 \\
\end{align*}
\]

\[
\begin{align*}
\text{Input space split: } & b = 0 \\
& \begin{cases}
M: 0.25 & E: 0.38 \\
M: 0 & E: 0 \\
M: 0.50 & E: 0.50
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{ ASSERT1: } & \neg b \Rightarrow z \\
\text{ ASSERT2: } & (a \& \neg b) \Rightarrow z \\
\text{ ASSERT3: } & (a \& b) \Rightarrow z
\end{align*}
\]

\[
\begin{align*}
\text{Design function: } & z = (a(\neg c)) \& b \\
\text{Splitting variable: } & a \\
\end{align*}
\]

\[
\begin{align*}
\text{Simulation trace: } & a \ b \ c \ z \\
& 0 \ 0 \ 0 \ 0 \\
& 0 \ 1 \ 1 \ 0 \\
& 1 \ 0 \ 1 \ 0 \\
& 1 \ 1 \ 0 \ 1 \\
\end{align*}
\]

\[
\begin{align*}
\text{Input space split: } & b = 1 \\
& \begin{cases}
M: 0 & E: 0 \\
M: 1 & E: 0
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{ ASSERT1: } & \neg b \Rightarrow z \\
\text{ ASSERT2: } & (a \& \neg b) \Rightarrow z \\
\end{align*}
\]

\[
\begin{align*}
\text{Design function: } & z = (a(\neg c)) \& b \\
\text{Splitting variable: } & a \\
\end{align*}
\]

\[
\begin{align*}
\text{Simulation trace: } & a \ b \ c \ z \\
& 0 \ 0 \ 0 \ 0 \\
& 0 \ 1 \ 1 \ 0 \\
& 1 \ 0 \ 1 \ 0 \\
& 1 \ 1 \ 0 \ 1 \\
\end{align*}
\]

\[
\begin{align*}
\text{Input space split: } & b = 0 \\
& \begin{cases}
M: 0 & E: 0 \\
M: 0.50 & E: 0.50
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{ ASSERT1: } & \neg b \Rightarrow z \\
\text{ ASSERT2: } & (a \& \neg b) \Rightarrow z \\
\text{ ASSERT3: } & (a \& b) \Rightarrow z
\end{align*}
\]

**Figure 2:** Decision Tree Building Process and Assertion Generation

### 2.4 Formal verifier and A-Val

The candidate assertions inferred by A-miner are based purely on statistical correlation metrics like mean and error. We restrict the candidate assertions we consider to those with 100% confidence. This means that even if a single example in the trace data does not subscribe to a rule generated by the tree, the rule will be discarded. Despite this strict restriction, A-miner may still infer candidate assertions that are true of the simulation data, but are not true of all possible inputs. To identify candidate assertions that are system-invariants, the design as well as the candidate assertions are passed to a formal verification engine. If a candidate assertion passes the formal check, it is a system invariant. Otherwise, the formal verifier generates a counterexample trace that shows a violation of the candidate assertion. The SMV [12] model checking engine is a part of GoldMine, alongside a commercial model checker. In the example in Figure 2, \( A_1 \) is declared false, while \( A_2 \) and \( A_3 \) are declared true. In GoldMine, A-val forms the evaluation phase for the assertion, to bridge the gap between the human and machine generated assertions.

GoldMine provides a radical, but powerful validation method. Through mining the simulation trace, it reports its findings in a human-digestible form (assertion) early on and with minimal manual effort. However, in GoldMine, there is no concept of feedback from any phase to the data miner. Given that data mining performs very effectively when given feedback, in this work we have incorporated feedback from the formal verification phase for enhancing the simulation test data.

### 3. COUNTEREXAMPLE-BASED INCREMENTAL DECISION TREES

The decision tree is a structure that captures the design model from the perspective of observable behavior. An assertion can be false due to two reasons—either some behavior has not been observed by the decision tree due to insufficient data, or some inference has been made erroneously due to selecting a correlated, but not causal splitting variable. A counterexample trace exposes both these situations by introducing scenarios that involve at least one new variable. If this new scenario is now included in the input pattern data observed by the decision tree, firstly it prevents the generation of the same spurious assertion. Secondly, it guides the decision tree to navigate regions of input space that have not been considered/observed so far. A beneficial side effect of this process is the increase in coverage of the input simulation data steadily with every iteration.

