EE 350 Class Notes

by

Nannapaneni Narayana Rao

This particular set of class notes was provided by Tony Zuccarino, a student in my EE 350 class in Summer 1983, 25 years ago, and presently in California. According to him, in an e-mail dated December 3, 2008: "The only notes I seem to have kept ... are those from your EE 350 summer of 1983!"

Thank you, Tony, for your kindness.

Narayana Rao
December 18, 2008
Urbana, IL

EE 350 CLASS NOTES

by

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EE 350
(NNR)

Summer 1983

Date	Day	Time	Lect. No.	Topic(s)	Problems
6/13	M	8		First Class	
6/13	M	1	1	Review and Introduction	

I. TIME DOMAIN ANALYSIS (8 lectures)

Due on day on dot

Date	Day	Time	Lect. No.	Topic(s)	Problems
6/14	Tu	8	2	General solution; semi-infinite line	①
6/15	W	8	3	Line terminated by resistance	(4b,c)
6/16	Th	8	4	Bounce diagram technique	⑤
6/17	F	8	5	Bounce diagram; Line discontinuity	⑦
6/20	M	8	6	Three lines in cascade (11,13)	
6/20	M	1	7	Inductive and nonlinear terminations	
6/21	Tu	8	8	Lines with initial conditions	⑮ ⑰
6/22	W	8	9	Lines with initial conditions (cont'd)	⑲
6/23	Th	8		REVIEW ㉓	
6/24	F	8		EXAM I	

II. FREQUENCY DOMAIN ANALYSIS (8 lectures)

Date	Day	Time	Lect. No.	Topic(s)	Problems
6/27	M	8	10	General solution; semi-infinite line	
6/27	M	1	11	Short-circuited line	
6/28	Tu	8	12	Natural oscillations ⑫	
6/29	W	8	13	Input impedance and resonance (3,4)	
6/30	Th	8	14	Line terminated by complex load 8 —	????
7/1	F	8	15	Line impedance and power flow 10 —	
7/5	Tu	8	16	Lines in cascade; λ/4 transformer 11 —	
7/6	W	8	17	Stub matching 12, 14a	
7/7	Th	8		REVIEW 17	
7/8	F	8		EXAM II	

Date	Day	Time	Lect. No.	Topic(s)	Problems
				III. SMITH CHART (6 lectures)	
7/11	M	8	18	Development	
7/11	M	1	19	Introductory applications	
7/12	Tu	8	20	Single stub matching	1,2
7/13	W	8	21	Double stub matching	4
7/14	Th	8	22	Matching with fixed elements	6
7/15	F	8	23	$\lambda/4$ transformer; Bandwidth	8
				IV. WAVEGUIDES (4 lectures)	
7/18	M	8	24	TM and TE waves	10
7/18	M	1	25	Characteristics of TM and TE waves	
7/19	Tu	8	26	Transmission line analogy	2
7/20	W	8	27	Dispersion and group velocity	5,7
7/21	Th	8		REVIEW	8
7/22	F	8		EXAM III	
				V. LOSSY LINES (4 lectures)	
7/25	M	8	28	Lossy line; standing waves	13
7/25	M	1	29	Determination of $\overline{Z}_0$ and $\overline{\gamma}$	32,33
7/26	Tu	8	30	Smith chart and matching	[illegible]
7/27	W	8	31	Matching; Distortionless line	34
				VI. ANTENNAS (5 lectures)	
7/28	Th	8	32	Hertzian dipole	
7/29	F	8	33	Radiation resistance; Directivity	
8/1	M	8	34	Half-wave dipole	
8/1	M	1	35	Antenna arrays	all the rest 38, 39, 40
8/2	Tu	8	36	Image antennas	
8/3	W	8		REVIEW	
8/5	F	3-5		FINAL EXAM	

REVIEW AND INTRODUCTION

MAXWELL'S EQUATIONS

Integral form:	Differential form
$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
$\oint \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s} + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{s}$	$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$
$\oint_S \vec{B}\cdot d\vec{s} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$
$\oint_S \vec{D}\cdot d\vec{s} = \int_V \rho\, dv$	$\vec{\nabla}\cdot\vec{D} = \rho$

Law of Conservation of Charge

$$\oint_S \vec{J}\cdot d\vec{s} + \frac{d}{dt}\int_V \rho\, dv = 0 \qquad \vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

Constitutive Relations

$\vec{D} = \epsilon\vec{E}$	Dielectrics
$\vec{H} = \vec{B}/\mu$	Magnetic materials
$\vec{J} = \vec{J}_c = \sigma\vec{E}$	Conductors

Boundary Conditions

On Perfect Conductor Surface

$$\vec{i}_n \times \vec{E} = 0 \qquad E_t = 0$$

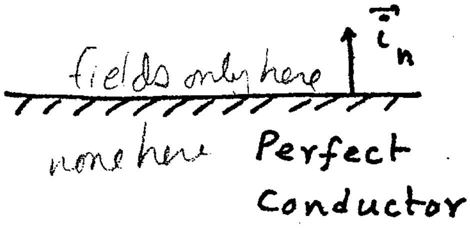
only normal comp ≠ 0

$$\vec{i}_n \times \vec{H} = \vec{J}_s \qquad H_t = J_s$$

$$\vec{i}_n \bullet \vec{D} = \rho_s \qquad D_n = \rho_s$$

dot

$$\vec{i}_n \bullet \vec{B} = 0 \qquad B_n = 0$$



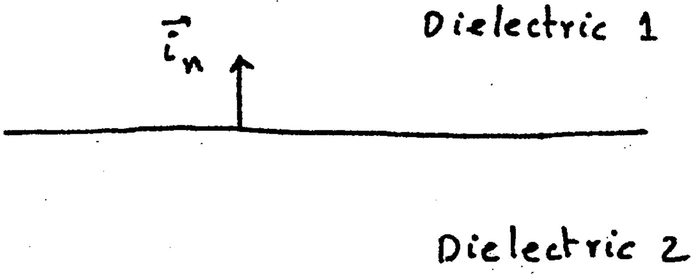
At a Dielectric Interface

$$\vec{i}_n \times (\vec{E}_1 - \vec{E}_2) = 0 \qquad E_{t1} = E_{t2}$$

$$\vec{i}_n \times (\vec{H}_1 - \vec{H}_2) = 0 \qquad H_{t1} = H_{t2}$$

$$\vec{i}_n \cdot (\vec{D}_1 - \vec{D}_2) = 0 \qquad D_{n1} = D_{n2}$$

$$\vec{i}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0 \qquad B_{n1} = B_{n2}$$



Uniform Plane Wave

$\vec{E} = E_x(z,t)\,\vec{i}_x$ $\qquad\qquad$ $\vec{H} = H_y(z,t)\,\vec{i}_y$

$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ $\qquad\qquad$ $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \dfrac{\partial \vec{D}}{\partial t}$

Assume $\sigma = 0$ (lossless medium)

$$\begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\frac{\partial \vec{B}}{\partial t} \qquad\qquad \begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix} = \frac{\partial \vec{D}}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} = -\mu \frac{\partial H_y}{\partial t} \qquad\qquad \frac{\partial H_y}{\partial z} = -\frac{\partial D_x}{\partial t} = -\epsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu \frac{\partial^2 H_y}{\partial z \partial t} = -\mu \frac{\partial}{\partial t}\left(-\epsilon \frac{\partial E_x}{\partial t}\right) = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

Wave Equation

$$E_x(z,t) = A\,f(t - z/v_p) + B\,g(t + z/v_p)$$

$$H_y(z,t) = \frac{1}{\eta}\Big[\underbrace{A\,f(t - z/v_p)}_{(+)\ \text{wave}} - \underbrace{B\,g(t + z/v_p)}_{(-)\ \text{wave}}\Big]$$

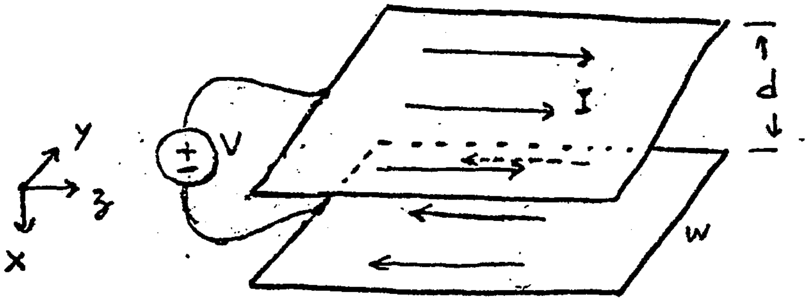
$v_p = \dfrac{1}{\sqrt{\mu\epsilon}}$ = Velocity of propagation

$\eta = \sqrt{\dfrac{\mu}{\epsilon}}$ = Intrinsic impedance

Parallel-Plate or Parallel-Strip Line

$$E_x = \frac{V}{d}$$

$$H_y = \frac{I}{w}$$



$$\frac{\partial}{\partial z}\left(\frac{V}{d}\right) = -\mu\frac{\partial}{\partial t}\left(\frac{I}{w}\right) \qquad \frac{\partial}{\partial z}\left(\frac{I}{w}\right) = -\epsilon\frac{\partial}{\partial t}\left(\frac{V}{d}\right)$$

$$\frac{\partial V}{\partial z} = -\underset{\downarrow}{\left(\frac{\mu d}{w}\right)}\frac{\partial I}{\partial t} \qquad \frac{\partial I}{\partial z} = -\underset{\downarrow}{\left(\frac{\epsilon w}{d}\right)}\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial z} = -\underset{\substack{\text{Inductance} \\ \text{unit length}}}{\mathcal{L}}\frac{\partial I}{\partial t} \qquad \frac{\partial I}{\partial z} = -\underset{\substack{\text{Capacitance} \\ \text{unit length}}}{\mathcal{C}}\frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial z^2} = -\mathcal{L}\frac{\partial^2 I}{\partial z\,\partial t} = -\mathcal{L}\frac{\partial}{\partial t}\left(-\mathcal{C}\frac{\partial V}{\partial t}\right) = \mathcal{L}\mathcal{C}\frac{\partial^2 V}{\partial t^2}$$

$$V(z,t) = A\,f(t - z/v_p) + B\,g(t + z/v_p)$$

$$I(z,t) = \frac{1}{Z_0}\Big[\underbrace{A\,f(t - z/v_p)}_{(+)\text{ wave}} \underbrace{- B\,g(t + z/v_p)}_{(-)\text{ wave}}\Big]$$

$$v_p = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{1}{\sqrt{\mu\epsilon}} = \text{Velocity of propagation}$$

$$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} : \text{Characteristic impedance}$$

Analogy:

$$E_x \longleftrightarrow V \qquad \epsilon \longleftrightarrow \mathcal{C} \qquad v_p \longleftrightarrow v_p$$

$$H_y \longleftrightarrow I \qquad \mu \longleftrightarrow \mathcal{L} \qquad \eta \longleftrightarrow Z_0$$

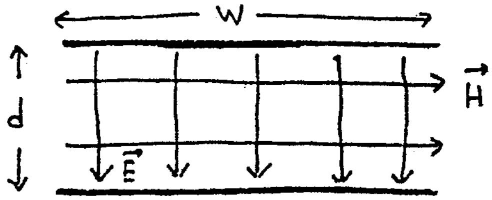
Line Parameters

Parallel-strip Line

$$\mathcal{L} = \mu \frac{d}{w}$$

$$\mathcal{C} = \epsilon \frac{w}{d} \qquad d \ll w$$

$$Z_0 = \eta \frac{d}{w}$$

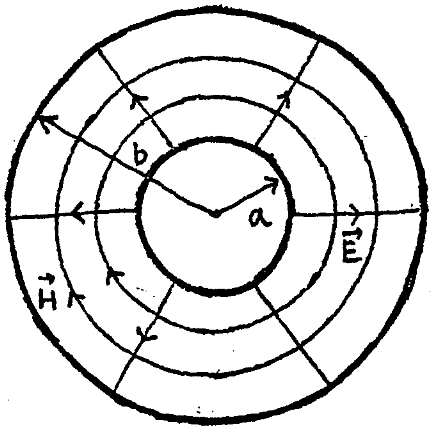


Coaxial Cable

$$\mathcal{L} = \frac{\mu}{2\pi} \ln \frac{b}{a}$$

$$\mathcal{C} = \frac{2\pi\epsilon}{\ln \frac{b}{a}}$$

$$Z_0 = \frac{\eta}{2\pi} \ln \frac{b}{a}$$

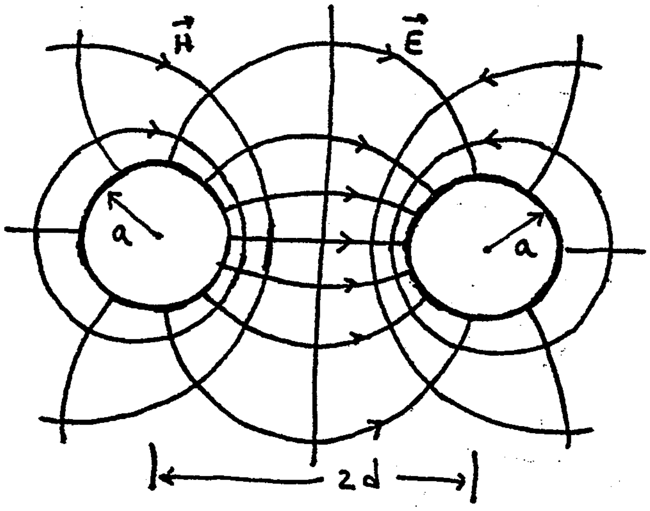


Parallel-Wire Line

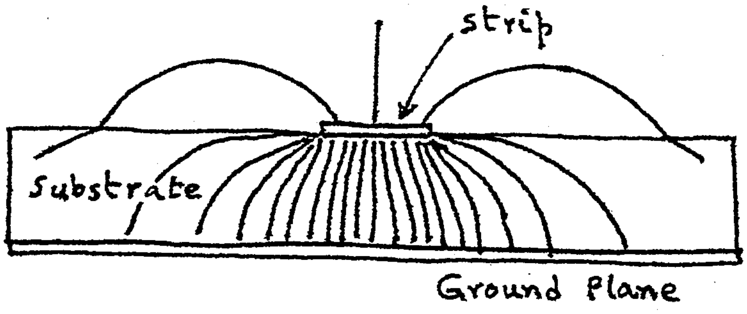
$$\mathcal{L} = \frac{\mu}{\pi} \cosh^{-1} \frac{d}{a}$$

$$\mathcal{C} = \frac{\pi\epsilon}{\cosh^{-1} \frac{d}{a}}$$

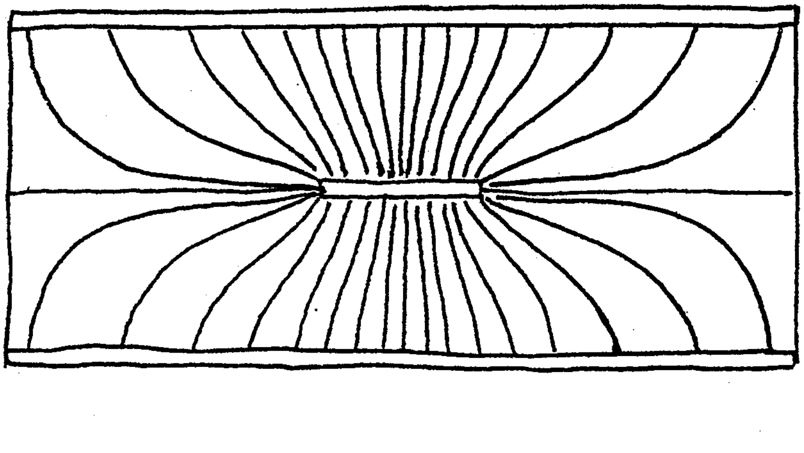
$$Z_0 = \frac{\eta}{\pi} \cosh^{-1} \frac{d}{a}$$



Microstrip Line



Shielded Strip Line



TIME DOMAIN ANALYSIS

(2 - 9)

Time Domain Solution for Lossless Line:

$$V(z,t) = V^+(t - z/v_p) + V^-(t + z/v_p)$$

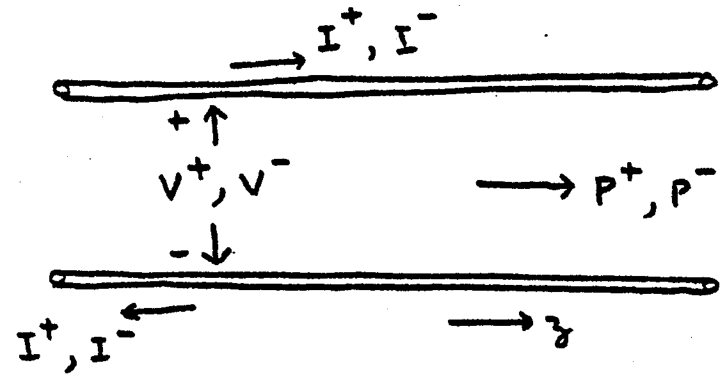
$$I(z,t) = \frac{1}{Z_0}\left[V^+(t - z/v_p) - V^-(t + z/v_p)\right]$$

$$v_p = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} \qquad Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$$

$$\boxed{\begin{gathered} V = V^+ + V^- \\ I = I^+ + I^- \\ I^+ = \frac{V^+}{Z_0} \qquad I^- = -\frac{V^-}{Z_0} \end{gathered}}$$

Sum of currents but

because of this sign

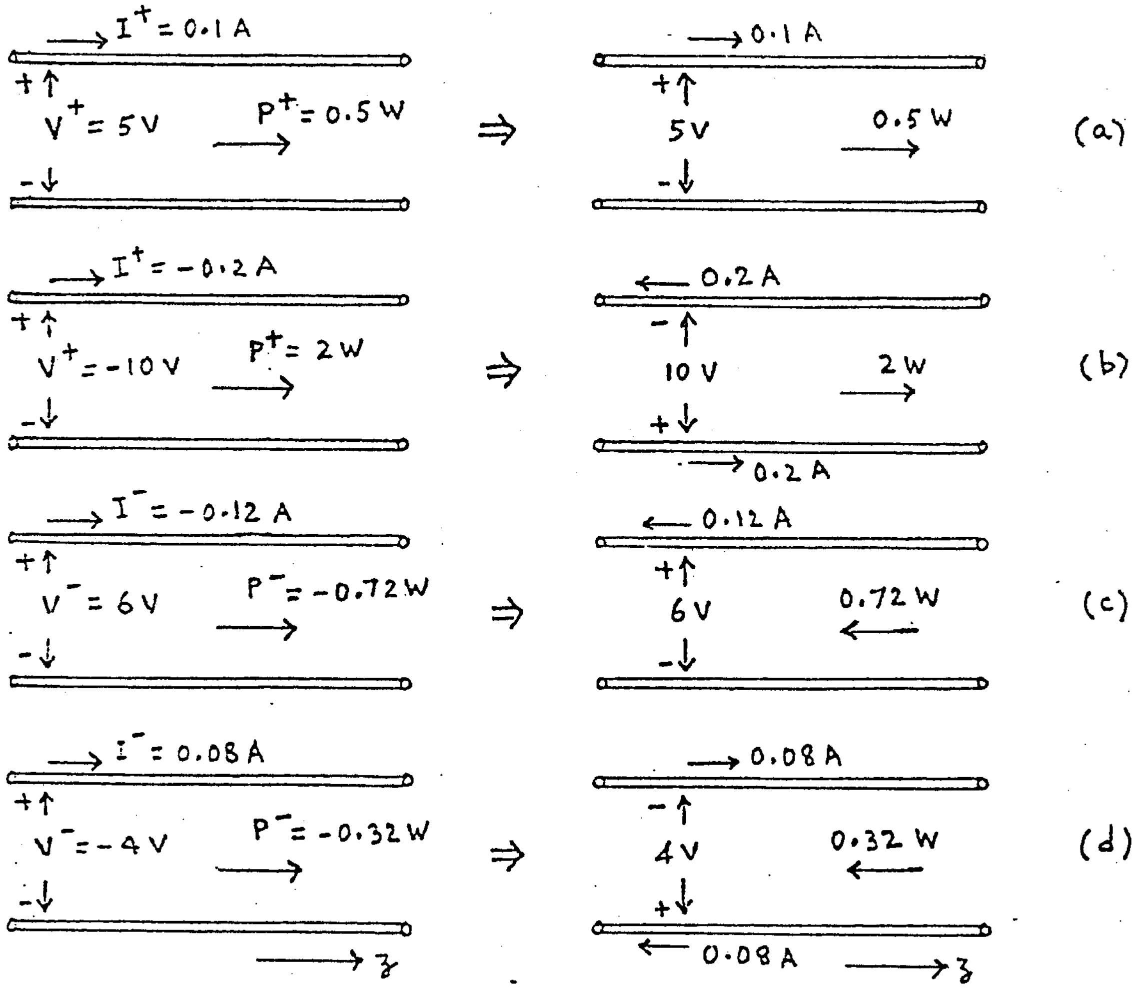


$$P^+ = V^+ I^+ = V^+\left(\frac{V^+}{Z_0}\right) = \frac{(V^+)^2}{Z_0}$$

$$P^- = V^- I^- = V^-\left(-\frac{V^-}{Z_0}\right) = -\frac{(V^-)^2}{Z_0}$$

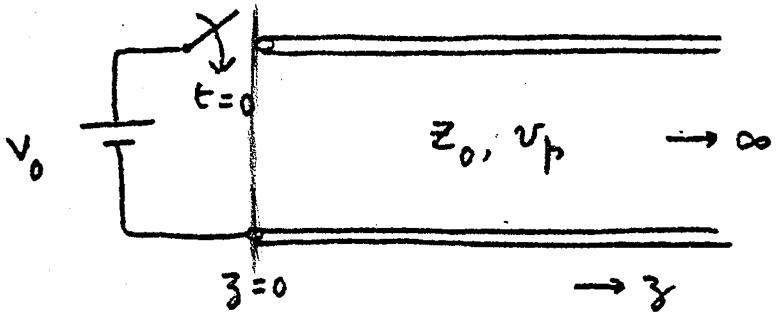
Numerical examples for illustrating polarities:

Assume $Z_0 = 50\Omega$



3.1.2. Examples involving numerical values of voltage, current and power flow associated with (+) and (−) waves.

Semi-infinite Line



$$\boxed{\begin{aligned} V(z,t) &= V^+(t - z/v_p) \\ I(z,t) &= \frac{1}{Z_0} V^+(t - z/v_p) \end{aligned}}$$

$V^- \equiv 0$

$$V(0,t) = V_0\, u(t) \leftarrow \text{since closing switch for } t \geq 0$$

B.C.