In order to disprove an assertion, the new data instance consists of all antecedent variables of the assertion and some new additional variables. The antecedent variables’ values are also identical to that in the false assertion and the implied variable’s value is different from that in the false assertion. This characteristic of a counterexample enables a natural way to add it as new data instance to incrementally build a decision tree instead of rebuilding a decision tree from scratch every iteration.

In order to keep track of the improvisation of the decision tree for a given output, we devised an incremental version of the decision tree. The iterative algorithm using GoldMine (depicted in Figure 3) incrementally builds a decision tree for an output until it reaches the goal of generating only true assertions (no counterexamples). The full set of correct assertions, plus the new test patterns created from counterexamples during iterations comprise the tangible outputs of the algorithm.

In the recursive incremental decision tree algorithm described in Figure 4, the parts different from GoldMine (lines 4, 7, 8) are outlined. Figure 5 shows the a regular decision tree and an incremental version of it.

A decision tree corresponds to a design output. The formal verification in line 4 is employed to check the correctness of assertion whenever a leaf node is reached during the incremental building of decision tree. If a candidate assertion is true on design, the algorithm returns as in the regular decision tree. In the example, assertions \( A_1 \) and \( A_2 \) generated from original simulation traces are true on the design. If the checked assertion is false/spurious, a counterexample is reported by formal verification. A counterexample: \( a = 0, b = 1, c = 0 \) and \( z = 1 \) is generated to contradict the assertion \( A_0 \) on the decision tree on the left.

The \( \text{Ctx_simulation}() \) function simulates the input pattern created by the counterexample. This lends concrete values to all the splitting variables in previous iterations of the decision tree in the new simulation run.

Since the counterexample follows the same path as the failed assertion, the decision tree continues splitting when it reaches the leaf node corresponding to that false assertion. All other paths of the decision tree are kept unchanged. Due to the new data instance, the mean and error values for each node need to be recomputed using the \( \text{Recompute_error}() \) function. The error value of the leaf node
1. Incr_Decision_Tree_Building(TreeNode Node)
2. begin
3. if(Error(Node)==0) begin
4. [If(Formal_verif(Node.assertion))==true]
5. return;
6. else begin
7. Ctx_simulation();
8. Recompute_error(Node);
9. end
10. end
11. Node.left=New_node();
12. Node.right=New_node();
13. Select_splitting_variable();
14. Incr_Decision_Tree_Building(Node.left);
15. Incr_Decision_Tree_Building(Node.right);
16. end

Figure 4: Incremental Decision Tree Building Algorithm. The dotted lines represent parts that are different from GoldMine’s decision tree building approach.

will no longer be equal to zero. In the example, the incremental decision tree continues to split on the leaf node corresponding to false assertion A0 in the regular decision tree. It can also be observed that the mean and error value are recomputed in this iteration on the path from the root to the leaf. The algorithm exits when all the assertions at the leaf nodes of an incremental tree are true.

3.1 Stimulus Generation for Sequential Behavior

During the building of a decision tree, the design should be unrolled until the mining window length, as defined in Section 2.1. The simulation trace used for assertion mining may have internal register state visible. It may be desirable to have assertions form a single-cycle flat picture of the design, where assertions on the outputs are functions of internal state values and primary inputs. Assertions can also be formed for the internal state variables themselves, as functions of other state registers and inputs. Such a view of the design gives a “next cycle” model, where the assertions describe internal registers and primary outputs in a similar manner. On the other hand, it may be desirable to have temporal assertions on the design that capture only input-output behavior over some number of cycles.

We can generate assertions of both types with this algorithm, based on the mining window length and visible state provided. Although the assertion span sequential behavior over a given length, the generated counterexample may be longer than the mining window length. This may be to expose sequential behavior where an intermediate state variable can be driven to a specific value over several cycles starting from the primary input. In this case, the incremental decision tree algorithm considers only the variables until the farthest back temporal stage, i.e. unrolled until the mining window length. The concrete values of these variables can be acquired through simulation of the counterexample by the data generator. The result is a temporal assertion that spans the mining window length, bolstered by single-cycle assertions using internal state registers to describe the behavior. We discuss an example of sequential logic coverage in Section 6.

3.2 Final Decision Tree and Unreachable States

Our counterexample based incremental decision tree building algorithm is a process of approximation and refinement of an output function. If the complete functionality of an output was available to the decision tree in the form of simulation data, it would completely represent the output function. Such a truth table (or state transition relation for sequential designs) would result in a complete decision tree. However, such an exhaustive enumeration of input patterns is not feasible to obtain as test data. Therefore, the decision tree tries to approximately predict the logic function of an output with available data. Faulty predictions are exposed and used for corrective purposes through counterexamples. This makes future predictions more accurate. At the point where all the predictions are accurate is where all the assertions of the decision tree are true. At this point of convergence, this final decision tree represents the complete functionality of an output in the design. The input patterns required to generate such a final decision tree are sufficient for completely covering the functionality of that output.

It is important to note that final decision trees include only the legal, reachable states of the design. This is a subset of the state space that is obtained by statically enumerating input combinational or sequential patterns. Static traversal of states does not account for illegal inputs or dynamically unreachable state. However, since the decision trees are constructed out of dynamic simulation data, it only observes the behavior that is executable, thereby eliminating unreachable states. For sequential logic, the algorithm captures the behavior in the assertions for a given length. The constraints on register variables from previous cycles are also captured by the decision tree. Although the assertions are captured for only a bounded number of cycles, the formal verification ensures that the temporal assertions will exclude unreachable or illegal states in the design. This means that the input test patterns that generate a final decision tree comprise exactly the necessary stimulus to capture the output logic. There are no superfluous patterns that reach illegal state in our methodology.

When all the assertions generated from the decision tree are true, either all expressions for an output are completely covered or the
uncovered logic in the design will be redundant logic.

4. ALGORITHM COMPLETENESS AND CONVERGENCE ANALYSIS

In this section, we prove that our counterexample based test generation algorithm converges and at the point of convergence for any output, the corresponding decision tree for that output represents the complete functionality of that output.

We present some definitions that are required for proving the mentioned claims. Let us consider an RTL design whose state transition graph (Kripke structure [12]) model is depicted by $M$. We will use $M$ synonymously for the design as well its model. Let there be $N$ inputs in $M$. An input pattern is a unique assignment of values to inputs of $M$. Input patterns can be combinational (single cycle) or sequential (across multiple cycles). An input pattern set is a set of all such input patterns in use for a design validation effort.

The input pattern set for $M$ forms the data for the decision tree algorithm. We define decision trees as used in our context.

**Definition 1.** A decision tree $D^*$ for an output $z$ is a binary tree where each node corresponds to a unique splitting variable that is statistically correlated to $z$. A path for a decision tree is a sequence of nodes from the root node to a leaf node.

In general, decision trees need not be binary trees, but since our variables are in the Boolean domain, there are only two possible values of each (one-bit) variable. A decision tree is a data structure used in predictive modeling to map observations about a variable of interest to inferences about the variable’s target value. In our case, every output of $M$ is a variable of interest. Every output has a corresponding decision tree that makes inferences about the output’s target value (true and false). These inferences are made at the leaves of the decision tree, where the branches leading from the root to the leaf represent conjunctions of splitting (correlated) variables. These inferences are also considered likely or candidate assertions for the concerned output.

**Definition 2.** A candidate assertion $A_C$ of $D^*$ is a Boolean conjunction of propositions (variable, value pairs) along a path in $D^*$.

In the next phase of GoldMine, model checking [12] is used to compute the truth or falsehood of a candidate assertion. In case a candidate assertion is false, a counterexample or simulation trace through the design is generated, that exemplifies the violation of the assertion.

**Definition 3.** A true assertion $A_T$ is a candidate assertion such that $M \models A_T$.

**Definition 4.** The support of a Boolean conjunction $y$, which is denoted as support$(y)$, is the set of variables in $y$.

**Definition 5.** If $M \not\models A_C$, the conjunction of variable value pairs in the counterexample is represented by $\chi_{A_C}$ such that support$(\chi_{A_C}) \supset$ support$(A_C)$.

Since the counterexample represents a valid simulation trace through the design that is not yet a part of the current input pattern set, it is added to the input pattern set. An incremental version of the decision tree is used in order to keep track of the coverage. The incremental decision tree maintains the ordering of variables as the decision tree from a previous iteration for all the variables until the leaf nodes. If the counterexample in the current iteration coincides with a path in the incremental decision tree, the variable(s) added by the counterexample will now be used as the splitting variable(s) at the leaf nodes of the incremental decision tree.