Boundary Condition

$$\therefore \quad V^+(t) = V_0\, u(t)$$

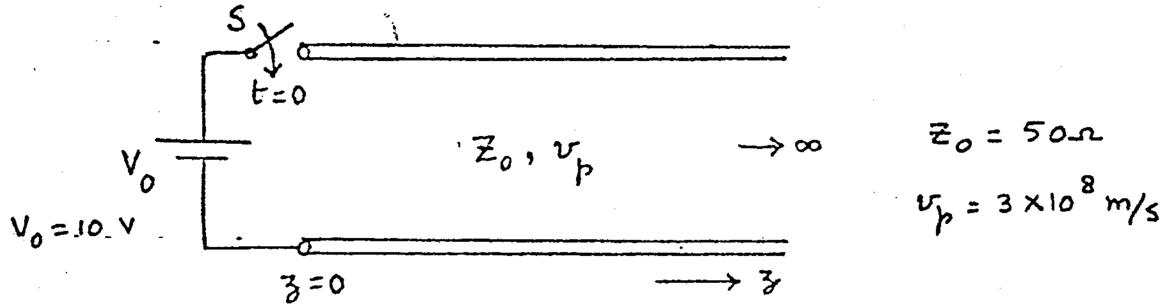
$$V(z,t) = V^+(t - z/v_p) = V_0\, u(t - z/v_p)$$

$$= \begin{cases} V_0 & \text{for } (t - z/v_p) > 0 \\ 0 & \text{for } (t - z/v_p) < 0 \end{cases}$$

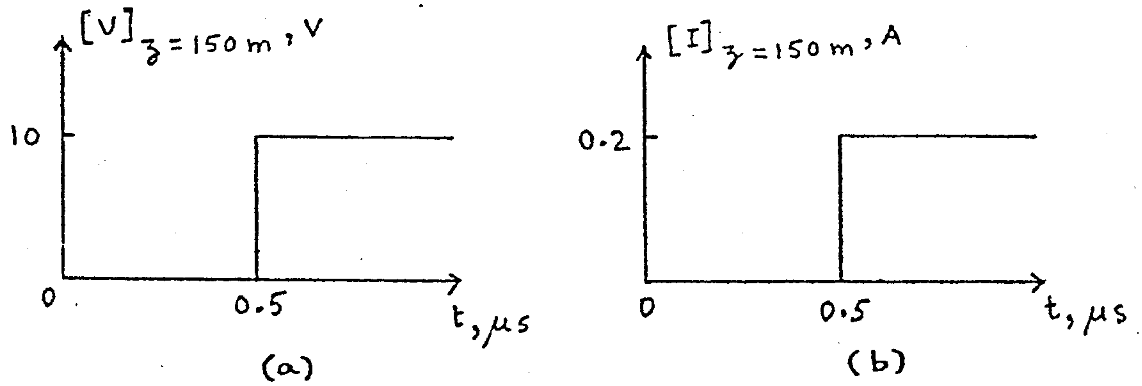
$$\boxed{V(z,t) = \begin{cases} V_0 & \text{for } t > z/v_p \\ 0 & \text{for } t < z/v_p \end{cases}}$$

$$\boxed{V(z,t) = \begin{cases} V_0 & \text{for } z < v_p t \\ 0 & \text{for } z > v_p t \end{cases}}$$

Example

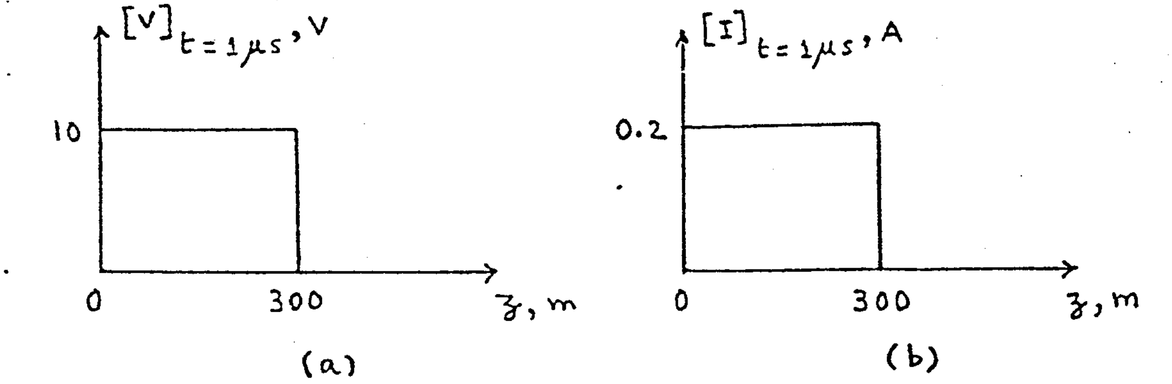


3.1.3. A semi-infinitely long line driven by a constant voltage source.



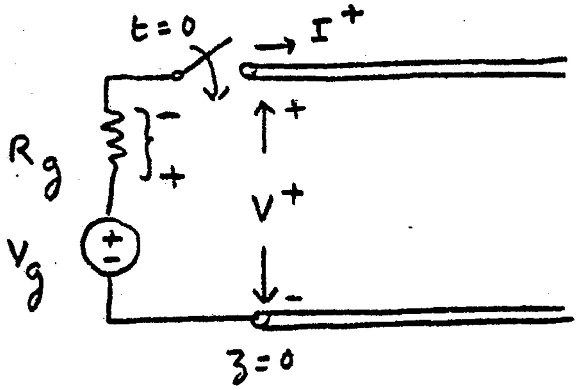
3.1.4. Variations with time of (a) voltage and (b) current at $z = 150$ m of the line of Fig. 3.1.3, for $V_0 = 10$ V, $Z_0 = 50\ \Omega$, and $v_p = 3 \times 10^8$ m/s.

$1\ \mu sec = 3 \times 10^8 \times 10^{-6} = 300m$



3.1.5. Variations with z of (a) voltage and (b) current for $t = 1\ \mu s$ along the line of Fig. 3.1.3, for $V_0 = 10$ V, $Z_0 = 50\ \Omega$, and $v_p = 3 \times 10^8$ m/s.

Effect of Source Resistance



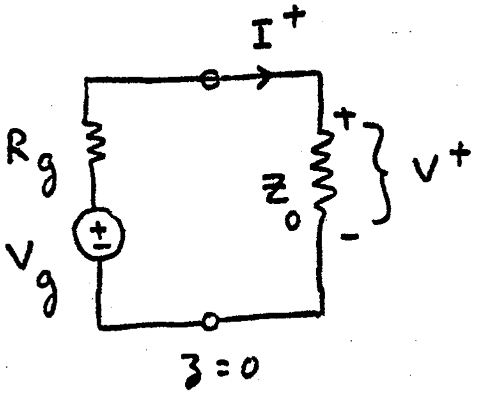
$$V_g - I^+ R_g - V^+ = 0 \qquad \text{B.C.}$$

$$I^+ = \frac{V^+}{Z_0} \qquad \text{(+) wave characteristic}$$

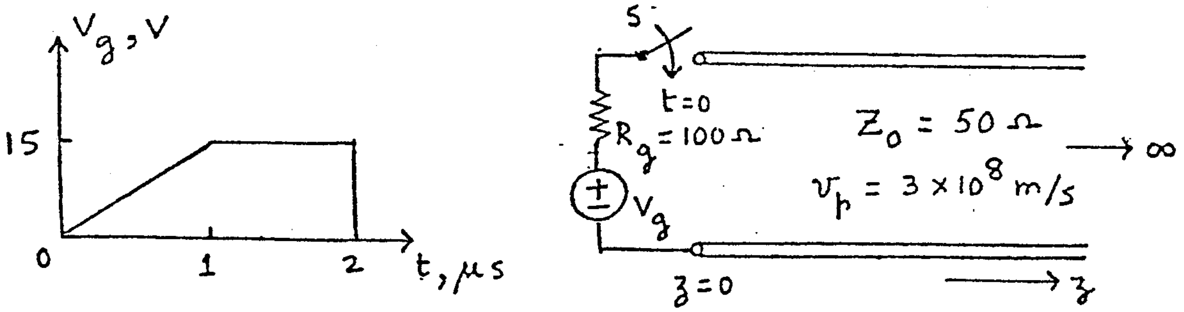
$$\therefore \; V_g - \frac{V^+}{Z_0} R_g - V^+ = 0$$

$$V_g = V^+\left(\frac{R_g}{Z_0} + 1\right)$$

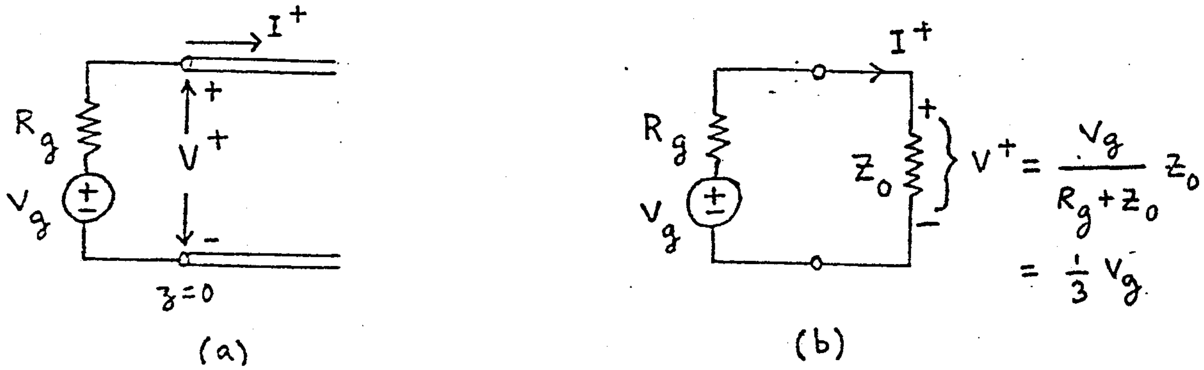
$$\boxed{\begin{aligned} V^+ &= V_g \frac{Z_0}{R_g + Z_0} \\ I^+ &= \frac{V^+}{Z_0} = \frac{V_g}{R_g + Z_0} \end{aligned}}$$



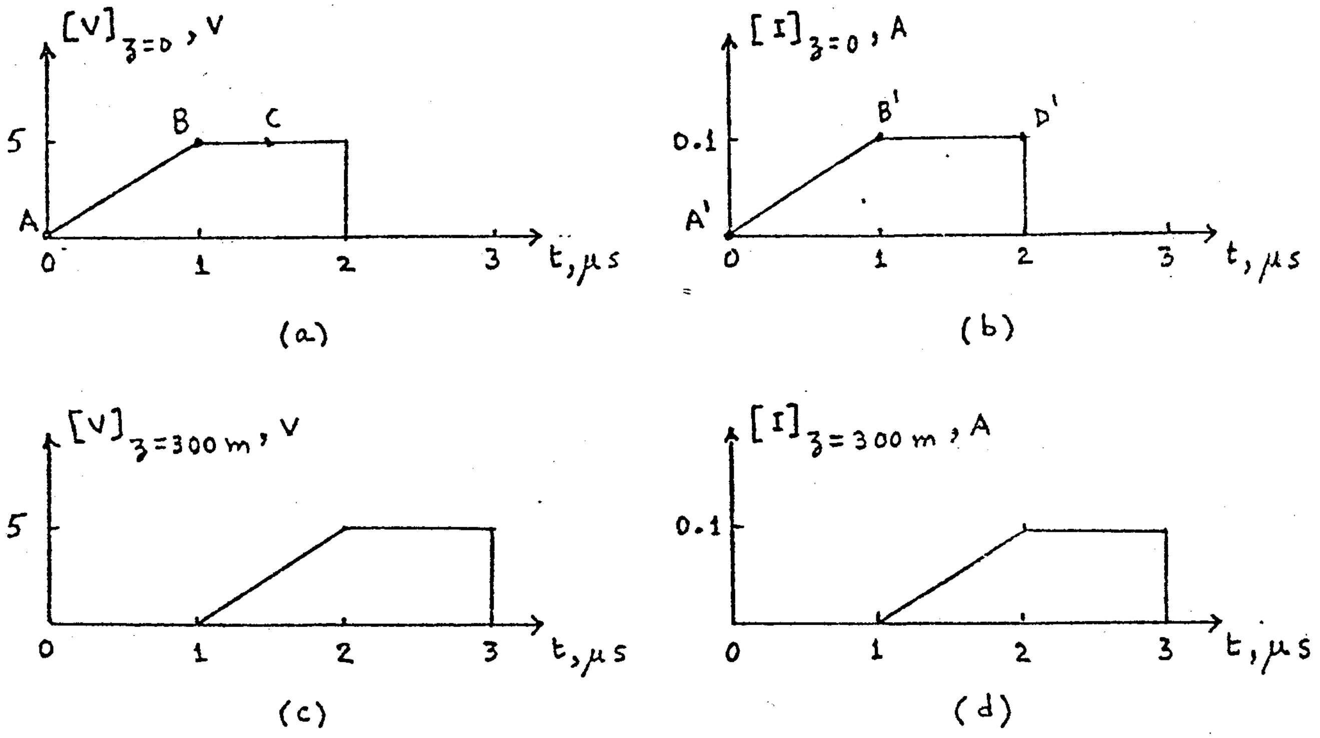
Example



3.1.6. A semi-infinitely long line driven by a time varying voltage source V_g in series with an internal resistance.

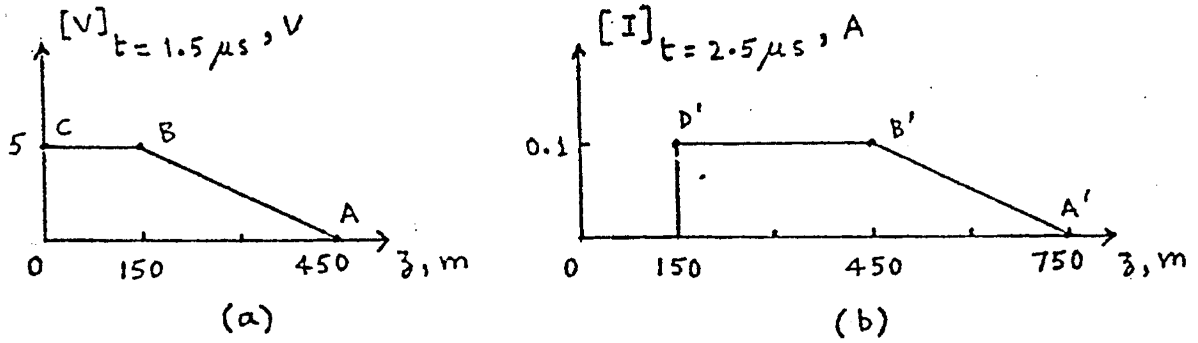


3.1.7. (a) For obtaining the (+) wave voltage and current at z = 0 for the line of Fig. 3.1.6. (b) Equivalent circuit for (a).



3.1.8. (a) and (b) Time variations of line voltage and current, respectively, at z = 0 for the line of Fig. 3.1.6. (c) and (d) Same as (a) and (b), respectively, except at z = 300 m.

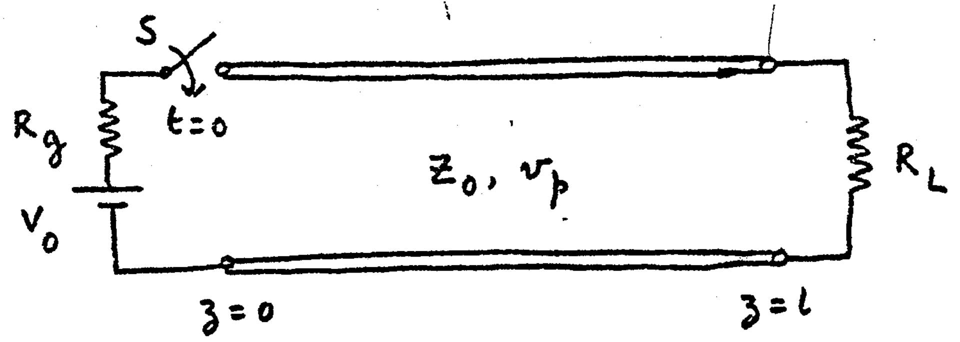
1.5us, 300m / μs 150m



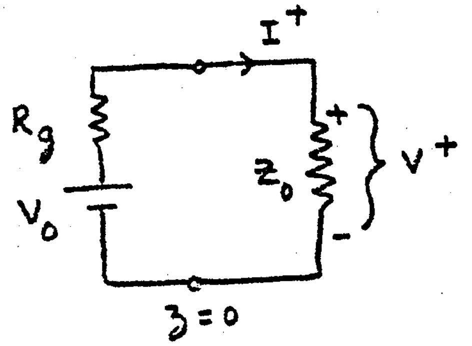
3.1.9. (a) Line voltage versus z along the line of Fig. 3.1.6 for t = 1.5 μs. (b) Line current versus z along the line of Fig. 3.1.6 for t = 2.5 μs.

Line Terminated by Resistance

wave reaches here at l/v_p



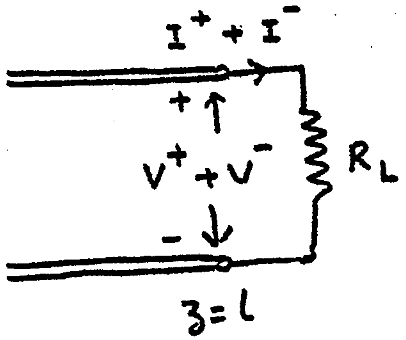
t = 0+



$$I^+ = V_0 \frac{1}{R_g + Z_0}$$

$$V^+ = V_0 \frac{Z_0}{R_g + Z_0}$$

t = l/v_p



$$V^+ + V^- = R_L(I^+ + I^-) \quad \text{B.C.}$$

$$I^+ = \frac{V^+}{Z_0}, \quad I^- = -\frac{V^-}{Z_0}$$

$$V^+ + V^- = R_L\left(\frac{V^+}{Z_0} - \frac{V^-}{Z_0}\right)$$

$$V^-\left(1 + \frac{R_L}{Z_0}\right) = V^+\left(\frac{R_L}{Z_0} - 1\right)$$

$$\boxed{V^- = V^+ \frac{R_L - Z_0}{R_L + Z_0}}$$

Define $\Gamma = \dfrac{V^-}{V^+}$ = voltage Reflection coefficient

$$\boxed{\Gamma = \frac{R_L - Z_0}{R_L + Z_0}}$$

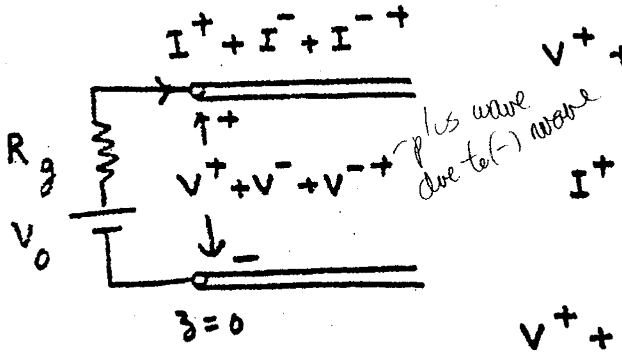
Then, current Reflection Coefficient

$$= \frac{I^-}{I^+} = \frac{-V^-/Z_0}{V^+/Z_0} = -\frac{V^-}{V^+} = -\Gamma$$

= − (voltage Reflection coefficient).

now reflected (−) wave from $z=\ell$, $t=\ell/v_p$ arrives at $z=0$, $t=2\ell/v_p$

<u>$t = 2l/v_p$</u>



$$V^+ + V^- + V^{-+} = V_0 - R_g(I^+ + I^- + I^{-+})$$

B.C.

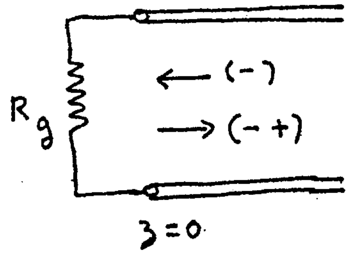
$$I^+ = \frac{V^+}{Z_0}, \quad I^- = -\frac{V^-}{Z_0}, \quad I^{-+} = \frac{V^{-+}}{Z_0}$$

$$V^+ + V^- + V^{-+} = V_0 - \frac{R_g}{Z_0}(V^+ - V^- + V^{-+})$$

$$V^+\left(1 + \frac{R_g}{Z_0}\right) + V^{-+}\left(1 + \frac{R_g}{Z_0}\right) = V_0 + V^-\left(\frac{R_g}{Z_0} - 1\right)$$

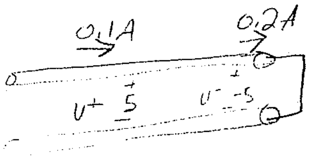
$$V_0 \frac{Z_0}{R_g + Z_0}\left(\frac{Z_0 + R_g}{Z_0}\right) + V^{-+}\left(1 + \frac{R_g}{Z_0}\right) = V_0 + V^-\left(\frac{R_g}{Z_0} - 1\right)$$

$$\boxed{\frac{V^{-+}}{V^-} = \frac{R_g - Z_0}{R_g + Z_0}}$$



at t=

Discussion of Reflection Coefficient



(a) $R_L = 0$, short circuited line

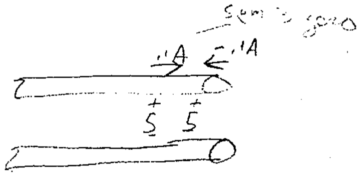
$$\Gamma = \frac{0 - Z_0}{0 + Z_0} = -1$$

Reflected Voltage $\overset{\to^{+}\ \nwarrow = 0}{=}$ − (Incident voltage) voltage is in opposite direction

Reflected current $\overset{\to^{+}\ \nwarrow = 2I}{=}$ Incident current so current is in same direction

∴ Voltage across R_L is zero, whereas current is doubled.

(b) $R_L = \infty$, open circuited line



$$\Gamma = \frac{\infty - Z_0}{\infty + Z_0} = 1$$

Reflected Voltage = Incident voltage

Reflected current = − (Incident current)

∴ Current through R_L is zero, whereas voltage is doubled.

(c) $R_L = Z_0$, line terminated by its characteristic impedance.

$$\Gamma = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

This corresponds to no reflection. ∴ This case is equivalent to a semi-infinite line in so far as the source is concerned.