**Definition 6.** An incremental decision tree $I^*$ for an output $z$ and a previous decision tree $D^*$, is a decision tree such that the variable ordering of all variables in $D^*$ is preserved until a leaf node. Every variable $v$ in support$(\chi_{A_C})$ support$(A_C)$ becomes a splitting variable at the leaf node of $I^*$ along the path of $A_C$.

**Definition 7.** The final decision tree $F^*$ is an incremental decision tree such that for all assertions $A_C$ of $F^*$, $M \models A_C$.

**Definition 8.** The logic cone of an output $z$ in $M$ is the set of variables that affect $z$.

The logic cone is deciphered by computing the transitive closure of all variables pertaining to an output. In GoldMine, we do a logic cone analysis for every output. The decision tree for an output is therefore restricted to the variables in its logic cone, or the relevant variables with respect to that output.

**Theorem 1.** It takes finite iterations to reach $F^*$ for any given $I^*$.

**Proof:** Let us run the incremental algorithm for $k$ iterations, then the minimum number of new nodes added to $I^*$ is $2k$. The minimum total number of nodes in $I^*$ after $k$ iterations is $2k + 1$. Let $n \subseteq N$ be the number of variables in the logic cone of $z$. The maximum size for $D^*$ by construction and by definition of binary trees is $2^{n+1} - 1$. Therefore, $2k + 1 \leq 2^{n+1} - 1$. This bounds the size of the incremental decision tree.

It may be noted that since we are restricting the decision tree for an output to focus only on the relevant variables, the maximum size of the decision tree is not exponential in the size of the entire set of inputs $N$, but in $n$. In practice, we observe that $n << N$.

**Theorem 2.** The final decision tree $F^*$ corresponds to the entire functionality of $z$.

**Proof:** Assuming a final decision tree $F^*$ does not correspond to the entire functionality of $z$, then there is at least one input pattern to reach a state of $z$ that does not correspond to a path in $F^*$, so at least one $A_C$ of $F^*$ should be such that $M \not\models A_C$. But this is false by definition of $F^*$. Therefore, the assumption is contradicted.

The above theorem makes a powerful statement about the coverage of our method. When all the assertions are true, the complete functionality of an output is captured.

In practice, the learning-based data mining algorithm is able to generate compact assertions, each of which represents several satisfiable input patterns for the corresponding output. The incremental decision tree algorithm can converge quickly to cover all the logic function of corresponding output.

5. COVERAGE ANALYSIS

In the simplest terms, what we want from a coverage effort is expose the entire legal, reachable design behavioral space to examination so that this space can be validated against a statement of desired behavior. We posit that our algorithm using GoldMine
and iterative refinement of the decision tree achieves exactly that property: when the final decision tree for an output has been constructed, the entire reachable design space for that output is captured by the tree. The combination of input patterns and assertions generated by the tree are artifacts that represent the complete functionality of that output. Our notion of coverage, then is output space directed, as opposed to traditional input space directed notions of coverage. With respect to this notion of coverage, we can achieve functional coverage closure with respect to every output in the design.

Our test generation strategy automatically computes and explores only the reachable state space since it is dynamically derived from simulation data. This is distinct from traditional functional coverage notions that are input-space directed, like expression coverage or conditional coverage. These are not constrained by reachable state space or legal states. So, frequently, we can achieve complete coverage in our methodology, but not complete expression coverage.

GoldMine's counterexample based approach for test generation ensures a monotonic decrease of the uncovered design space with each iterative refinement. In each iteration, the generated counterexample is able to cover a new design function which has not been covered before by previous patterns. The newly activated function can be in the form of conditional expression, branch or assignment statements in the RTL design. Moreover, the existence of a final decision tree as a goal provides a deterministic metric of progress through the refinement process. This is a significant improvement over random testing, whose coverage graph can be arbitrarily shaped, often resulting in plateaus where no progress is being made. In fact, due to the frequent lack of feedback in the random test generation process, it is difficult to acquire a satisfactory functional coverage picture in this process.

6. EXAMPLE: TWO PORT ARBITER

In this section, we will demonstrate GoldMine's incremental counterexample refinement using a 2 port arbiter. This arbiter uses round robin logic with priority on port 0. In our example, we will unroll the circuit 1 cycle in GoldMine to capture some temporal properties of the of the port 0 access signal, gnt0. The simulation data below represents a directed test that a validation engineer might write. We will show how the A-Miner makes inferences about the design and is aided by the counterexample refinement to improve assertion and directed test quality.