$t = \infty$ (steady state situation),

$$V_{ss} = V^+\left(1 + \Gamma_R + \overset{\text{at load end}}{\Gamma_R}\,\overset{\text{at source end}}{\Gamma_S} + \Gamma_R^2\Gamma_S + \Gamma_R^2\Gamma_S^2 + \cdots\right)$$

$$= \frac{V_0 Z_0}{R_g + Z_0}\left[\left(1 + \Gamma_R\Gamma_S + \Gamma_R^2\Gamma_S^2 + \cdots\right) + \Gamma_R\left(1 + \Gamma_R\Gamma_S + \Gamma_R^2\Gamma_S^2 + \cdots\right)\right]$$

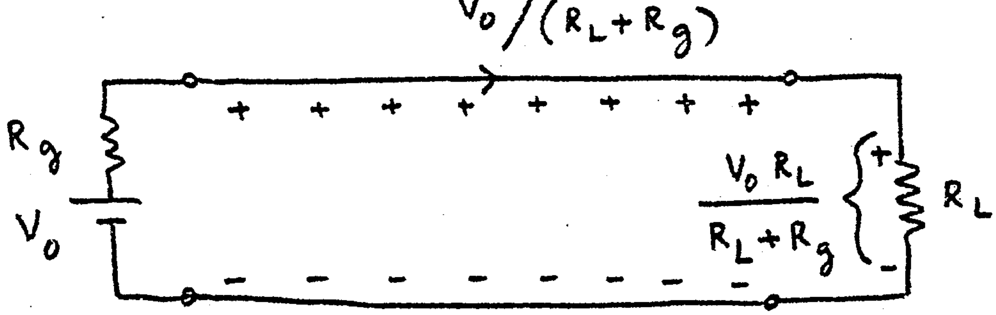
$$= \frac{V_0 Z_0}{R_g + Z_0}\,\frac{1 + \Gamma_R}{1 - \Gamma_R\Gamma_S} \qquad 1 + a + a^2 + \cdots = \frac{1}{1-a}$$

$$= \frac{V_0 Z_0}{R_g + Z_0}\,\frac{1 + \left(\frac{R_L - Z_0}{R_L + Z_0}\right)}{1 - \left(\frac{R_L - Z_0}{R_L + Z_0}\right)\left(\frac{R_g - Z_0}{R_g + Z_0}\right)} = V_0\,\frac{R_L}{R_L + R_g}$$

$$I_{ss} = I^+\left[1 + (-\Gamma_R) + (-\Gamma_R)(-\Gamma_S) + (-\Gamma_R)^2(-\Gamma_S) + \cdots\right]$$

$$= \frac{V_0}{R_g + Z_0}\left[\left(1 + \Gamma_R\Gamma_S + \cdots\right) - \Gamma_R\left(1 + \Gamma_R\Gamma_S + \cdots\right)\right]$$

$$= \frac{V_0}{R_g + Z_0}\,\frac{1 - \Gamma_R}{1 - \Gamma_R\Gamma_S} = \frac{V_0}{R_L + R_g}$$

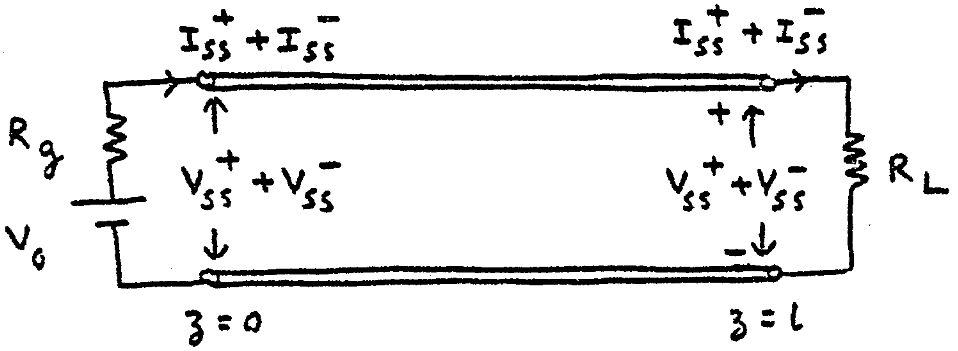


Actual Situation in the Steady state

One (+) wave + One (−) wave

Superposition of all transient (+) waves

superposition of all transient (−) waves



(1) $$V_{ss}^+ + V_{ss}^- = V_0 - R_g\left(I_{ss}^+ + I_{ss}^-\right)$$ B.C. at $z = 0$

(2) $$V_{ss}^+ + V_{ss}^- = R_L\left(I_{ss}^+ + I_{ss}^-\right)$$ B.C. at $z = l$

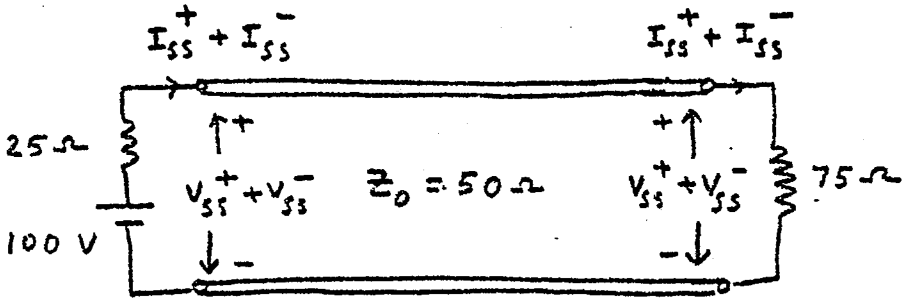
(3) $$I_{ss}^+ = \frac{V_{ss}^+}{Z_0}$$ (+) wave characteristic

(4) $$I_{ss}^- = -\frac{V_{ss}^-}{Z_0}$$ (−) wave characteristic

Four equations, four unknowns V_{ss}^+, V_{ss}^-, I_{ss}^+, I_{ss}^-. Hence solving for them, we can find

$$V_{ss} = V_{ss}^+ + V_{ss}^- \quad \text{and} \quad I_{ss} = I_{ss}^+ + I_{ss}^- .$$

A Numerical Example



$$(1) \qquad V_{ss}^+ + V_{ss}^- = 100 - 25\left(I_{ss}^+ + I_{ss}^-\right)$$

$$(2) \qquad V_{ss}^+ + V_{ss}^- = 75\left(I_{ss}^+ + I_{ss}^-\right)$$

$$(3) \qquad I_{ss}^+ = \frac{V_{ss}^+}{50}$$

$$(4) \qquad I_{ss}^- = -\frac{V_{ss}^-}{50}$$

$$V_{ss}^+ + V_{ss}^- = 100 - \frac{25}{50}\left(V_{ss}^+ - V_{ss}^-\right)$$

$$V_{ss}^+ + V_{ss}^- = \frac{75}{50}\left(V_{ss}^+ - V_{ss}^-\right)$$

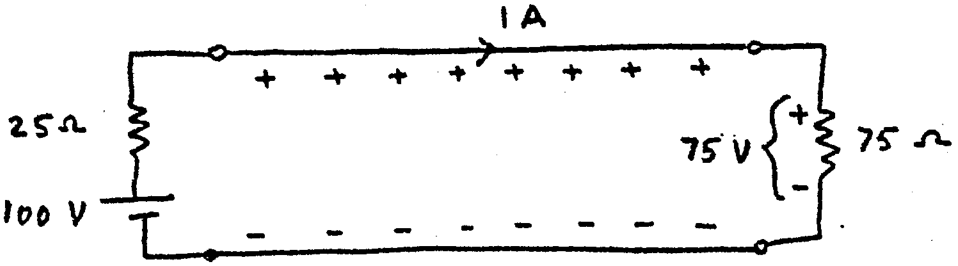
$$1.5\ V_{ss}^+ + 0.5\ V_{ss}^- = 100$$

$$-0.5\ V_{ss}^+ + 2.5\ V_{ss}^- = 0 \longrightarrow V_{ss}^+ = 5\ V_{ss}^-$$

$$8\ V_{ss}^- = 100 \longrightarrow V_{ss}^- = 12.5\ , \quad V_{ss}^+ = 62.5$$

$$V_{ss} = V_{ss}^+ + V_{ss}^- = 75\ \text{V}.$$

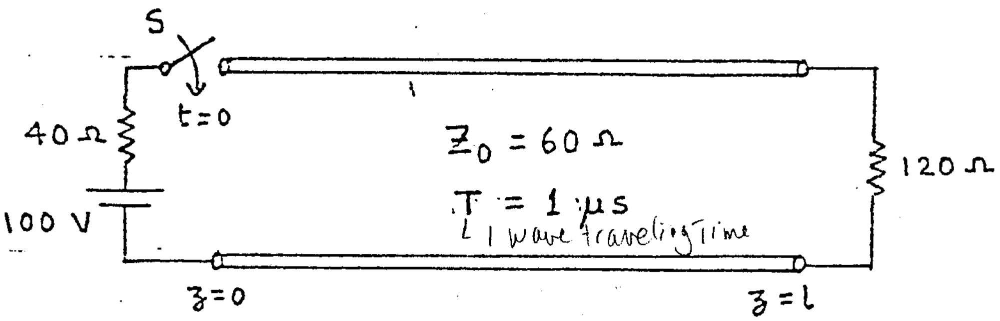
$$I_{ss} = I_{ss}^+ + I_{ss}^- = \frac{62.5}{50} - \frac{12.5}{50} = 1\ \text{A}$$



Bounce Diagram Technique

is a graphical technique for keeping track of the bouncing back and forth of the transient waves

Example for Constant Voltage Source:



3.2.5. Transmission line system for illustrating the bounce diagram technique of keeping track of the transient phenomenon.

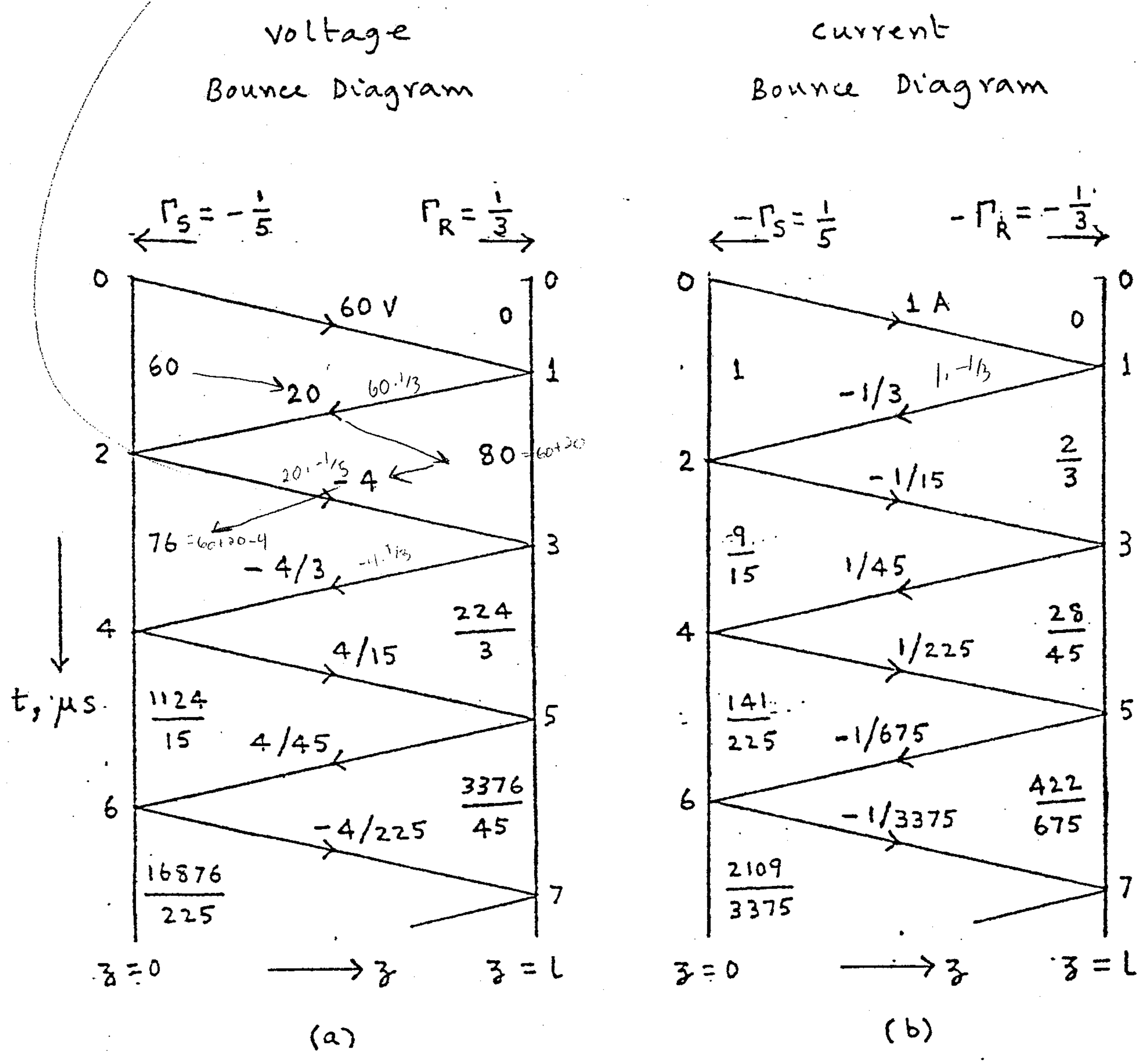
Voltage carried by the initial (+) wave $= 100 \dfrac{60}{40+60} = 60\text{V}$

Current carried by the initial (+) wave $= \dfrac{60}{60} = 1\text{ A}$

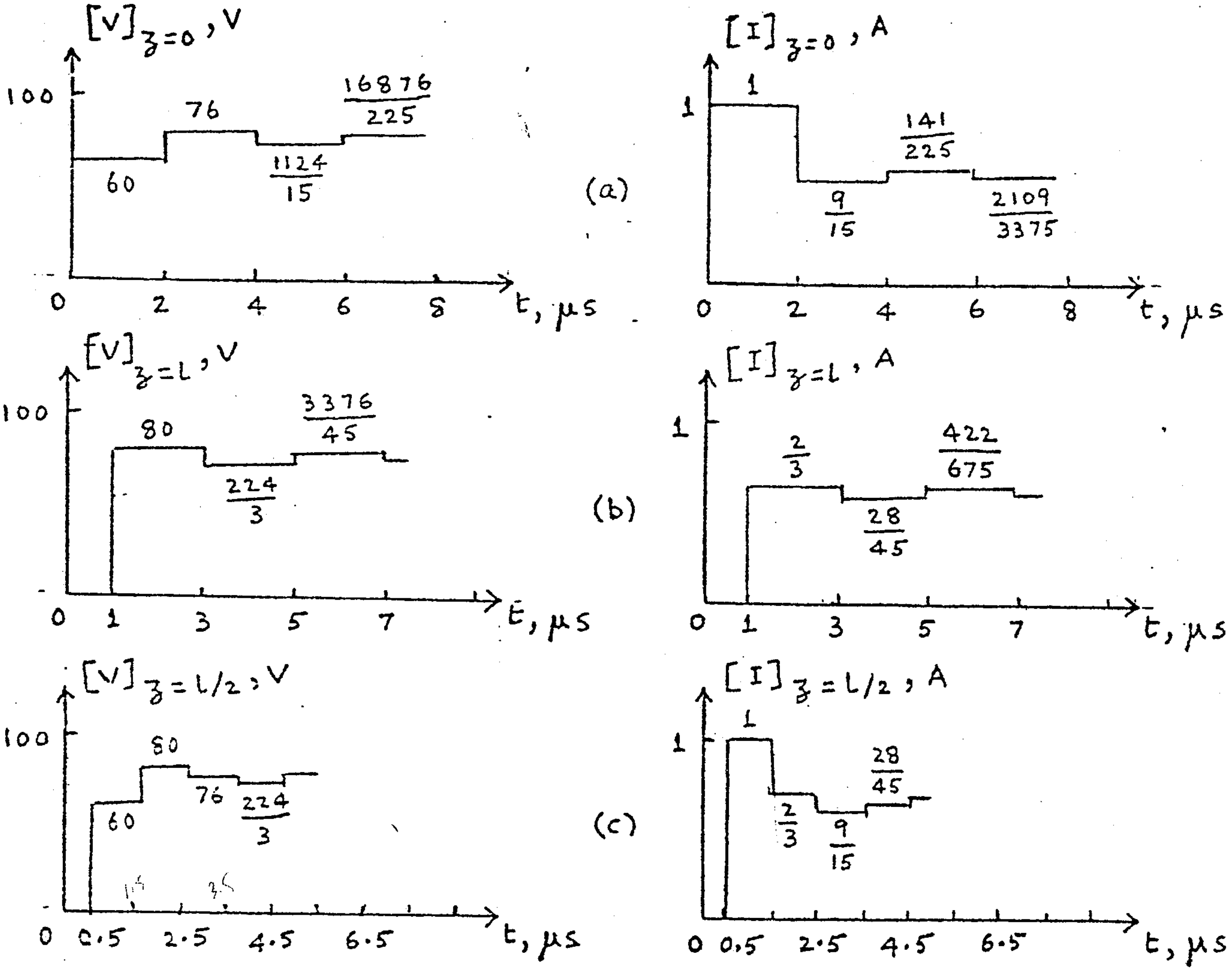
Voltage reflection coefficient at load, $\Gamma_R = \dfrac{120-60}{120+60} = \dfrac{1}{3}$

Voltage reflection coefficient at source, $\Gamma_S = \dfrac{40-60}{40+60} = -\dfrac{1}{5}$

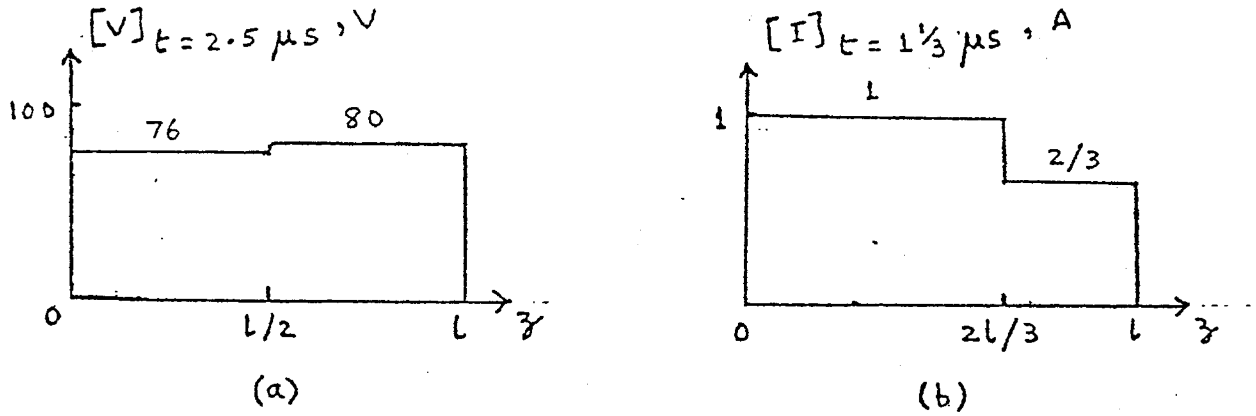
Zig-zag down instead of going from the top.



3.2.6. (a) and (b) Voltage and current bounce diagrams, respectively, depicting the bouncing back and forth of the transient waves for the system of Fig. 3.2.5.



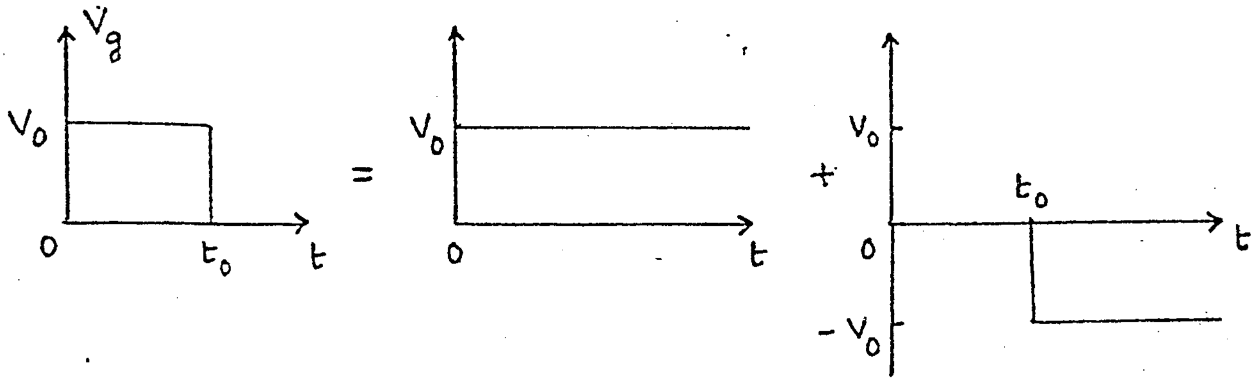
3.2.7. Time variations of line voltage and line current at (a) $z = 0$, (b) $z = l$, and (c) $z = l/2$, for the system of Fig. 3.2.5.



(a) (b)

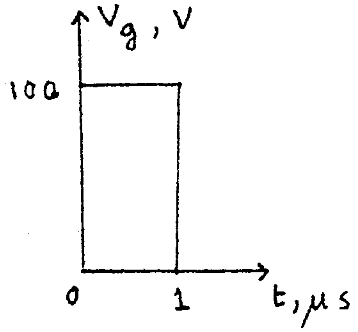
3.2.8. Variations with z of (a) line voltage for $t = 2.5$ μs, and (b) line current for $t = 1\text{-}1/3$ μs, for the system of Fig. 3.2.5.