![Figure 7: Arbiter: RTL and simulation trace](image)

The goal of the A-Miner is to partition our simulation data, also known as our example set, into subsets that all display some common behavior based on statistics. Our decision tree data structure starts with a root node which contains all examples and the examples are partitioned into likely behavior by the time they reach the leaf nodes. The initial structure of the decision tree is represented below.

![Figure 8: Initial Decision Tree](image)
is because there is a state outside of the window that affects the output, $gnt0$. At this point, we allow the A-Miner to search the registers and primary outputs in the farthest back temporal state for a suitable split. In our example, we add the signal $gnt0(t-1)$ to the search. The A-Miner makes this split and produces the full tree below and A11 and A12 are newly generated true assertions.

Figure 9: First Iteration: Counterexamples and Refined Tree

A0: $\neg req0 \Rightarrow X^gnt0$
A1: $req0 \Rightarrow X^gnt0$

The two candidate assertions generated above are proven false by formal verification. A counterexample is produced for each failed assertion containing the series of states that will contradict this assertion. We simulate these counterexamples and add the results to our example set as shown below. The decision tree continues to grow since the error is greater than 0 for each node. This means that the confidence is no longer 100% for A0 and A1. The A-Miner finds four more counter examples based on the new data.

A2: $\neg req0 \wedge (X \neg req0) \Rightarrow X^gnt0$
A3: $\neg req0 \wedge (X req0) \Rightarrow X^gnt0$
A4: $req0 \wedge (X \neg req1) \Rightarrow X^gnt0$
A5: $req0 \wedge (X req1) \Rightarrow X^gnt0$

After one iteration, A2 and A3 are verified to be true. However, A4 and A5 both fail formal verification and a counterexample is produced for each. We again simulate the counterexamples and add them to our data set. The refined tree is shown below.

7. EXPERIMENTAL RESULTS

To evaluate the quality of our method, we implement the incremental decision tree building algorithm and generate validation stimulus and assertions for several design modules. These include some simple synthetic blocks we created to test various features, and some blocks from the Rigel RTL design [11] and the ITC Benchmark Suite of designs. The simple blocks include a small combinational example block (cex_small), a 2-input arbiter (arbiter2), and a 4-input arbiter with more internal state (arbiter4). Specific signals are sometimes indicated in the results or figures, such as $arbiter2.gnt0$ for the output signal $gnt0$ of the arbiters2 module. From the Rigel design three key modules in the processor are chosen: Instruction Fetch (fetch), Instruction Decode (decode), and Instruction Writeback (wbstage). These modules are used for the following experiments:

1. Plot expression coverage of simple modules as GoldMine test generation proceeds
2. Limit studies of the counterexample method
   (a) Zero-pattern seed, start with no test patterns and iterate
   (b) Full-coverage seed, start with patterns that provide
3. Bug finding, inject random faults and use the previously derived assertions as a regression suite to find the bugs

4. Compare and contrast to standard coverage metrics

The runtime for this algorithm is proportional to number of Gold-Mine tests generated. The size of the design, number of initial samples, and maximum number of iterations all affect the number of counterexamples. A more complex design will require more counterexamples to be generated because in the worst case, the assertion will only be refined by one variable per iteration. The number of initial samples affects how complete the decision tree is before counterexamples are added because a larger number of examples gives a better indication of the design and therefore a more accurate decision tree. As the completeness of the initial tree increases, the number of counterexamples decreases because fewer candidate assertions will need to be refined. The maximum counterexample depth determines how many iterations before the decision tree stops trying to refine an assertion. As the counterexample depth decreases, the runtime decreases as does the accuracy of the decision tree. Experimental results show the average time per formal verification of an assertion to be 1.5 seconds. If a counterexample is produced, the time to produce that counterexample is an additional 1.5 seconds. Most tests completed within 24 hours on an Intel Core 2 Quad Q6600 with 4GB of memory. Memory usage is proportional to the number of examples, which increases as the number of counterexamples increases. All tests used well below the 4GB of the machine.

Our implementation for this algorithm is a very naïve approach and there are many potential performance optimizations. Every time a candidate assertion is produced, that assertion is formally verified. This means that the formal verifier must compile the full design every time a candidate assertion is produced. An improved approach would collect all candidate assertions and formally verify all assertions at once, drastically decreasing the time spent during formal verification. Counterexamples are also produced at the same time that an assertion is run through formal verification. Combining the counterexamples into one large test bench could drastically cut down on simulation time as well. In addition, only the current node in the decision tree and its descendants can benefit from a counterexample. Using the batched approach, all nodes with failing assertions would benefit from all counterexamples produced, rather than just their own counterexample. Given these enhancements, it would be very reasonable to expect at least a 100% speed increase.

7.1 Coverage Increase

The first experiment demonstrates the increase in expression coverage as the counterexample algorithm progresses, showing a monotonic increase in coverage. We have summarized these results in Figure 13 and Figure 14. Expression coverage was chosen as a representative example of an industry standard metric, though as explained in Section 5 this metric is often unable to achieve 100% coverage on its own. Redundant statements, unreachable states, and other RTL characteristics often limit expression coverage effectiveness. A steady increase in such coverage is however an indicator of progress in the quality of the assertions and tests created by this algorithm.

This experiment was performed on several typical circuit designs. The first step of the test is to simulate the original test suite. The original test suite can be in the form of a directed test or a completely random input stimulus test. Any spurious assertions are refined using counterexamples until the A-Miner has generated a true assertion or until the counterexample exceeds the length of the unrolled circuit. We can easily calculate the input space covered by an assertion as \( \frac{1}{2^{\text{depth of node}}} \). We accumulate the coverage of all system invariants to determine the input space coverage of our set of assertions. This experiment can be split into several groups.

- Combinational, directed test: cex_small
- Combinational, random stimulus test: wb_stage
- Sequential, directed test: arbiter2, arbiter4
- Sequential, random stimulus test: fetch_stage

The coverage of the true assertions is evaluated by considering the percentage of the truth table that is covered by that assertion. We consider the possible input space as \( 2^{(\text{number of inputs} \times \text{length of assertion})} \).

We can consider the inputs specified in the assertion as concrete inputs and the rest as don’t care inputs. This means that the number of input combinations covered by an assertions is equal to \( 2^{(\text{number of don’t care inputs})} \).

In our decision tree, the number of concrete inputs in an assertion is equal to the depth of the node containing it. Based on this information, we can see that the input space covered by a potential assertion is cut in half every time the depth increases. This shows that the input space covered is \( \frac{1}{2^{\text{depth}}} \).

The results show a consistent increase in the input space covered by the assertions in each iteration. For the wb_stage and cex_small modules, we note that incremental refinement converges to 100% input space covered. We also note that since the cex_small module is a simple design and the wb_stage module is a complex design, there is a direct correlation between the complexity of the module and the number of counterexample iterations required to converge.

We also notice that there is an exponential increase in the input space covered in the early iterations but only a logarithmic increase in input space covered in the later iterations. This shows that even if incremental refinement is only applied up to a certain depth, there will still be a relatively large coverage gain.
7.2 Zero Initial Patterns

The second experiment is a limit study showing that the counterexample algorithm works even when no directed tests exist. The lack of any patterns would begin the procedure with a simple assertion of the form “output always 0”, for example, which the formal verification would show false and provide a counterexample, which would be the first functional pattern. Table 1 shows the increase in coverage for each tested design as the algorithm progresses. Even with no initial test patterns, the counterexample method is able to create a test suite that achieves good coverage with few iterations. This indicates that counterexamples may be a useful methodology to jump start a module design environment by creating many tests that can then be run on the testbench model checkers.

Table 1: Coverage Percentage by Iteration Starting From Zero Patterns

<table>
<thead>
<tr>
<th>Iterations</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>12</th>
<th>15</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>arbiter2.gnt0</td>
<td>0%</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>arbiter4.gnt0</td>
<td>0%</td>
<td>51.25</td>
<td>69.53</td>
<td>97.29</td>
<td>99.97</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>fetchstage.valid</td>
<td>100</td>
<td>25</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

7.3 Complete Coverage Initial Patterns

The third experiment explores GoldMine test development on a module that already has full coverage by at least some of the common coverage metrics. The goal is to see if GoldMine tests can find any of the uncovered state in the design by finding counterexamples. If a block already has full coverage on some metrics, and very high coverage on others, it is often difficult to get to higher coverage or to know if higher coverage is even possible. We evaluated such a condition and were able to derive counterexample tests that did indeed improve expression coverage that was already quite high. Figure 15 shows that a block with 100% line and branch coverage, and high condition coverage, achieved higher condition coverage after GoldMine test generation.