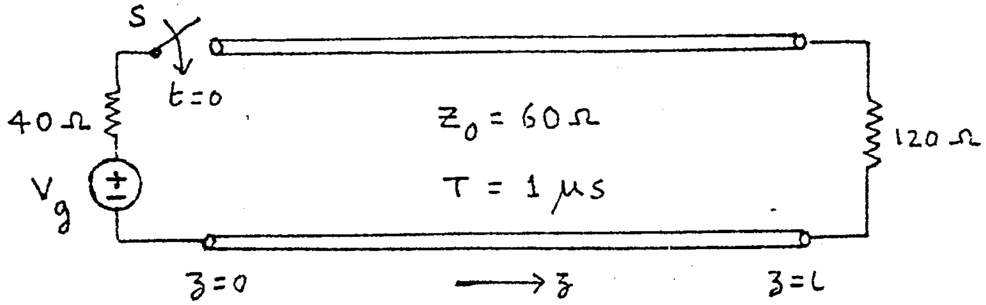
Use of superposition for Rectangular Pulse Voltage Source:



3.2.9. Representation of a rectangular pulse as the superposition of two step functions.

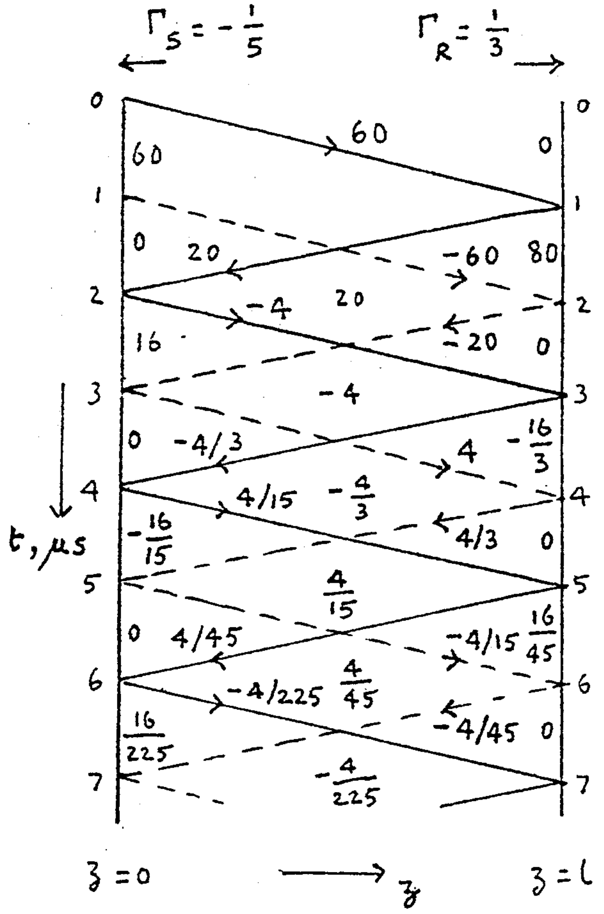
Example

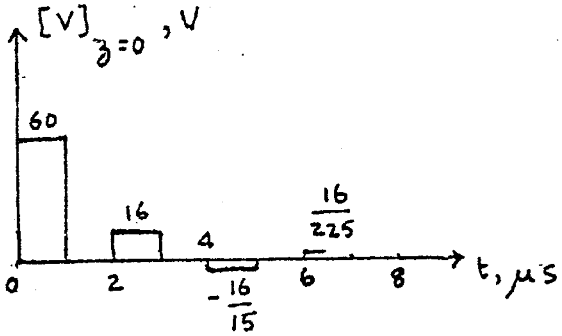


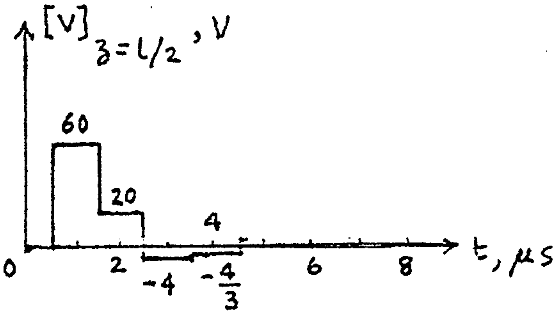


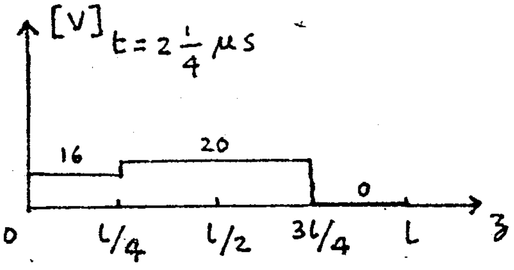
Superposition of two 100 V [illegible]

Shift solid line down 1 μs to get dotted line.





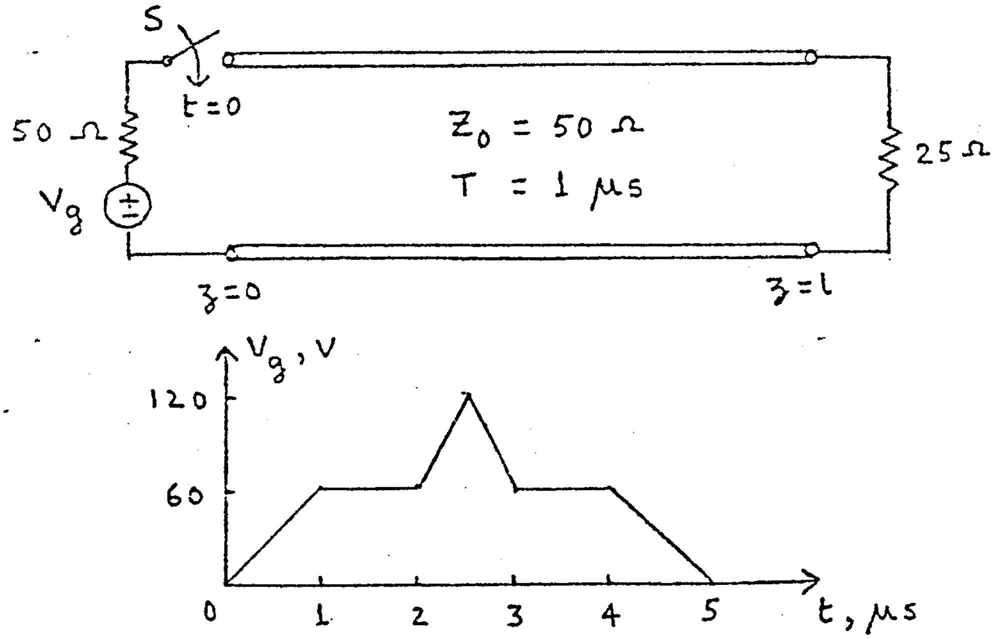




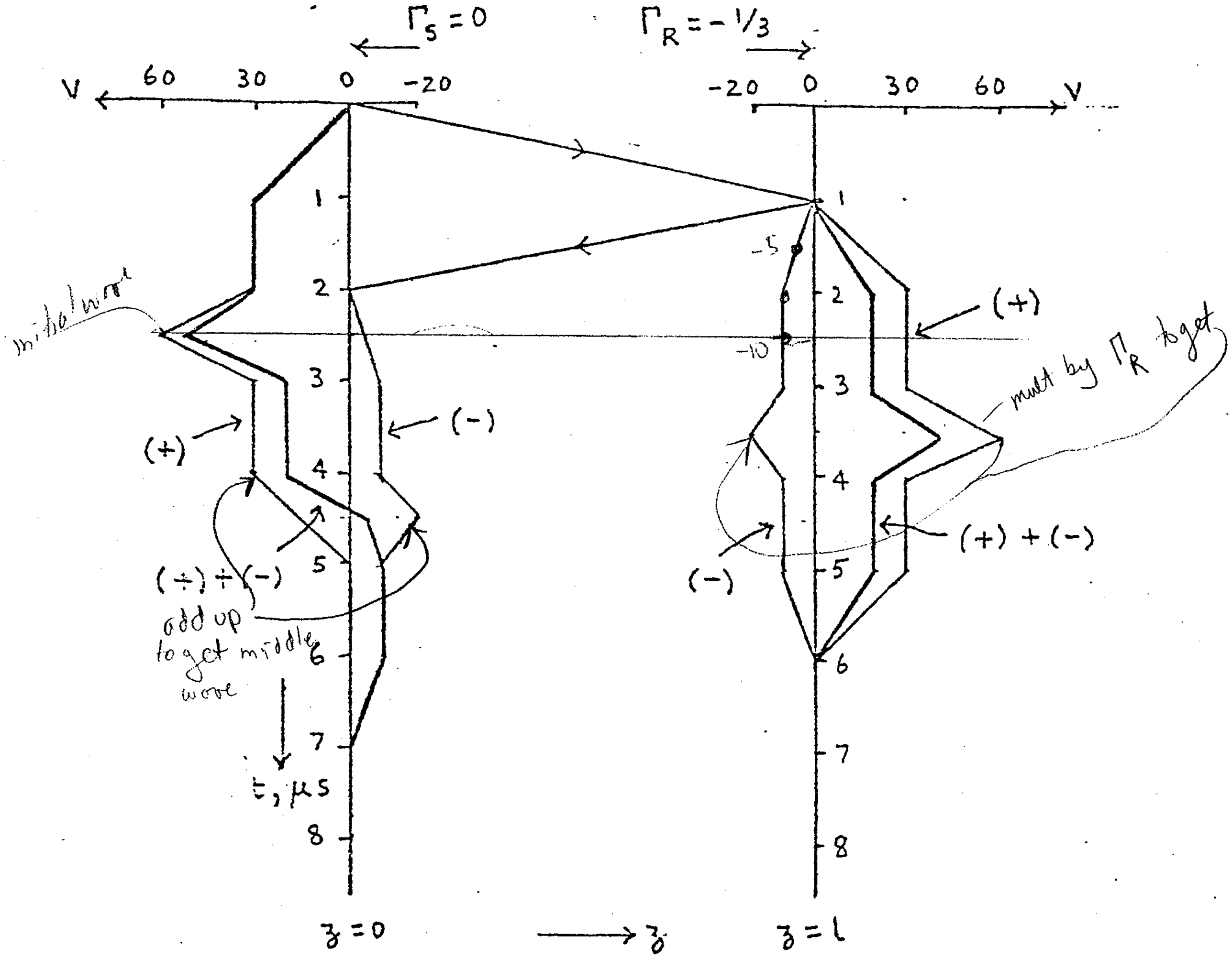
Bounce Diagram Technique for Arbitrary Voltage Source

Example:

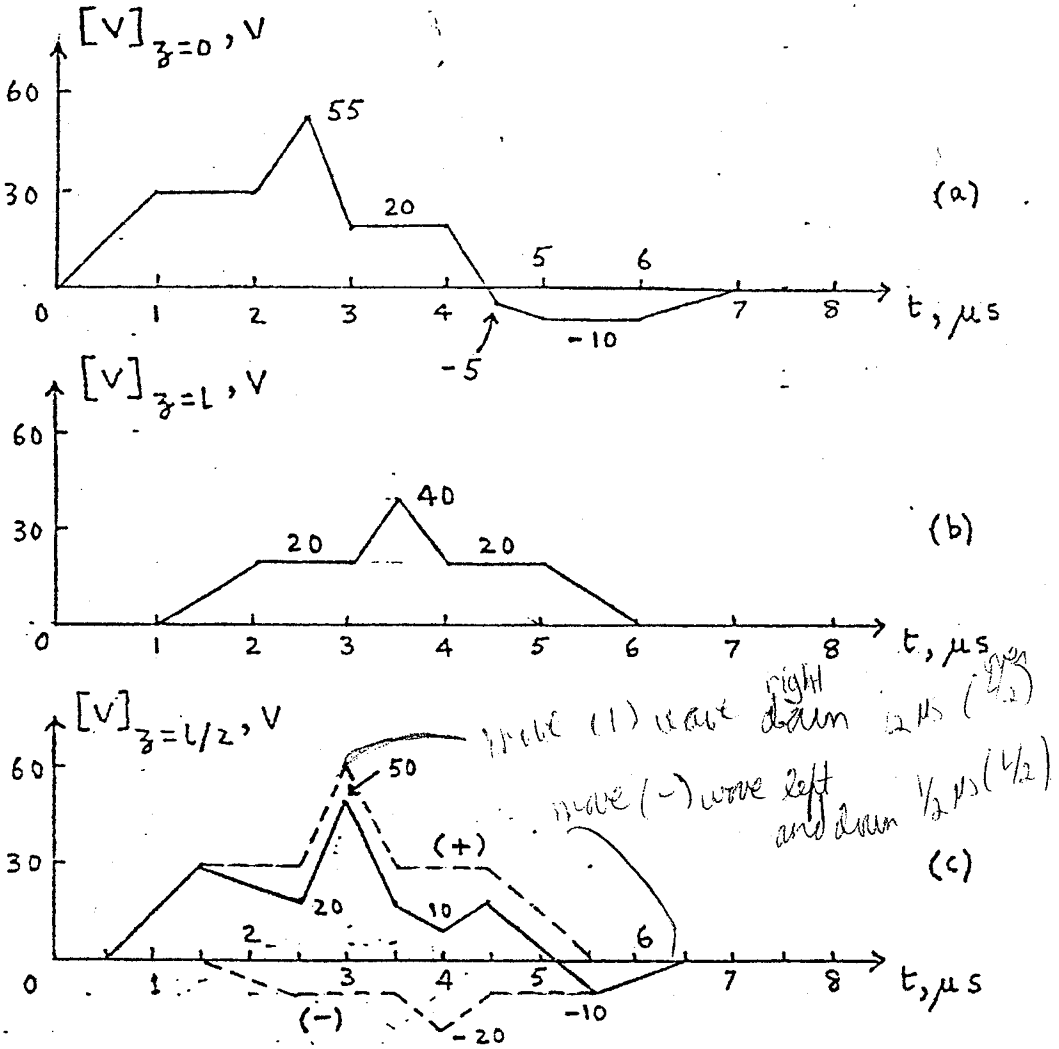
here $\Gamma_S = 0$ since $R_{TERM} = Z_0$
so only one reflection.



3.2.10. Transmission line system for illustrating the bounce diagram technique for an arbitrary voltage source.



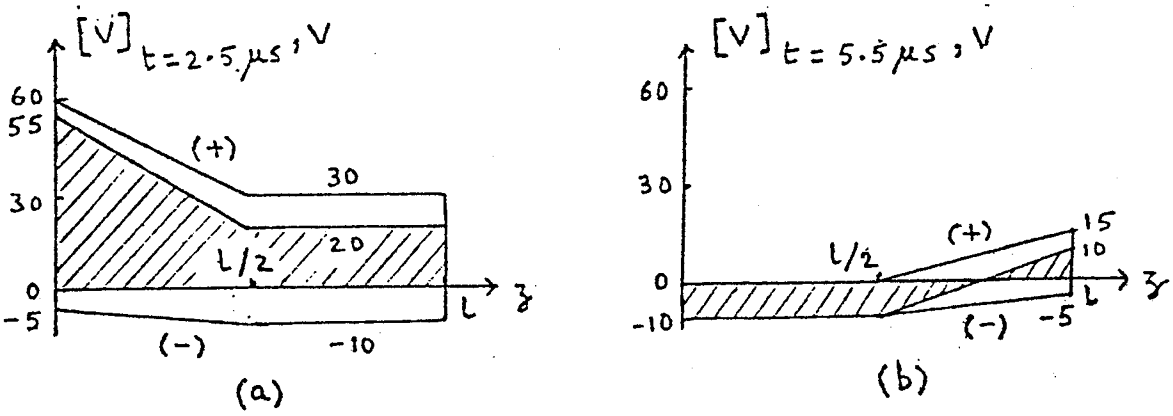
3.2.11. Voltage bounce diagram for the system of Fig. 3.2.10.



move (+) wave right down ½ μs (l/2)

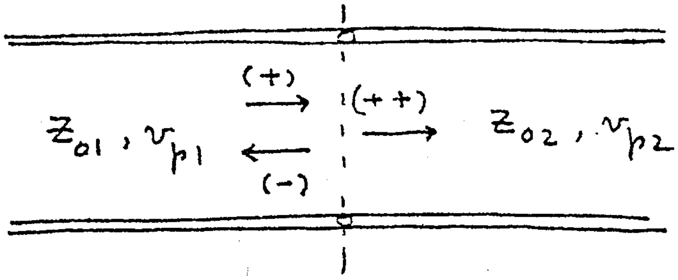
move (−) wave left and down ½ μs (l/2)

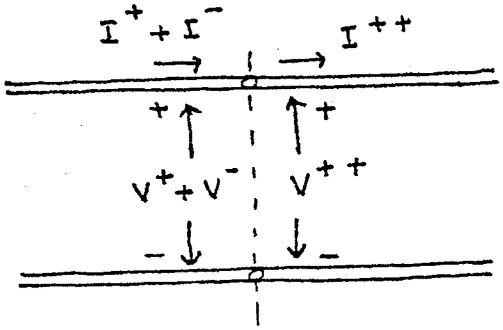
3.2.12. Time variations of line voltage at (a) z = 0, (b) z = l, and (c) z = $l/2$, for the system of Fig. 3.2.10.



3.2.13. Variations with z of line voltage for (a) t = 2.5 μs, and (b) t = 5.5 μs, for the system of Fig. 3.2.10.

Transmission Line Discontinuity





$$\left.\begin{aligned} &(1) \quad V^+ + V^- = V^{++} \\ &(2) \quad I^+ + I^- = I^{++} \end{aligned}\right\} \quad B \cdot C$$

$$I^+ = \frac{V^+}{Z_{01}}, \quad I^- = -\frac{V^-}{Z_{01}}, \quad I^{++} = \frac{V^{++}}{Z_{02}}$$

$$(2) \rightarrow \quad \frac{V^+}{Z_{01}} - \frac{V^-}{Z_{01}} = \frac{V^{++}}{Z_{02}}$$

$$\frac{Z_{02}}{Z_{01}}\left(V^+ - V^-\right) = V^{++}$$

$$V^+ + V^- = \frac{Z_{02}}{Z_{01}}\left(V^+ - V^-\right)$$

$$V^-\left(1 + \frac{Z_{02}}{Z_{01}}\right) = V^+\left(\frac{Z_{02}}{Z_{01}} - 1\right)$$

$$\boxed{\Gamma = \frac{V^-}{V^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}}$$

Define $\tau_V = \dfrac{V^{++}}{V^{+}}$ = Voltage Transmission Coefficient

$$= \frac{V^{+} + V^{-}}{V^{+}} = 1 + \frac{V^{-}}{V^{+}}$$

$$\boxed{\tau_V = 1 + \Gamma}$$

τ_v greater than 1 ($V^{++} > V^{+}$) however $\tau_c < 1$ and power decreases

$\tau_c = \dfrac{I^{++}}{I^{+}}$ = Current Transmission Coefficient

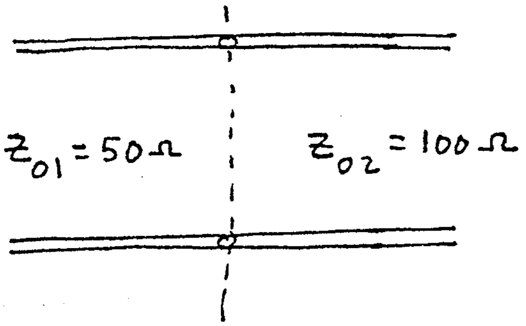
$$= \frac{I^{+} + I^{-}}{I^{+}} = 1 + \frac{I^{-}}{I^{+}}$$

$$\boxed{\tau_c = 1 - \Gamma}$$

Note that

$$\begin{aligned}
P^{++} &= V^{++} I^{++} \\
&= \tau_V V^{+} \cdot \tau_c I^{+} \\
&= (1+\Gamma) V^{+} \cdot (1-\Gamma) I^{+} \\
&= (1-\Gamma^2) V^{+} I^{+} \\
&= (1-\Gamma^2) P^{+} \\
&\leq P^{+}
\end{aligned}$$

Examples



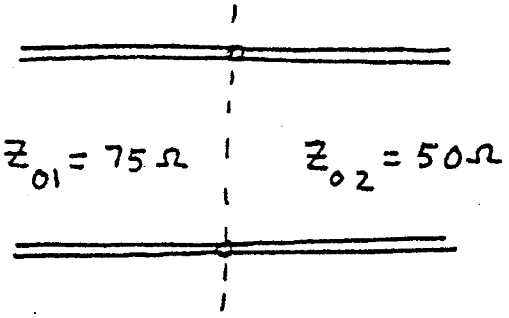
$$\Gamma = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3}$$

$$\tau_V = 1 + \Gamma = \frac{4}{3}$$

$$\tau_C = 1 - \Gamma = \frac{2}{3}$$

$$\therefore V^{++} = \frac{4}{3} V^{+} \quad , \quad I^{++} = \frac{2}{3} I^{+}$$

$$P^{++} = \frac{8}{9} P^{+}$$



$$\Gamma = \frac{50-75}{50+75} = -\frac{25}{125} = -\frac{1}{5}$$

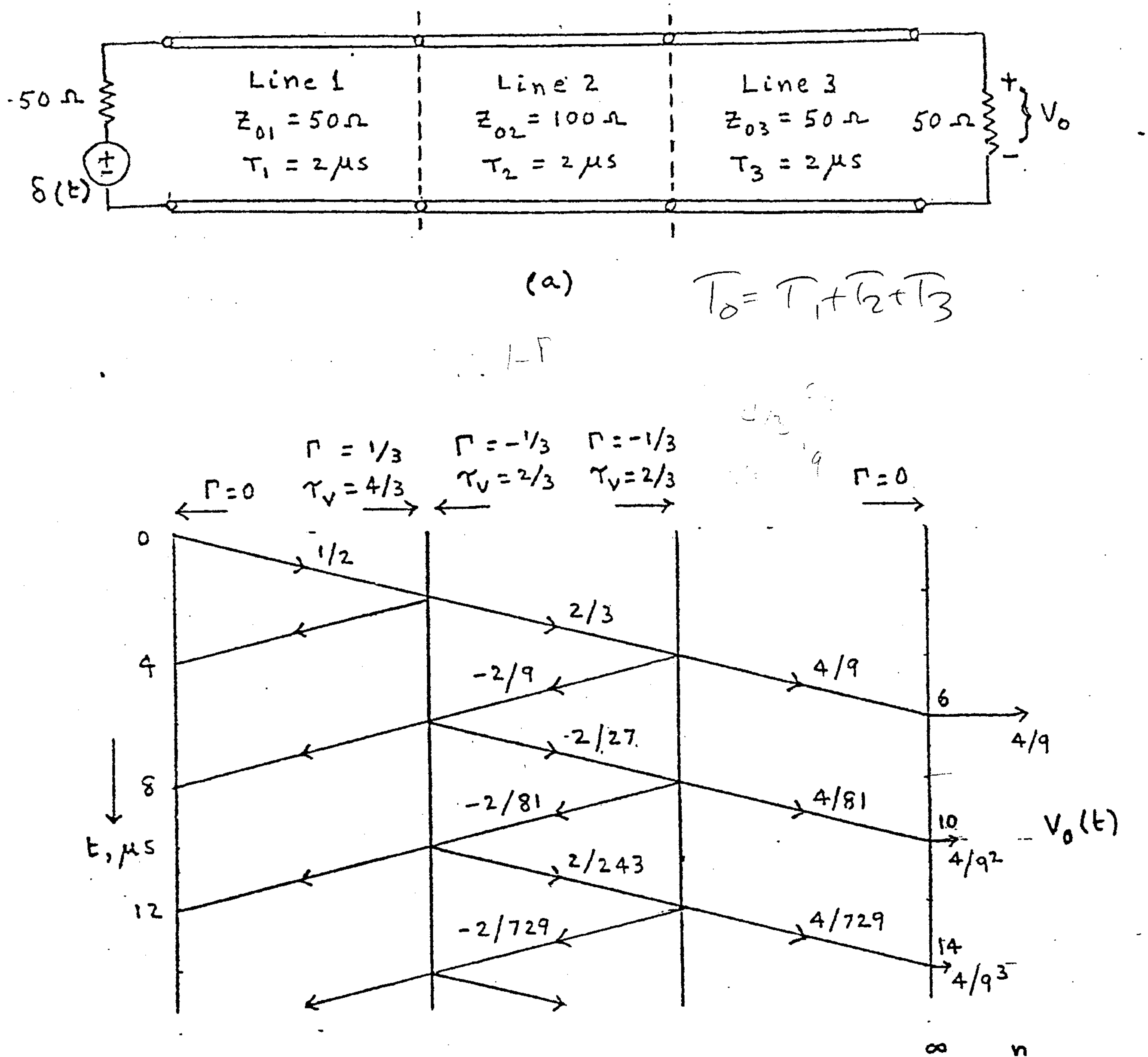
$$\tau_V = 1 + \Gamma = \frac{4}{5}$$

$$\tau_c = 1 - \Gamma = \frac{6}{5}$$

$$V^{++} = \frac{4}{5} V^{+} \quad , \quad I^{++} = \frac{6}{5} I^{+}$$

$$P^{++} = \frac{24}{25} P^{+}$$

Three Lines in Cascade



(a)

$T_0 = T_1 + T_2 + T_3$

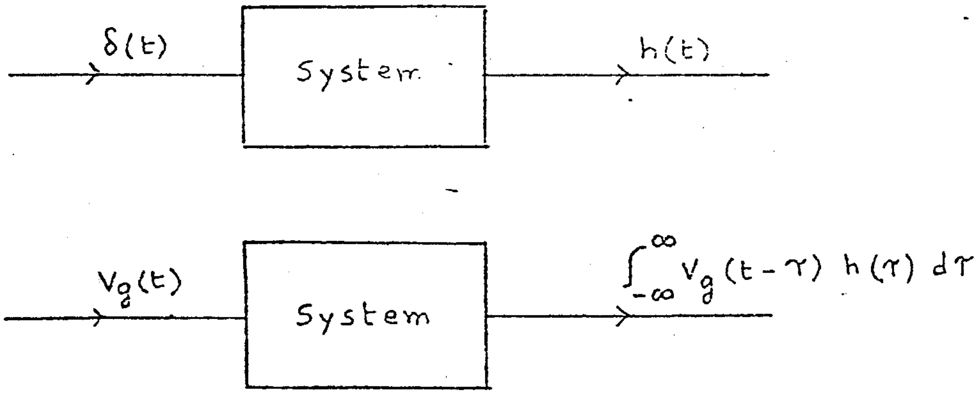
$1-\Gamma$

(b)

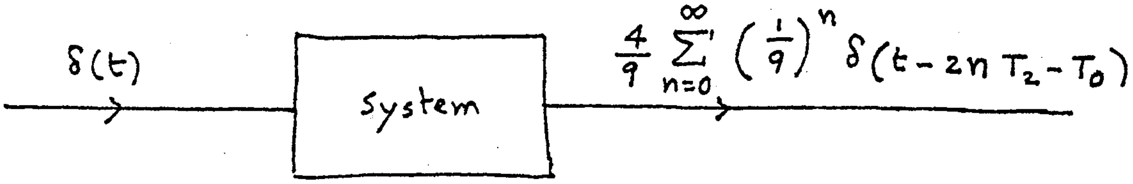
$$V_0(t) = \frac{4}{9} \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n \delta(t - 2nT_2 - T$$

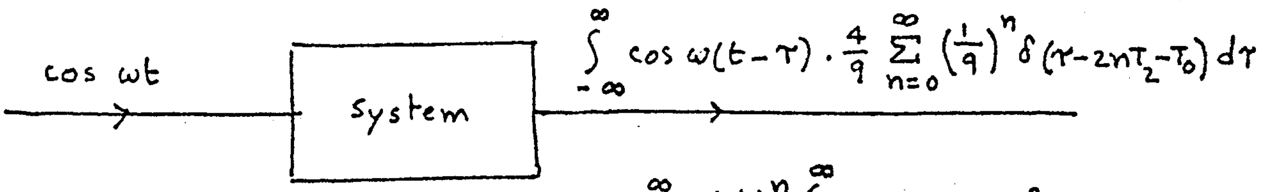
3.3.3. (a) A system of three lines in cascade driven by a unit impulse voltage source. (b) Voltage bounce diagram for finding the output voltage $V_o(t)$ for the system of (a).

Frequency Response from Unit Impulse Response



3.3.4. Block diagrams illustrating the application of the convolution integral for finding the response of a system to an arbitrary excitation $V_g(t)$ from its unit impulse response h(t).





$$= \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n \int_{-\infty}^{\infty} \cos\omega(t-\tau)\cdot\delta(\tau-2nT_2-T_0)\, d\tau$$

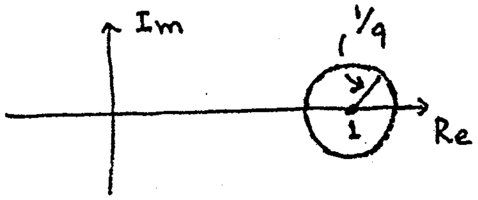
$$= \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n \cos\omega(t-2nT_2-T_0)$$

$$\bar{V}_0(\omega) = \frac{4}{9} \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n e^{-j\omega(2nT_2+T_0)}$$

$$= \frac{4}{9} e^{-j\omega T_0} \sum_{n=0}^{\infty} \left(\frac{1}{9} e^{-j2\omega T_2}\right)^n$$

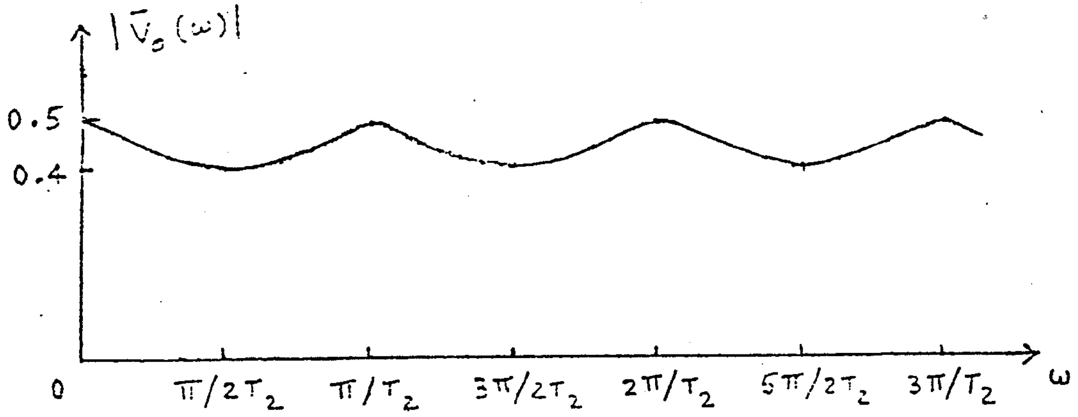
$$= \frac{\frac{4}{9} e^{-j\omega T_0}}{1-\frac{1}{9} e^{-j2\omega T_2}}$$

$$|\bar{V}_0(\omega)| = \frac{4/9}{\left|1-\frac{1}{9} e^{-j2\omega T_2}\right|}$$



$$|\bar{V}_0(\omega)|_{max} = \frac{4/9}{1-1/9} = \frac{4/9}{8/9} = 0.5$$

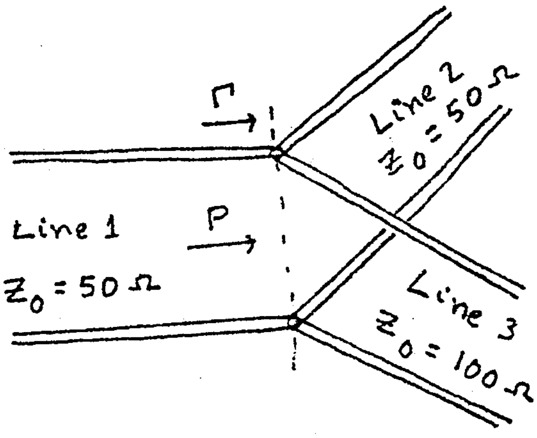
$$|\bar{V}_0(\omega)|_{min} = \frac{4/9}{1+1/9} = \frac{4/9}{10/9} = 0.4$$

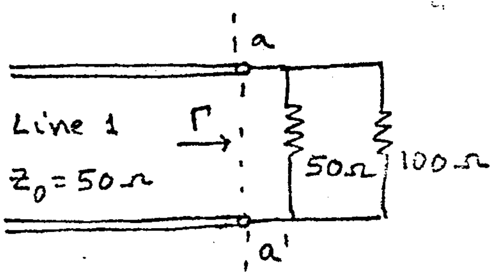


3.3.5. Rough sketch of amplitude response versus frequency for the system of Fig. 3.3.3 for sinusoidal excitation.

150/500 4/151

Junction of three lines :





$$50 \parallel 100 = \frac{50 \times 100}{50 + 100} = \frac{100}{3}$$

$$\Gamma = \frac{\frac{100}{3} - 50}{\frac{100}{3} + 50} = \frac{100 - 150}{100 + 150} = -\frac{50}{250} = -\frac{1}{5}$$

$$\tau_v = 1 + \Gamma = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\tau_c = 1 - \Gamma = 1 - \left(-\frac{1}{5}\right) = \frac{6}{5}$$

$$(\tau_c)_{eff\,2} = \tau_c \cdot \frac{100}{50 + 100} = \frac{2}{3}\tau_c = \frac{2}{3} \cdot \frac{6}{5} = \frac{12}{15}$$

$$(\tau_c)_{eff\,3} = \tau_c \cdot \frac{50}{50 + 100} = \frac{1}{3}\tau_c = \frac{1}{3} \cdot \frac{6}{5} = \frac{6}{15}$$

[illegible]

$$P^- = V^- \cdot I^-$$

$$P_{ref\,1} = (\Gamma V^+)(-\Gamma I^+) = -\Gamma^2 P^+ = \underset{\uparrow}{-}\frac{1}{25} P$$

signifies reflected power

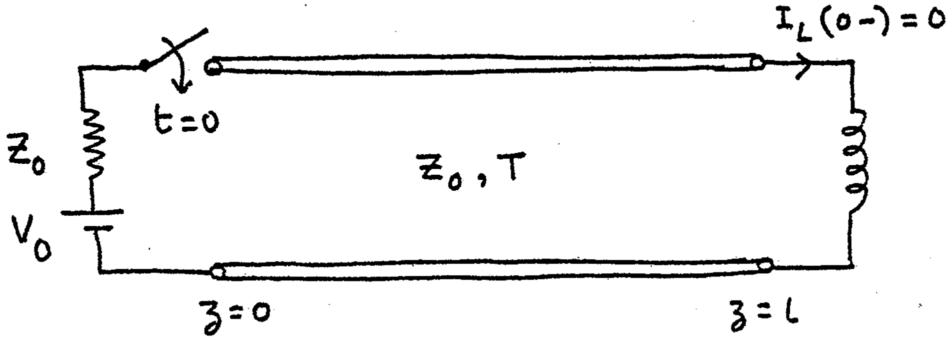
$$P_{trans\,2} = (\tau_v V^+)(\tau_{c\,eff\,2} I^+) = \frac{4}{5} \cdot \frac{12}{15} P = \frac{48}{75} P$$

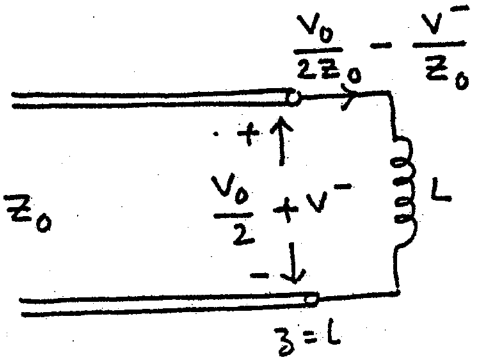
$$P_{trans\,3} = (\tau_v V^+)(\tau_{c\,eff3} I^+) = \frac{4}{5} \cdot \frac{6}{15} P = \frac{24}{75} P$$

Note that $\frac{1}{25} + \frac{48}{75} + \frac{24}{75} = \frac{3 + 48 + 24}{75} = 1.$

See Back

Line Terminated by an Inductor





$$\frac{V_0}{2} + V^- = L \frac{d}{dt}\left(\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right) \qquad \text{B.C.}$$

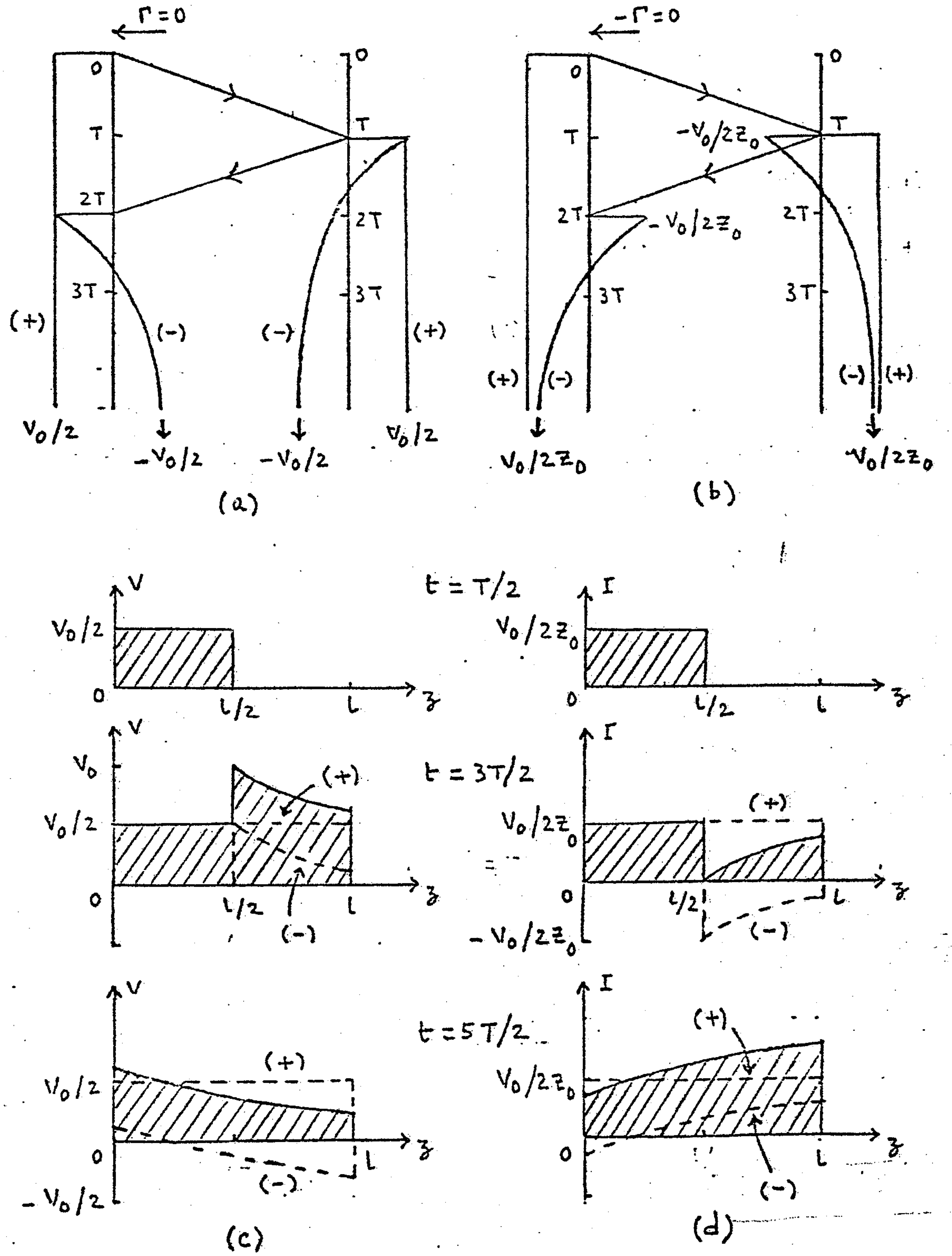
$$\left[\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right]_{t=T} = 0 \longrightarrow [V^-]_{t=T} = \frac{V_0}{2} \qquad \text{I.C.}$$

$$\frac{L}{Z_0}\frac{dV^-}{dt} + V^- = -\frac{V_0}{2}$$

$$V^- = -\frac{V_0}{2} + A e^{-\frac{Z_0}{L}t}$$

$$\frac{V_0}{2} = -\frac{V_0}{2} + A e^{-\frac{Z_0}{L}T} \longrightarrow A = V_0 e^{\frac{Z_0}{L}T}$$

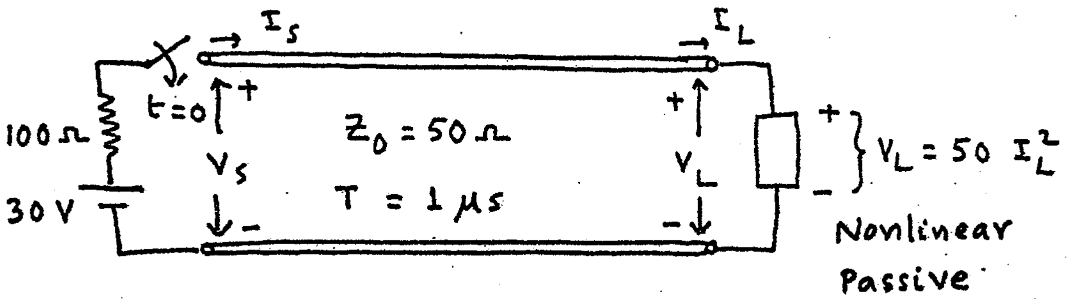
$$\boxed{\begin{aligned} V^-(l,t) &= -\frac{V_0}{2} + V_0 e^{-\frac{Z_0}{L}(t-T)} && t>T \\ I^-(l,t) &= -\frac{V^-(l,t)}{Z_0} = \frac{V_0}{2Z_0} - \frac{V_0}{Z_0} e^{-\frac{Z_0}{L}(t-T)} && t>T \end{aligned}}$$



3.4.3. (a) and (b) Voltage and current bounce diagrams, respectively, for the system of Fig. 3.4.1. (c) and (d) Sketches of line voltage and line current, respectively, versus z for the system of Fig. 3.4.1, for several values of t.

Line terminated by a nonlinear element

Example:



$t=0, z=0$: $30 = 100\, I_S + V_S$ — (1)

$V_S = V^+$

$I_S = \frac{V^+}{Z_0} = \frac{V_S}{50}$ — (2)

Solution point A

$t = 1\mu s, z = l$: $V_L = 50\, I_L^2$ —— (1)

$V_L = V^+ + V^-$

$I_L = \frac{V^+ - V^-}{Z_0} = \frac{V^+ - (V_L - V^+)}{Z_0} = \frac{2V^+ - V_L}{50}$ — (2)

For $V_L = V^+$ in (2), $I_L = \frac{V^+}{50}$

∴ (2) represents a straight line of slope $-\frac{1}{50}$ and passing through point A.

Thus solution is at point B.

$t = 2\mu s, \ z = 0: \quad 30 = 100\, I_s + V_s$ — (1)

$$V_s = v^+ + v^- + v^{-+}$$

$$I_s = \frac{v^+ - v^- + v^{-+}}{z_0}$$

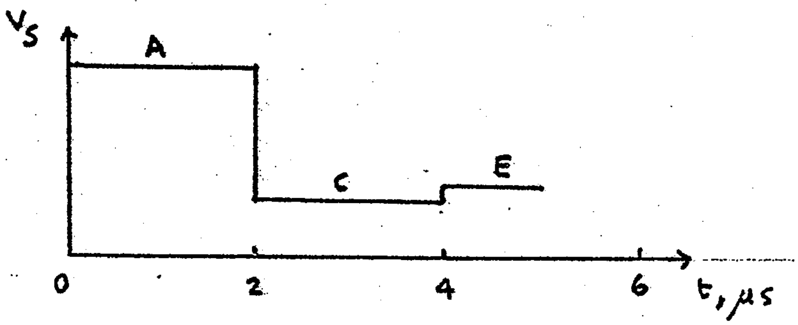
$$= \frac{v^+ - v^- + (V_s - v^+ - v^-)}{z_0} = \frac{-2v^- + V_s}{50} \quad - (2)$$

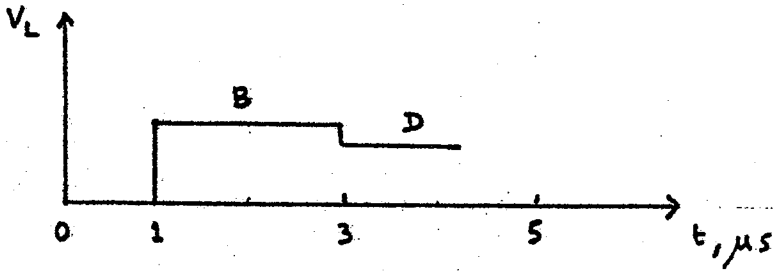
For $V_s = v^+ + v^-$ in (2), $I_s = \dfrac{v^+ - v^-}{50}$.

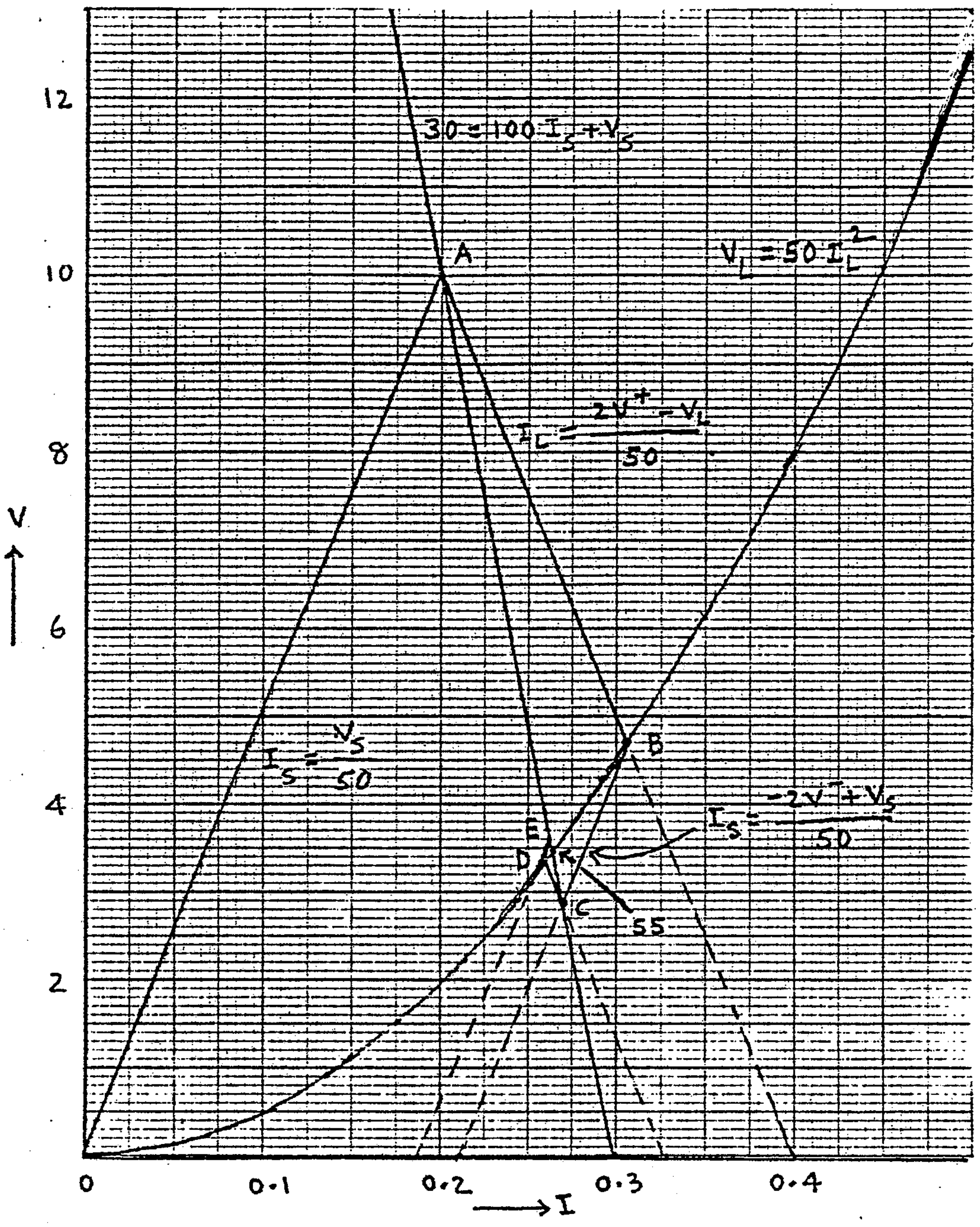
$\therefore$ (2) represents a straight line of slope $\frac{1}{50}$ and passing through point B.