![Figure 15: Increasing Coverage on High Coverage Block](image)

7.4 Fault Detection by Assertions

The fourth experiment is an example of using the provided assertions in a regression testing environment by injecting and finding bugs in a design that has previously had assertions built on the correct version. We implement a systematic mutation-based method to test the assertions’ ability to detect bugs. The internal design signal is selected to mutate and all generated assertions are then formally checked on the mutated design model. The failed assertions are considered able to cover the corresponding bug on the mutated design. Since we do not have actual block-level testbench code with monitors and checkers, we have used assertions as the regression vehicle in this experiment, but since the generated test vector suite also has very high coverage it would also be an effective regression suite.

Table 2 shows the results for several randomly chosen signals in the Rigel RTL modules. For a randomly chosen signal in a design, the figure shows the number of assertions that detected the fault. In each case, the assertion suite is able to detect the faults.

<table>
<thead>
<tr>
<th>Signal</th>
<th>stuck at 0</th>
<th>stuck at 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>stall_in</td>
<td>269</td>
<td>94</td>
</tr>
<tr>
<td>branch_pc</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>branch_mispredict</td>
<td>8</td>
<td>66</td>
</tr>
<tr>
<td>tcache_rdlv1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2: Faults Covered by Assertions

7.5 Comparison to Standard Coverages

In the second experiment, we show the output of several standard coverage analyses comparing standard directed and random tests with tests generated by the counterexample algorithm. Final coverage values for both Rigel designs and ITC Benchmark designs are included, showing the coverage achieved by the various methods.

![Figure 16: Coverage Comparison Between Random Tests and GoldMine Tests](image)

Table 3 and Figure 16 show comparisons to the GoldMine generated test method and both directed and random test pattern methods, applied to different designs. The Cycles column of each table lists the number of simulation cycles run to achieve the given test coverage for the Directed and Random examples, and the number of final test pattern cycles created for the Counterexamples section. From these tables we see that some of the coverage metrics remain low even after 1.5M cycles. In all cases, the final coverage results for tests created by the counterexample method are very high.

8. RELATED WORK

We discuss work that is related to different aspects of our approach. Our counterexample based stimulus generation approach distinguishes itself from all the existing coverage guided test generation approaches in that the generated counterexample is able to automatically explore logic not covered by previous stimulus. Counterexample- based refinement of abstractions for verification has been studied widely [4]. The idea of generating tests from counterexamples using model checking has been explored in software testing and hardware validation [2] [3] [7]. These methods require a predefined set of properties and then formally verify these properties. In our work, the set of properties are generated automatically, minimizing human intervention in the loop. Many techniques in prior art automatically generate validation patterns by incorporating coverage feedback [6] [13] dynamically. However, they do not use a flow similar to GoldMine for generating feedback.

Statistical methods have been adopted in hardware validation for assertion generation [9] [10] [15] and test generation [17] [8] [3]. IODINE [9] tries to automatically infer likely invariants by hypothesizing a set of predefined invariant pattern across one or more variables in the design and then analyzing the design’s dynamic behavior during simulation. The generated assertions need not be sound,
as well as they are usually simple assertions like one-hot encoding. In the field of data mining, incremental decision tree algorithms like VFDT (Very Fast Decision Trees) [5] are explored to allow an existing tree to be updated or revised using new data instances. These are typically applied to handling stream data whose characteristics change over time.

9. CONCLUSIONS

In conclusion, we have presented a completely automated stimulus generation methodology for systematic coverage closure based on GoldMine. The forward progress and termination properties of the algorithm make it a sound and practically attractive solution.

Our methodology starts from uncovered design space and covers it systematically. With every iteration, a newly uncovered region is converted to a covered region. This is different from covering all known state space and inching forwards toward the uncovered space.

Although we achieve coverage closure within an implementation, this does not imply adherence to a higher level specification or design intent. We believe that the enhanced test suite output from our methodology can be applied in a validation environment that includes traditional monitors and checkers. We also believe that together, the GoldMine assertions and test vectors have significant value to a design validation effort. The key contribution comes when a completed Final Decision Tree has been constructed. The existence of this structure itself implies a fully explored and validated design.

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10. REFERENCES