Thus solution is at point C.

and so on . . .

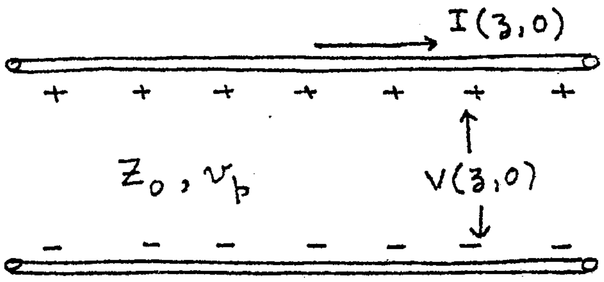






12
10
8
6
4
2
V
0
0.1
0.2
0.3
0.4
I
$30 = 100 I_S + V_S$
A
$V_L = 50 I_L^2$
$I_L = \frac{2V^+ - V_L}{50}$
$I_S = \frac{V_S}{50}$
B
$I_S = \frac{-2V^- + V_S}{50}$
E
D
K
C
55

Line with Initial Conditions



$$V^+(z,0) + V^-(z,0) = V(z,0)$$

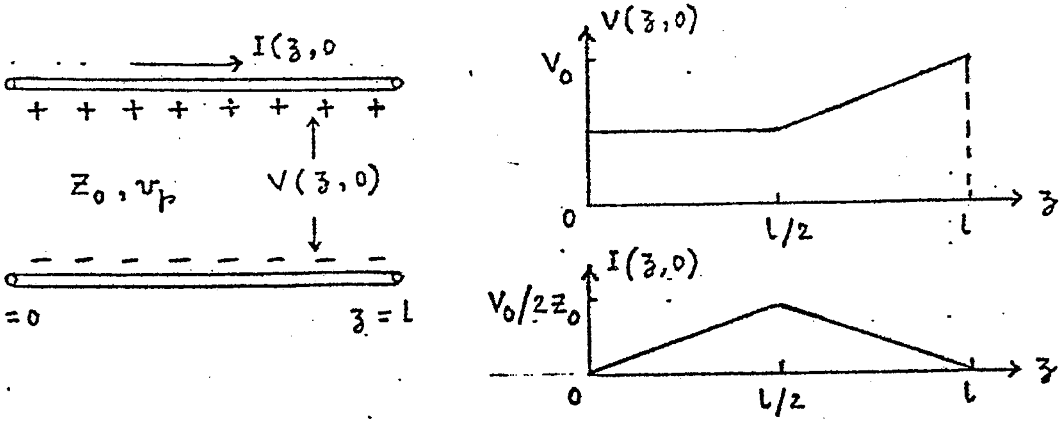
$$I^+(z,0) + I^-(z,0) = I(z,0)$$

$$I^+ = \frac{V^+}{Z_0}, \quad I^- = -\frac{V^-}{Z_0}$$

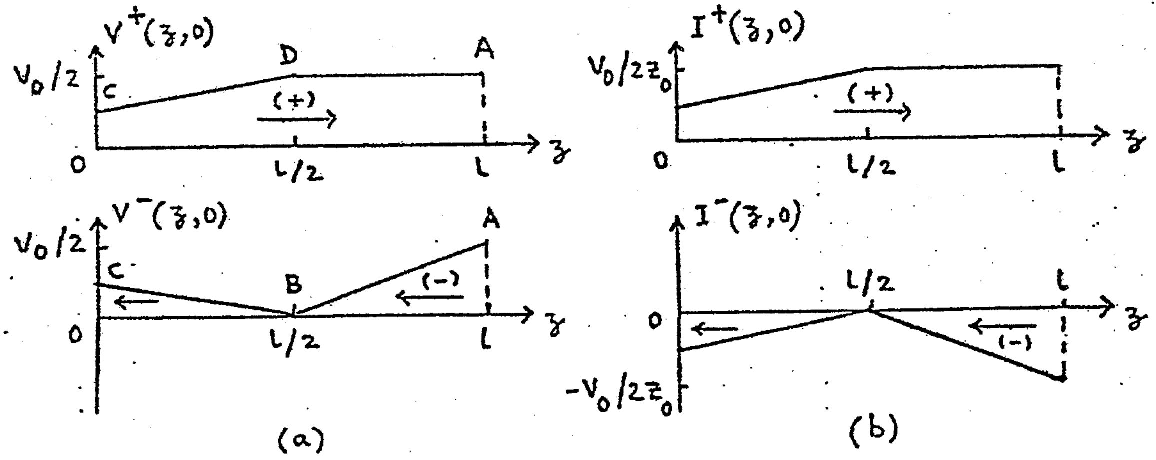
$$V^+(z,0) - V^-(z,0) = Z_0 I(z,0)$$

$$\boxed{\begin{aligned} V^+(z,0) &= \frac{1}{2}\left[V(z,0) + Z_0 I(z,0)\right] \\ V^-(z,0) &= \frac{1}{2}\left[V(z,0) - Z_0 I(z,0)\right] \end{aligned}}$$

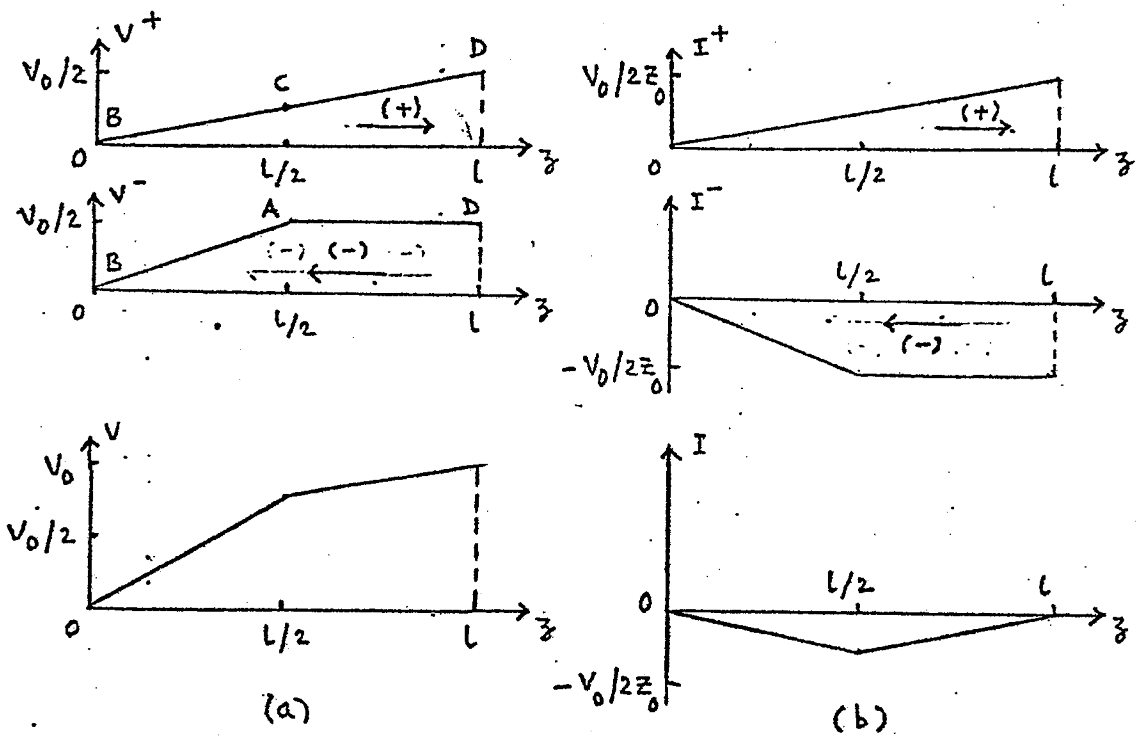
Example



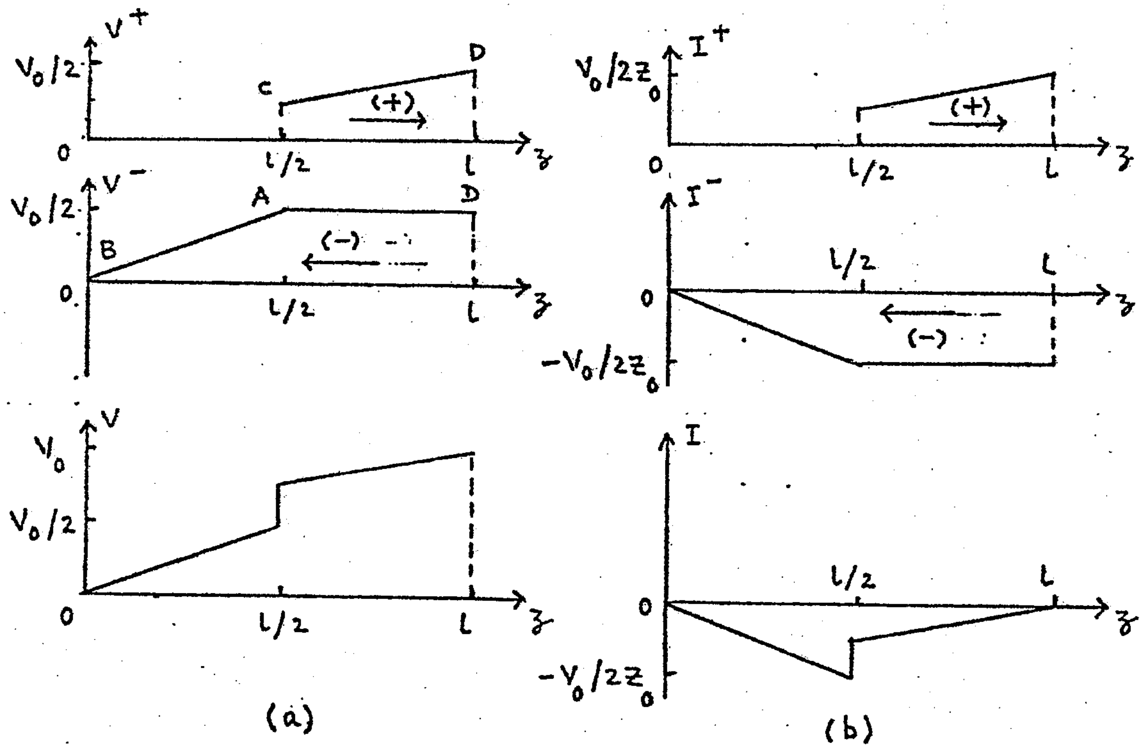
3.5.1. A line open-circuited at both ends and initially charged to the voltage and current distributions V(z,0) and I(z,0), respectively.



3.5.2. Distributions of (a) voltage and (b) current in the (+) and (-) waves obtained by decomposing the voltage and current distributions of Fig. 3.5.1.

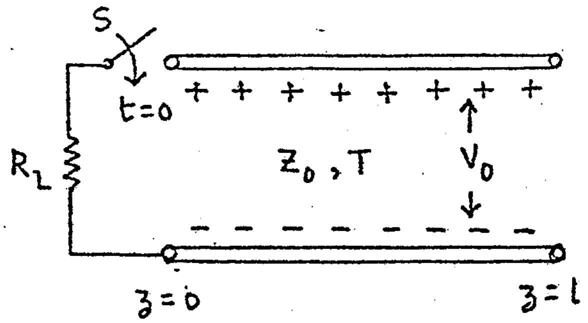


3.5.3. Distributions of (a) voltage and (b) current in the (+) and (−) waves and their sum for $t = l/2v_p$ for the initially charged line of Fig. 3.5.1.

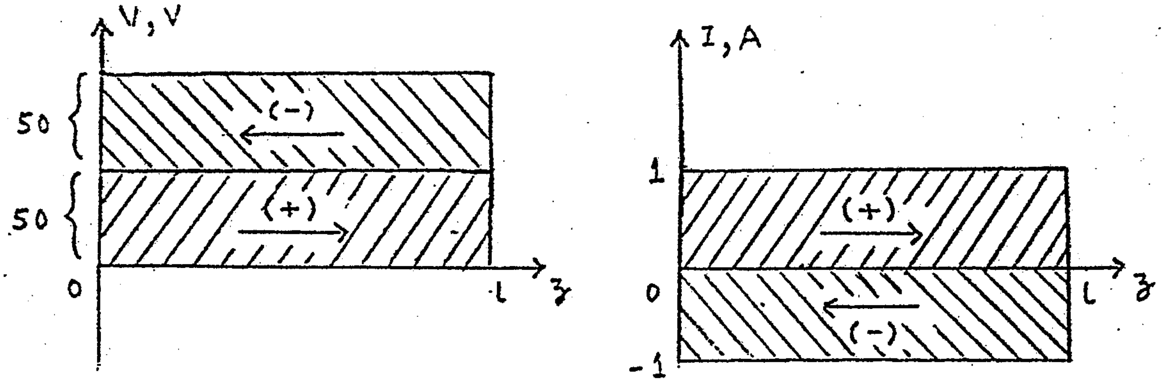


3.5.4. Distributions of (a) voltage and (b) current in the (+) and (−) waves and their sum for $t = l/2v_p$ for the initially charged line of Fig. 3.5.1, with a resistor of value Z_0 connected at the end $z = 0$ at $t = 0$.

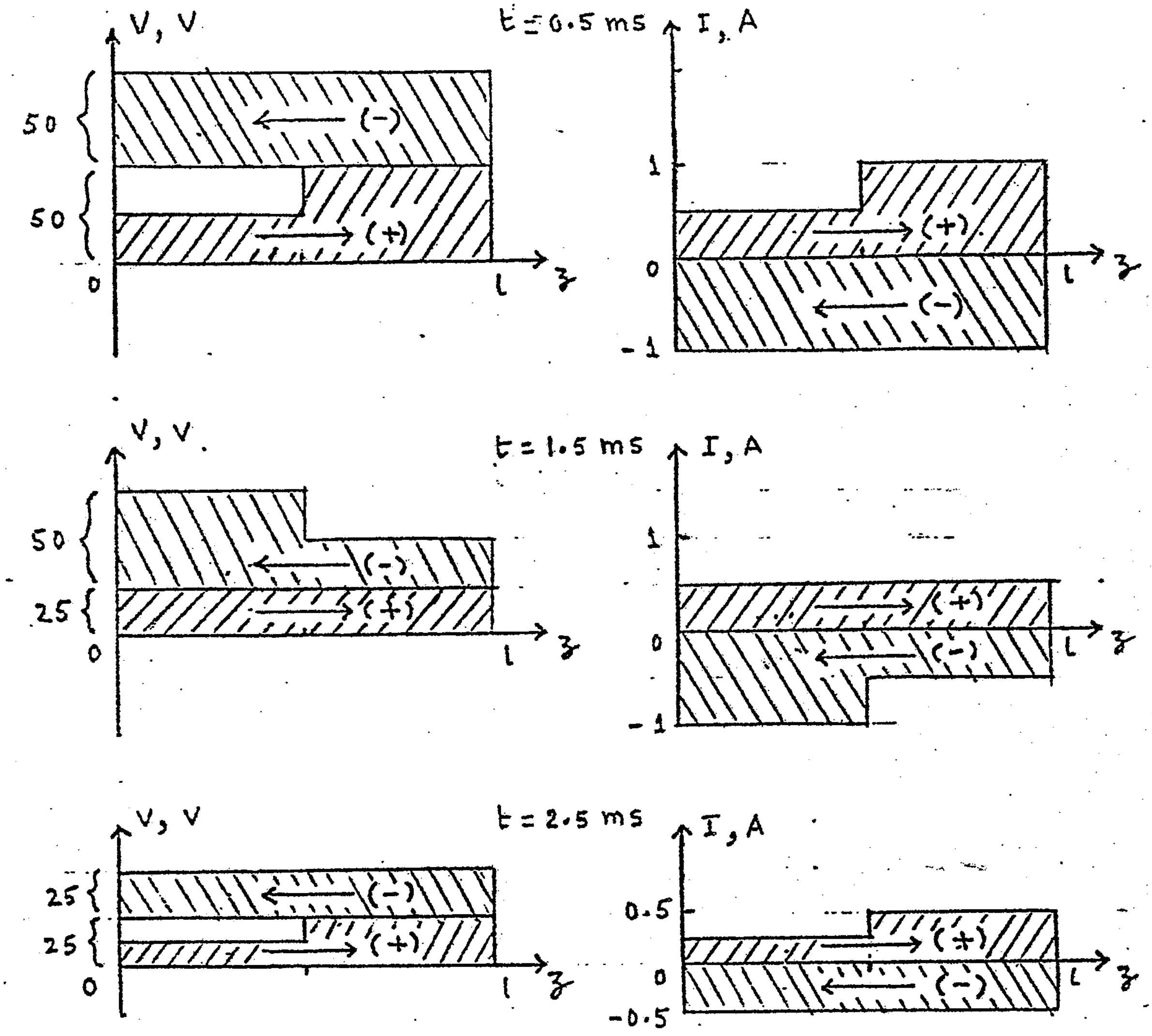
Uniform Distribution



3.5.5. A line charged initially to a uniform voltage V_0 and zero current and across one end of which a resistor is connected at t = 0.

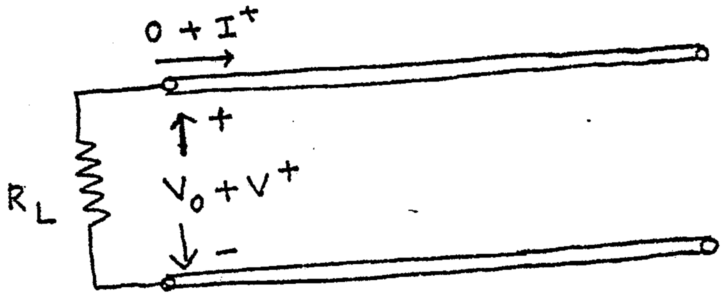


3.5.6. The continuous flow of (+) and (-) waves, representing the situation at t = 0- for the uniformly charged line of Fig. 3.5.5, for V_0 = 100 V and Z_0 = 50 Ω.



3.5.7. Distributions of voltage and current in the (+) and (−) waves for several values of $t > 0$ for the line of Fig. 3.5.5, for $V_0 = 100$ V, $Z_0 = 50\ \Omega$, $R_L = 150\ \Omega$, and $T = 1$ ms.

Bounce Diagram Technique for Uniform Distribution



$$V_0 + V^+ = -R_L\left(0 + I^+\right) \qquad \text{B.C.}$$

$$I^+ = \frac{V^+}{Z_0}$$

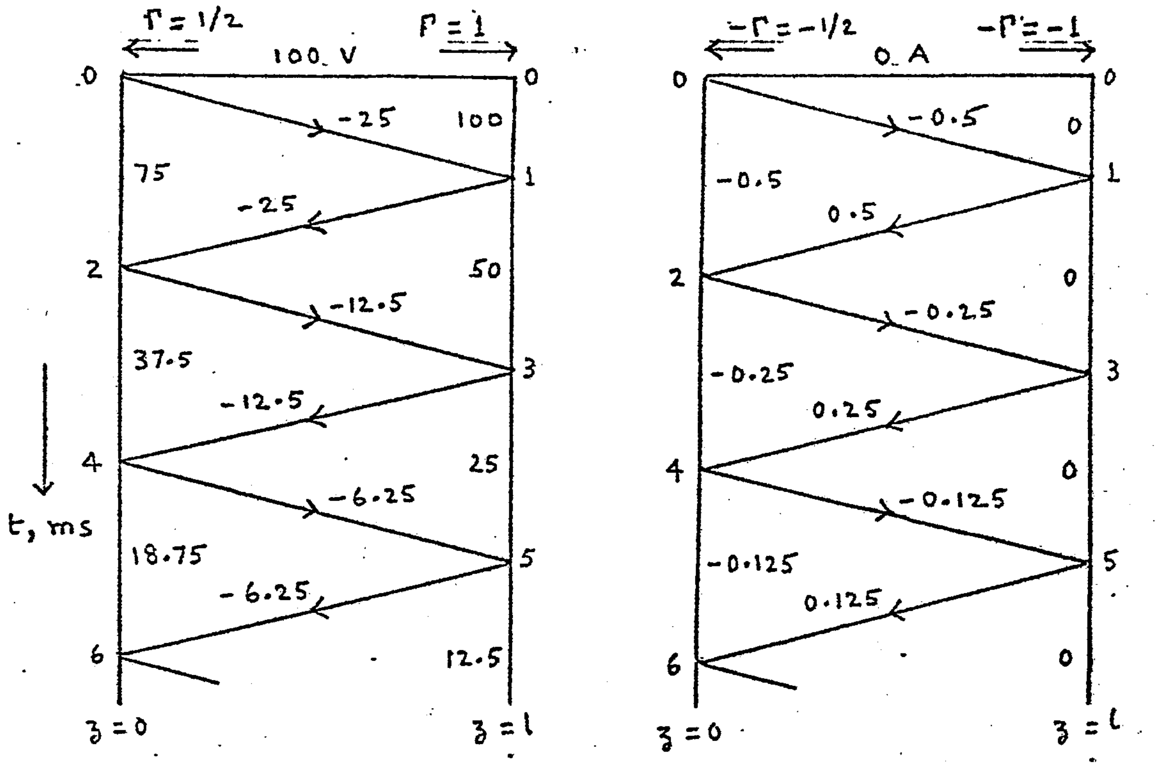
$$V_0 + V^+ = -\frac{R_L}{Z_0}V^+$$

$$V^+\left(1 + \frac{R_L}{Z_0}\right) = -V_0$$

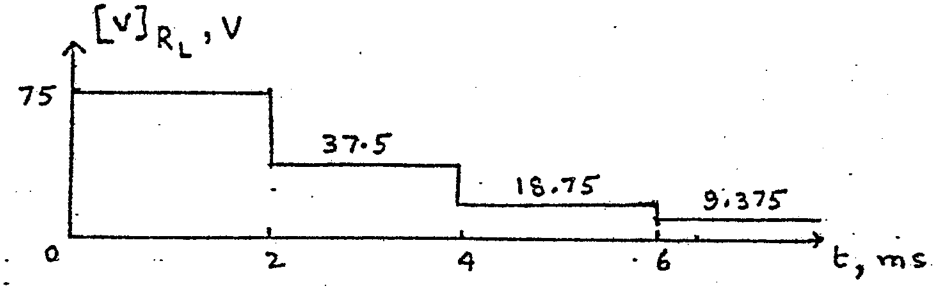
$$V^+ = -V_0\,\frac{Z_0}{R_L + Z_0}$$

For $V_0 = 100\text{V}$, $Z_0 = 50\,\Omega$, $R_L = 150\,\Omega$

$$V^+ = -100\,\frac{50}{150+50} = -25\text{ V}$$

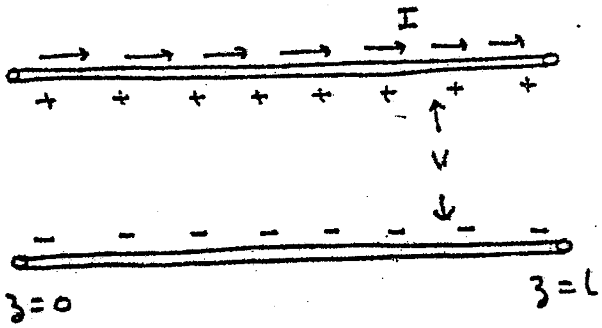


3.5.9. Voltage and current bounce diagrams depicting the transient phenomenon for $t > 0$ for the line of Fig. 3.5.5, for $V_0 = 100$ V, $Z_0 = 50\ \Omega$, $R_L = 150\ \Omega$, and $T = 1$ ms.



3.5.10. Time variation of voltage across R_L for $t > 0$ in Fig. 3.5.5 for $V_0 = 100$ V, $Z_0 = 50\ \Omega$, $R_L = 150\ \Omega$, and $T = 1$ ms.

Energy Storage in Transmission Lines



w_e = Electric stored energy density = $\frac{1}{2}\mathcal{C}V^2$

∴ W_e = Electric stored energy = $\int_{z=0}^{l} \frac{1}{2}\mathcal{C}V^2\,dz$

$$= \frac{1}{2}\mathcal{C}V_0^2\, l \quad \text{(for uniform distribution)}$$

$$= \frac{1}{2}\mathcal{C}V_0^2 v_p T = \frac{1}{2}\mathcal{C}V_0^2 \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} T = \boxed{\frac{1}{2}\frac{V_0^2}{Z_0} T}$$

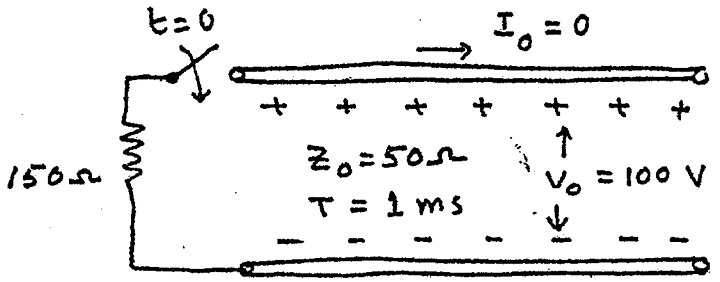
w_m = Magnetic stored energy density = $\frac{1}{2}\mathcal{L}I^2$

∴ W_m = Magnetic stored energy = $\int_{z=0}^{l} \frac{1}{2}\mathcal{L}I^2\,dz$

$$= \frac{1}{2}\mathcal{L}I_0^2\, l \quad \text{(for uniform distribution)}$$

$$= \frac{1}{2}\mathcal{L}I_0^2 v_p T = \frac{1}{2}\mathcal{L}I_0^2 \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} T = \boxed{\frac{1}{2} I_0^2 Z_0 T}$$

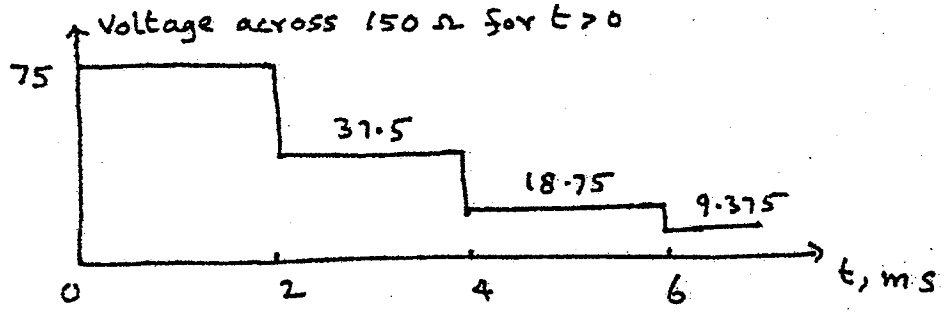
Check of Energy Balance :



$$W_e = \frac{1}{2}\frac{V_0^2}{Z_0}T = \frac{1}{2}\frac{(100)^2}{50} \times 10^{-3} = 0.1 \text{ J}$$

$$W_m = \frac{1}{2} I_0^2 Z_0 T = \frac{1}{2} (0)^2 \times 50 \times 10^{-3} = 0$$

$$\therefore \quad W = W_e + W_m = 0.1 + 0 = 0.1 \text{ J}$$



$$W_d = \int_{t=0}^{\infty} P_d \, dt = \int_{t=0}^{\infty} \frac{(V_{R_L})^2}{R_L} dt$$

$$= \int_0^{2\times10^{-3}} \frac{75^2}{150} dt + \int_{2\times10^{-3}}^{4\times10^{-3}} \frac{37.5^2}{150} dt + \int_{4\times10^{-3}}^{6\times10^{-3}} \frac{18.375^2}{150} dt + \cdots$$

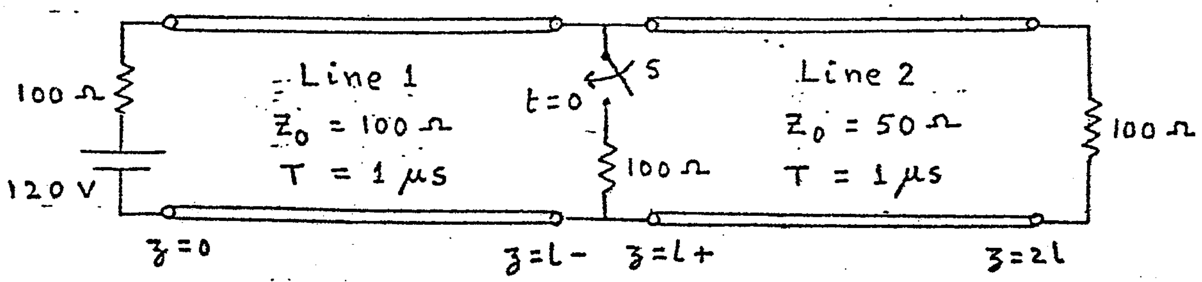
$$= \frac{2\times10^{-3}}{150} \left[75^2 + 37.5^2 + 18.375^2 + \cdots \right]$$

$$= \frac{2\times10^{-3}}{150} \times 75^2 \left(1 + \frac{1}{4} + \frac{1}{16} + \cdots \right)$$

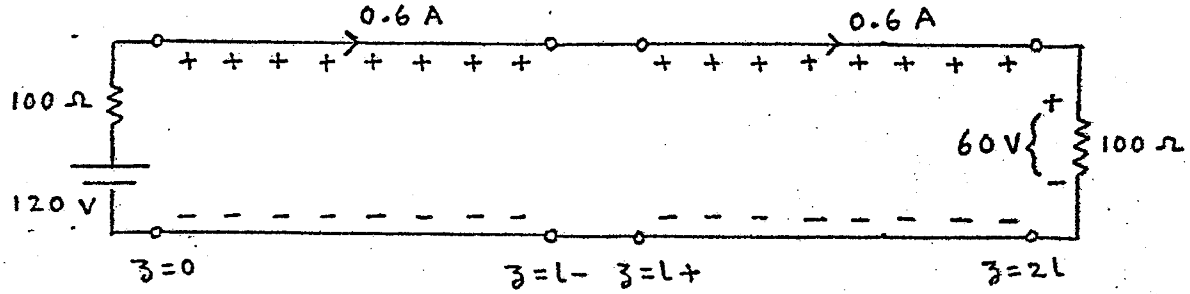
$$= \frac{2\times10^{-3}}{150} \times 75^2 \times \frac{1}{1 - 1/4}$$

$$= \frac{2\times10^{-3}}{150} \times 75^2 \times \frac{4}{3} = 0.1 \text{ J}$$

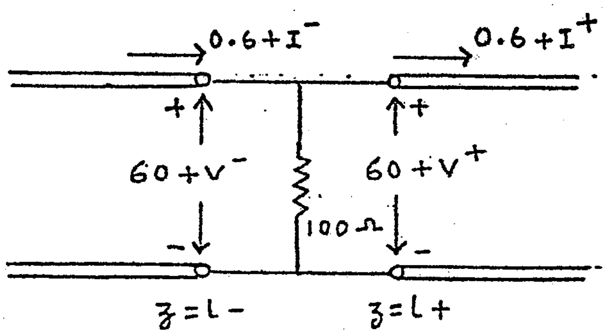
Another example of initially stored line system



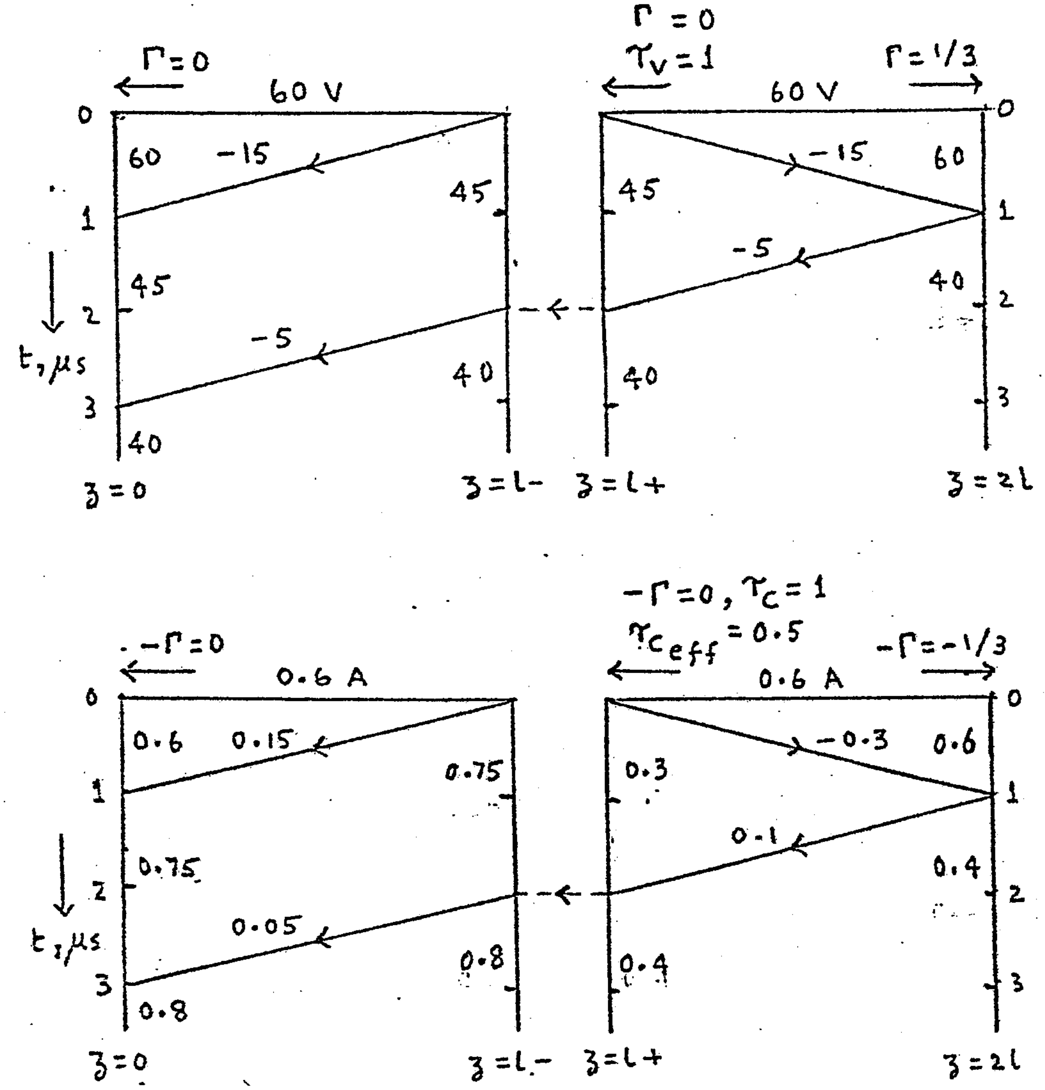
3.5.11. A transmission line system in steady state for t = 0 - and in which a 100 Ω resistor is connected across the junction between the lines at t = 0.



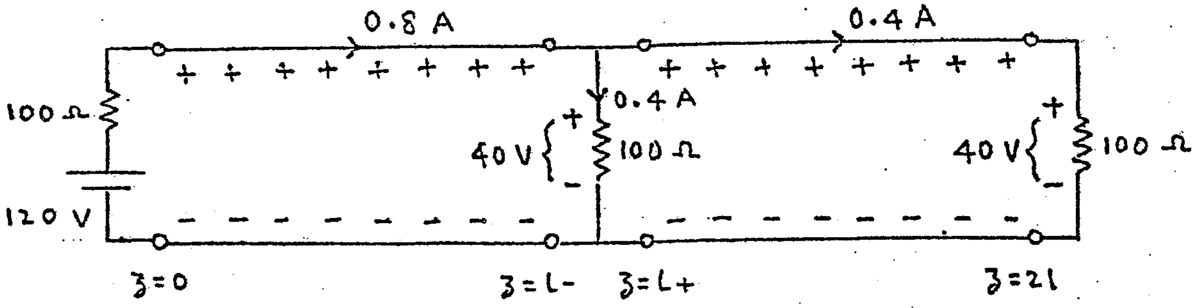
3.5.12. Equivalent circuit for the steady state situation at t = 0 - for the system of Fig. 3.5.11.



3.5.13. For obtaining the (+) and (-) wave voltages and currents resulting from the closure of the switch S in the initially chraged system of Fig. 3.5.11.



3.5.14. Voltage and current bounce diagrams depicting the transient phenomenon for t > 0 for the system of Fig. 3.5.11.



3.5.15. Steady state equivalent circuit for the system of Fig. 3.5.11, with the switch S closed.

FREQUENCY DOMAIN ANALYSIS

(10 - 17)

Frequency Domain Solution for Lossless Line

$$V(z,t) = V^+ \cos\left[\omega\left(t - \frac{z}{v_p}\right) + \theta\right] + V^- \cos\left[\omega\left(t + \frac{z}{v_p}\right) + \phi\right]$$

$$= V^+ \cos\left(\omega t - \omega\sqrt{\mathcal{L}\mathcal{C}}\, z + \theta\right) + V^- \cos\left(\omega t + \omega\sqrt{\mathcal{L}\mathcal{C}}\, z + \phi\right)$$

$$= \mathcal{R}e\left[V^+ e^{j(\omega t - \omega\sqrt{\mathcal{L}\mathcal{C}}\, z + \theta)} + V^- e^{j(\omega t + \omega\sqrt{\mathcal{L}\mathcal{C}}\, z + \phi)}\right]$$

$$= \mathcal{R}e\left[\left(V^+ e^{j\theta} e^{-j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z} + V^- e^{j\phi} e^{j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z}\right) e^{j\omega t}\right]$$

$$= \mathcal{R}e\left[\left(\bar{V}^+ e^{-j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z} + \bar{V}^- e^{j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z}\right) e^{j\omega t}\right]$$

$$\bar{V}(z) = \bar{V}^+ e^{-j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z} + \bar{V}^- e^{j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z}$$

$$I(z,t) = \frac{1}{Z_0}\left\{V^+ \cos\left[\omega\left(t - \frac{z}{v_p}\right) + \theta\right] - V^- \cos\left[\omega\left(t + \frac{z}{v_p}\right) + \phi\right]\right\}$$

$$= \mathcal{R}e\left[\frac{1}{Z_0}\left(\bar{V}^+ e^{-j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z} - \bar{V}^- e^{j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z}\right) e^{j\omega t}\right]$$

$$\bar{I}(z) = \frac{1}{Z_0}\left(\bar{V}^+ e^{-j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z} - \bar{V}^- e^{j\omega\sqrt{\mathcal{L}\mathcal{C}}\, z}\right)$$

$$\boxed{\begin{aligned} \bar{V}(z) &= \bar{V}^+ e^{-j\beta z} + \bar{V}^- e^{j\beta z} \\ \bar{I}(z) &= \frac{1}{Z_0}\left(\bar{V}^+ e^{-j\beta z} - \bar{V}^- e^{j\beta z}\right) \end{aligned}}$$

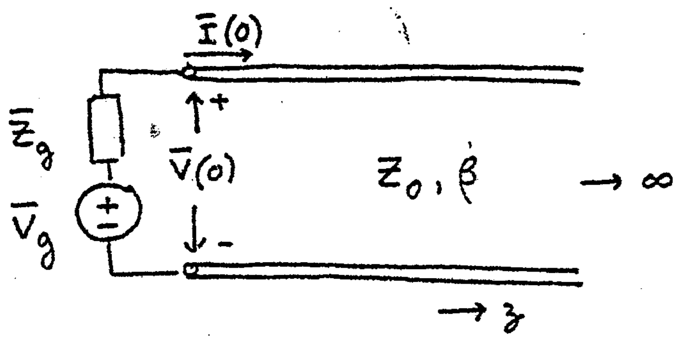
General solution

$$\beta = \omega\sqrt{\mathcal{L}\mathcal{C}} = \frac{\omega}{v_p} = \text{phase constant}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega/v_p} = \frac{v_p}{f}$$

$$v_p = \lambda f$$

The Semi-infinite Line



$$\boxed{V^- \equiv 0}$$

$$\bar{V}(z) = \bar{V}^+ e^{-j\beta z}$$

$$\bar{I}(z) = \frac{\bar{V}^+}{Z_0} e^{-j\beta z}$$

$$\bar{V}(0) = \bar{V}_g - \bar{Z}_g \bar{I}(0) \qquad \text{B.C.}$$

$$\bar{V}^+ e^{-j0} = \bar{V}_g - \bar{Z}_g \frac{\bar{V}^+}{Z_0} e^{-j0}$$

$$\bar{V}^+ = \bar{V}_g - \bar{Z}_g \frac{\bar{V}^+}{Z_0} \longrightarrow \bar{V}^+\left(1 + \frac{\bar{Z}_g}{Z_0}\right) = \bar{V}_g$$

$$\bar{V}^+ = \frac{Z_0}{\bar{Z}_g + Z_0} \bar{V}_g$$

$$|\bar{V}(z)| = |\bar{V}^+| \, |e^{-j\beta z}| = |\bar{V}^+|$$

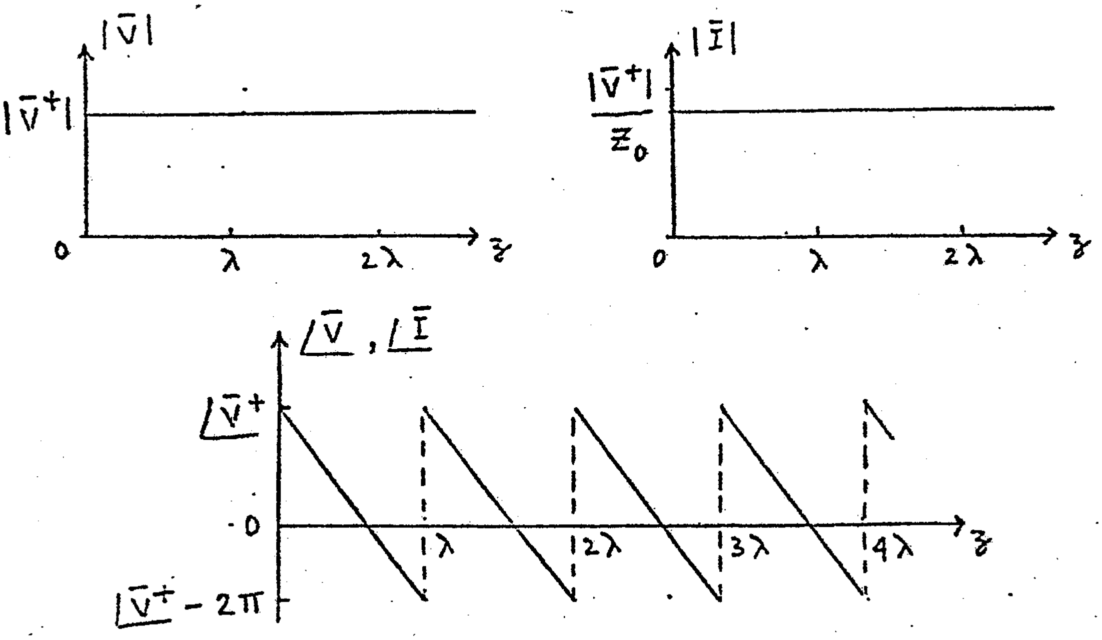
$$|\bar{I}(z)| = \frac{|\bar{V}^+|}{Z_0} \, |e^{-j\beta z}| = \frac{|\bar{V}^+|}{Z_0}$$

$$\angle \bar{V}(z) = \angle \bar{I}(z) = \angle \bar{V}^+ - \beta z = \angle \bar{V}^+ - \frac{2\pi}{\lambda} z$$

$$\langle P \rangle = \frac{1}{2} \operatorname{Re}\left[\bar{V}(z)\, \bar{I}^*(z)\right]$$

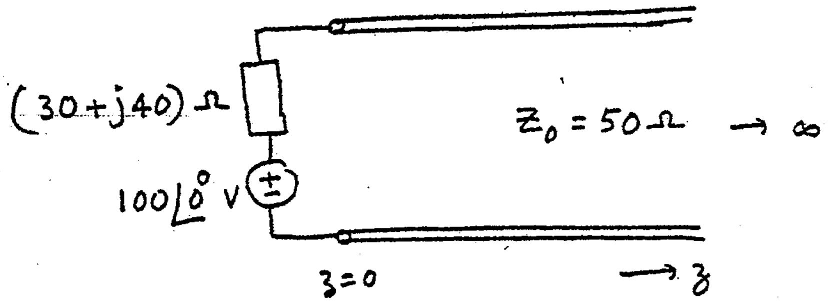
$$= \frac{1}{2} \operatorname{Re}\left[\bar{V}^+ e^{-j\beta z} \cdot \frac{\bar{V}^{+*}}{Z_0} e^{j\beta z}\right]$$

$$= \frac{1}{2} \frac{|\bar{V}^+|^2}{Z_0}.$$



4.1.2. Variations with z of the line voltage and current amplitudes ($|\bar{V}|$ and $|\bar{I}|$) and their phase ($\underline{/\bar{V}}$ and $\underline{/\bar{I}}$) for the semi-infinitely long line.

Example



$$\bar{V}^+ = \frac{50}{(30+j40)+50}\,100\angle 0^\circ = \frac{50\angle 0^\circ}{80+j40}\,100\angle 0^\circ$$

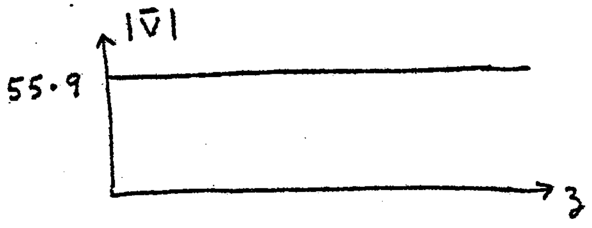
$$= \frac{50\angle 0^\circ}{89.44\angle 26.57^\circ}\,100\angle 0^\circ = \frac{5000\angle 0^\circ}{89.44\angle 26.57^\circ}$$

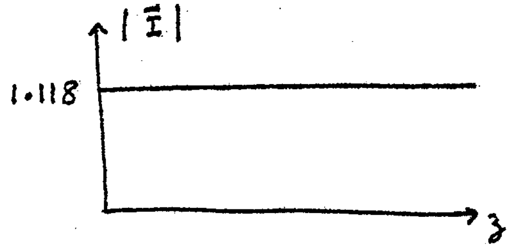
$$= 55.9\angle -26.57^\circ = 55.9\,e^{-j0.148\pi}$$

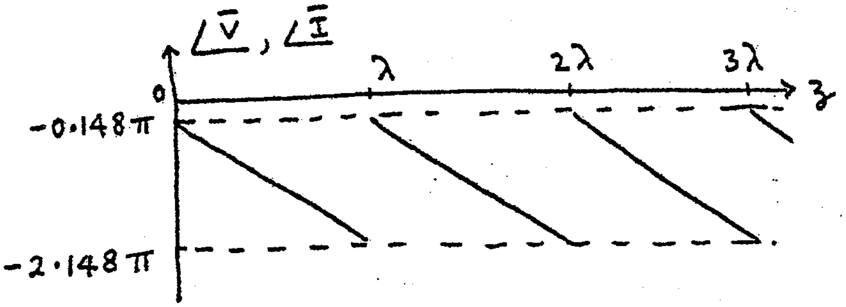
$$\therefore \bar{V}(z) = 55.9\,e^{-j0.148\pi}\,e^{-j\beta z} = 55.9\,e^{-j(\beta z + 0.148\pi)}\ \text{V}$$

$$\bar{I}(z) = \frac{55.9}{50}\,e^{-j0.148\pi}\,e^{-j\beta z} = 1.118\,e^{-j(\beta z + 0.148\pi)}\ \text{A}$$

$$\langle P\rangle = \frac{1}{2}\,\frac{55.9^2}{50} = 31.25\ \text{W}.$$







Nonuniform Line

$$\frac{\partial \bar{V}}{\partial z} = -j\omega \mathcal{L} \bar{I} \qquad \frac{\partial \bar{I}}{\partial z} = -j\omega \mathcal{C} \bar{V}$$

$$\frac{\partial^2 \bar{V}}{\partial z^2} = -j\omega \mathcal{L} \frac{\partial \bar{I}}{\partial z} - j\omega \bar{I} \frac{\partial \mathcal{L}}{\partial z}$$

$$= -j\omega \mathcal{L}(-j\omega \mathcal{C} \bar{V}) - j\omega\left(-\frac{1}{j\omega \mathcal{L}} \frac{\partial \bar{V}}{\partial z}\right)\frac{\partial \mathcal{L}}{\partial z}$$

$$\boxed{\frac{\partial^2 \bar{V}}{\partial z^2} - \frac{1}{\mathcal{L}} \frac{\partial \mathcal{L}}{\partial z} \frac{\partial \bar{V}}{\partial z} + \omega^2 \mathcal{L} \mathcal{C} \bar{V} = 0}$$

Specific Example: $\mathcal{L}(z) = \mathcal{L}_0 e^{-az}$, $\mathcal{C}(z) = \mathcal{C}_0 e^{az}$

$$\frac{\partial^2 \bar{V}}{\partial z^2} - \frac{1}{\mathcal{L}_0 e^{-az}}\left(-a \mathcal{L}_0 e^{-az}\right)\frac{\partial \bar{V}}{\partial z} + \omega^2 \mathcal{L}_0 e^{-az} \mathcal{C}_0 e^{az} \bar{V} = 0$$

$$\boxed{\frac{\partial^2 \bar{V}}{\partial z^2} + a\frac{\partial \bar{V}}{\partial z} + \omega^2 \mathcal{L}_0 \mathcal{C}_0 \bar{V} = 0}$$

Let $\bar{V} = A e^{sz}$

Then $s^2 + as + \omega^2 \mathcal{L}_0 \mathcal{C}_0 = 0$

$$s = \frac{-a \pm \sqrt{a^2 - 4\omega^2 \mathcal{L}_0 \mathcal{C}_0}}{2}$$

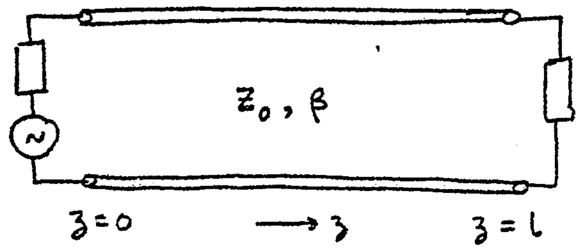
For propagation to occur, s must have an imaginary part, that is, $(a^2 - 4\omega^2 \mathcal{L}_0 \mathcal{C}_0)$ must be < 0, or

$$4\omega^2 \mathcal{L}_0 \mathcal{C}_0 > a^2$$

$$\omega > \frac{a}{2\sqrt{\mathcal{L}_0 \mathcal{C}_0}}$$

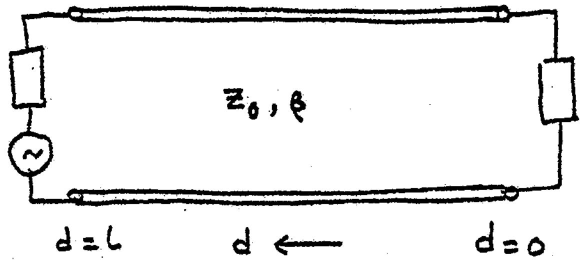
Thus there exists a cutoff frequency $f_c = \frac{a}{4\pi\sqrt{\mathcal{L}_0 \mathcal{C}_0}}$ below which propagation does not occur.

Change of Variable from z to d



$$\bar{V}(z) = \bar{V}^+ e^{-j\beta z} + \bar{V}^- e^{j\beta z}$$

$$\bar{I}(z) = \frac{1}{Z_0}\left(\underbrace{\bar{V}^+ e^{-j\beta z}}_{(+)\text{ wave}} - \underbrace{\bar{V}^- e^{j\beta z}}_{(-)\text{ wave}}\right)$$



$d = l - z$

$z = l - d$

$$\bar{V}(d) = \bar{V}^+ e^{-j\beta(l-d)} + \bar{V}^- e^{j\beta(l-d)}$$

$$= \bar{V}^+ e^{-j\beta l} e^{j\beta d} + \bar{V}^- e^{j\beta l} e^{-j\beta d}$$

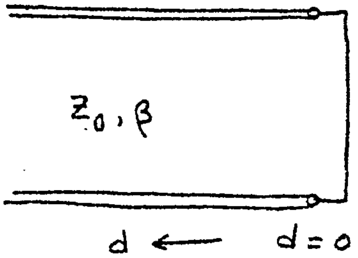
$$\bar{I}(d) = \frac{1}{Z_0}\left(\underbrace{\bar{V}^+ e^{-j\beta l}}_{\text{New } \bar{V}^+} e^{j\beta d} - \underbrace{\bar{V}^- e^{j\beta l}}_{\text{New } \bar{V}^-} e^{-j\beta d}\right)$$

$$\boxed{\begin{aligned} \bar{V}(d) &= \bar{V}^+ e^{j\beta d} + \bar{V}^- e^{-j\beta d} \\ \bar{I}(d) &= \frac{1}{Z_0}\left(\underbrace{\bar{V}^+ e^{j\beta d}}_{(+)\text{ wave}} - \underbrace{\bar{V}^- e^{-j\beta d}}_{(-)\text{ wave}}\right) \end{aligned}}$$

Short - Circuited Line

$$\bar{V}(d) = \bar{V}^+ e^{j\beta d} + \bar{V}^- e^{-j\beta d}$$

$$\bar{I}(d) = \frac{1}{Z_0}\left(\bar{V}^+ e^{j\beta d} - \bar{V}^- e^{-j\beta d}\right)$$



$$\boxed{\bar{V}(0) = 0} \quad \text{B.C.}$$

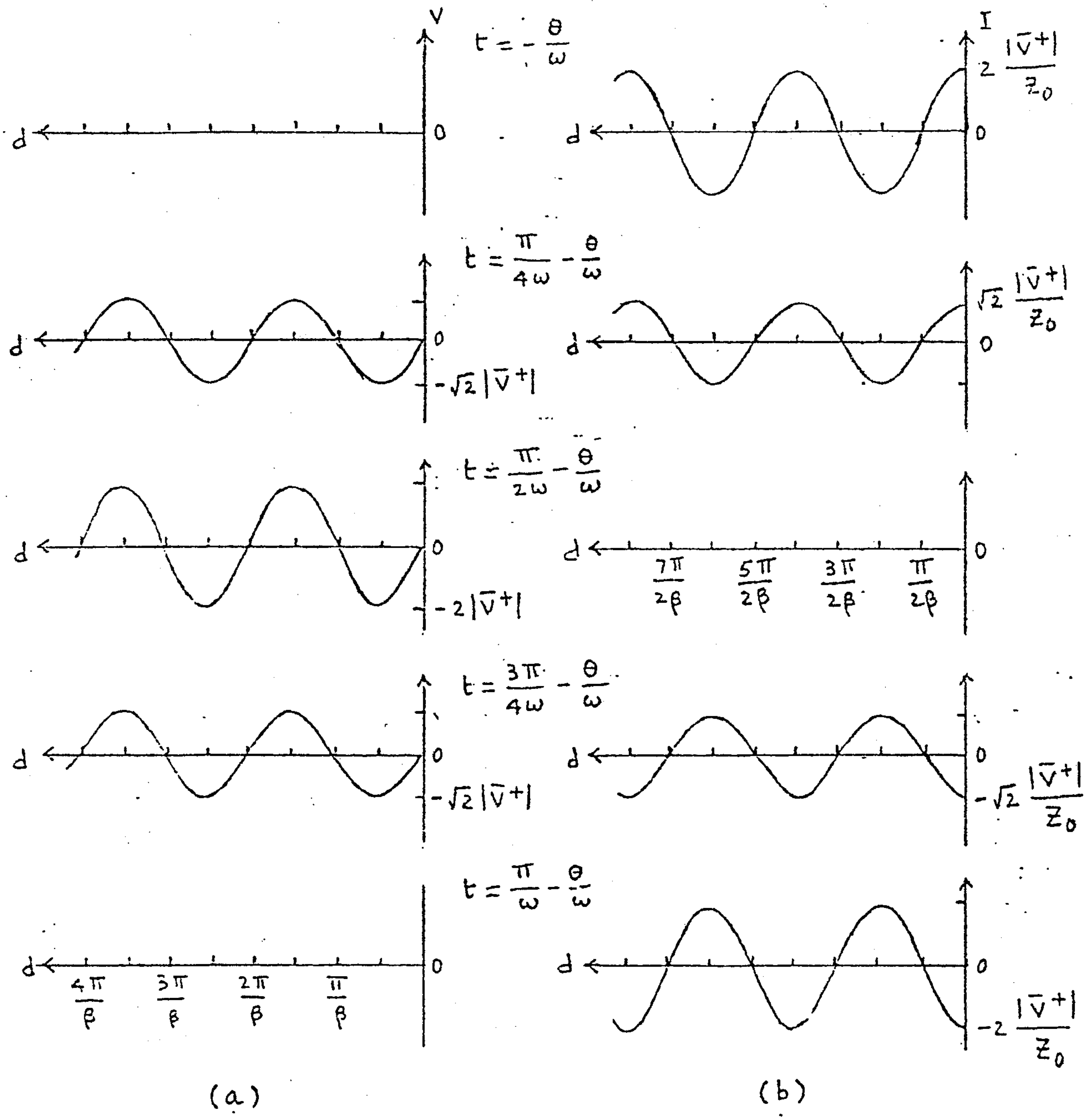
$$0 = \bar{V}^+ + \bar{V}^- \longrightarrow \bar{V}^- = -\bar{V}^+$$

$$\bar{V}(d) = \bar{V}^+ e^{j\beta d} - \bar{V}^+ e^{-j\beta d} = 2j\,\bar{V}^+ \sin\beta d$$

$$\bar{I}(d) = \frac{1}{Z_0}\left(\bar{V}^+ e^{j\beta d} + \bar{V}^+ e^{-j\beta d}\right) = \frac{2\bar{V}^+}{Z_0}\cos\beta d$$

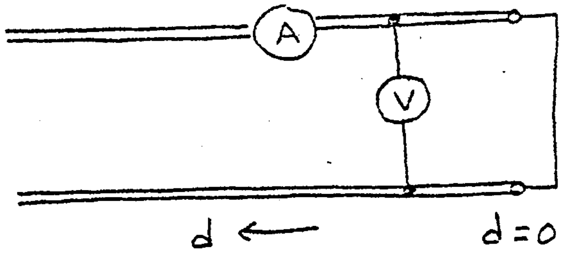
$$\begin{aligned} V(d,t) &= \text{Re}\left[\bar{V}(d)\cdot e^{j\omega t}\right] \\ &= \text{Re}\left[2j\,\bar{V}^+ \sin\beta d \cdot e^{j\omega t}\right] \\ &= \text{Re}\left[2\,e^{j\pi/2}\,|\bar{V}^+|\,e^{j\theta}\,e^{j\omega t}\sin\beta d\right] \\ &= 2\,|\bar{V}^+| \sin\beta d\,\cos\left(\omega t + \theta + \frac{\pi}{2}\right) \\ &= -2|\bar{V}^+| \sin\beta d\,\sin(\omega t + \theta) \end{aligned}$$

$$\begin{aligned} I(d,t) &= \text{Re}\left[\bar{I}(d)\cdot e^{j\omega t}\right] \\ &= \text{Re}\left[\frac{2}{Z_0}|\bar{V}^+|\,e^{j\theta}\cos\beta d \cdot e^{j\omega t}\right] \\ &= \frac{2\,|\bar{V}^+|}{Z_0}\cos\beta d\,\cos(\omega t + \theta) \end{aligned}$$



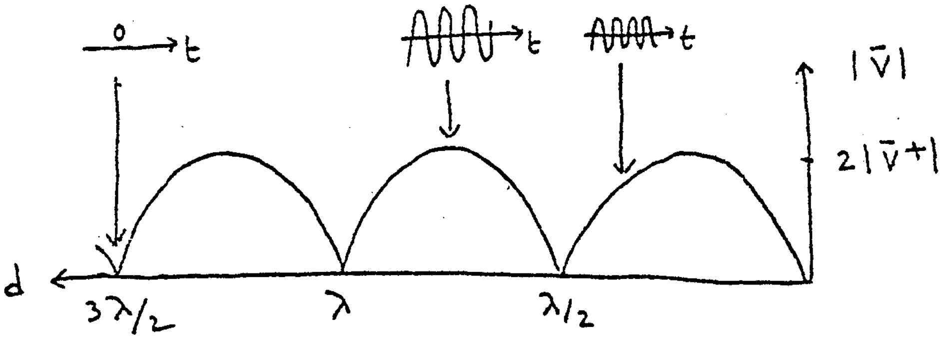
4.2.2. Sketches of (a) instantaneous line voltage, and (b) instantaneous line current, versus d for several values of t for the short-circuited line.

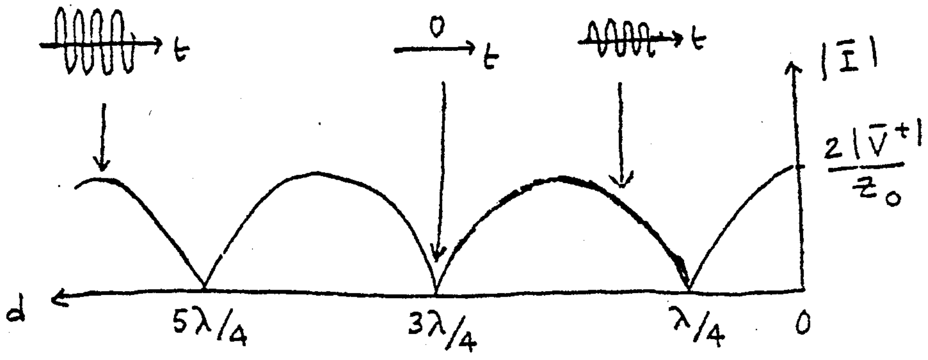
Standing Wave Patterns



$$|\bar{V}(d)| = 2\,|\bar{V}^{+}| \cdot |\sin\beta d|$$

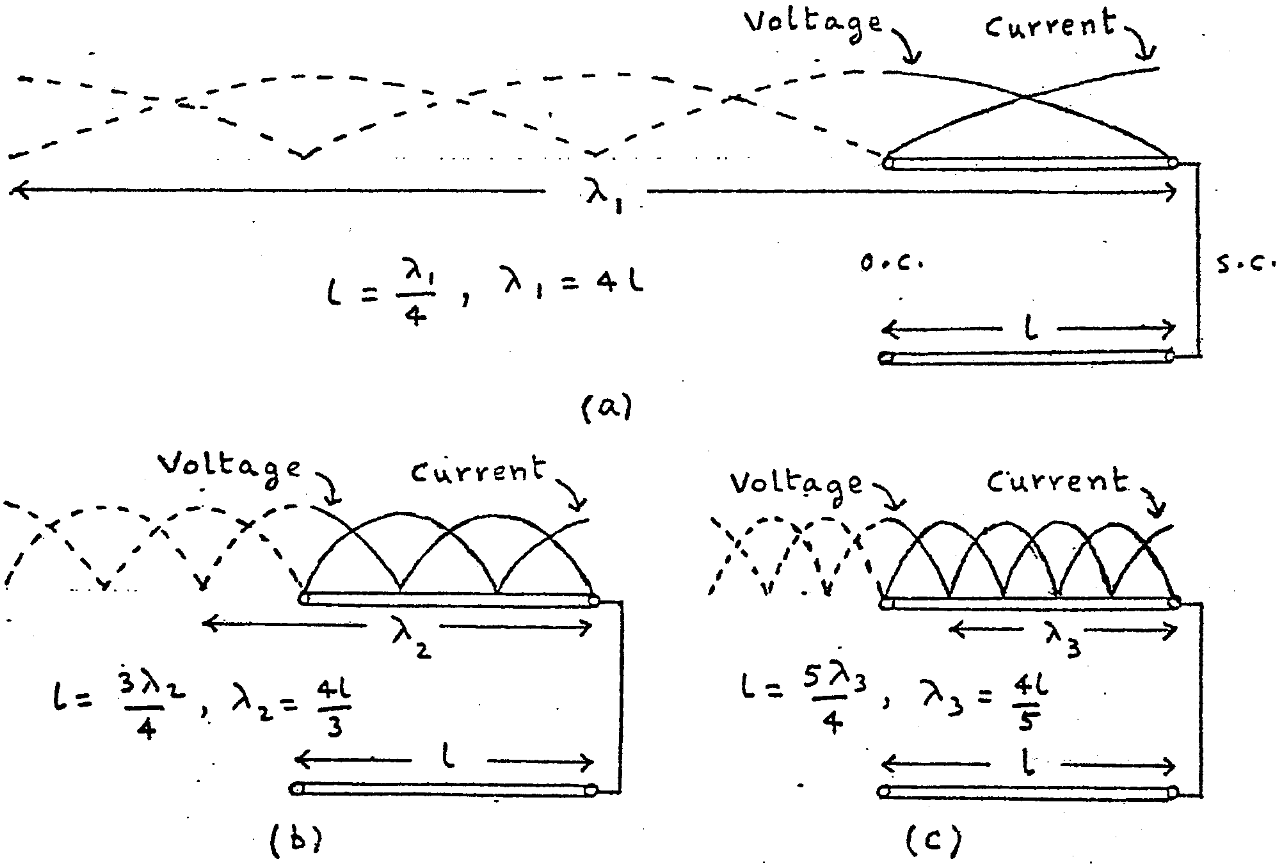
$$|\bar{I}(d)| = \frac{2\,|\bar{V}^{+}|}{Z_0} \cdot |\cos\beta d|$$





Natural Oscillations

Line short-circuited at one end and open-circuited at the other end:



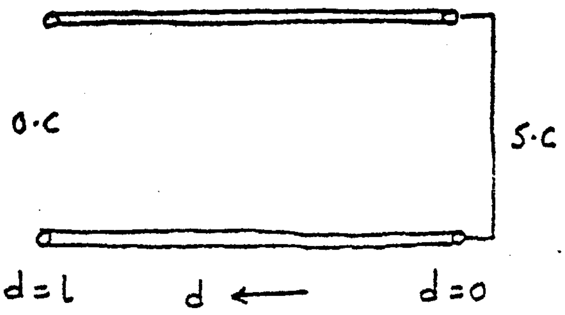
4.2.4. Standing wave patterns corresponding to (a) 1/4 cycle, (b) 3/4 cycle, and (c) 5/4 cycle, of a sine wave for the voltage and current amplitude distributions for a line of length l open-circuited at one end and short-circuited at the other end.

$$f_n = \frac{v_p}{\lambda_n} = \frac{v_p(2n-1)}{4l} \quad n=1,2,3$$

In general,

$$l = \frac{(2n-1)\lambda_n}{4}, \quad n = 1, 2, 3, \ldots$$

$$\therefore \lambda_n = \frac{4l}{(2n-1)}, \quad n = 1, 2, 3, \ldots$$



$$\bar{V}(d) = \bar{V}^+ e^{j\beta d} + \bar{V}^- e^{-j\beta d}$$

$$\bar{I}(d) = \frac{1}{Z_0}\left(\bar{V}^+ e^{j\beta d} - \bar{V}^- e^{-j\beta d}\right)$$

$$\left.\begin{array}{ll} (1) & \bar{V}(0) = 0 \\ (2) & \bar{I}(l) = 0 \end{array}\right\} \text{B.C.}$$

$$(1) \rightarrow \quad 0 = \bar{V}^+ + \bar{V}^- \quad \rightarrow \quad \bar{V}^- = -\bar{V}^+$$

$$(2) \rightarrow \quad 0 = \frac{1}{Z_0}\left(\bar{V}^+ e^{j\beta l} + \bar{V}^+ e^{-j\beta l}\right)$$

$$e^{j\beta l} + e^{-j\beta l} = 0$$

$$e^{j2\beta l} = -1$$

$$2\beta l = \pi, 3\pi, 5\pi, \dots$$

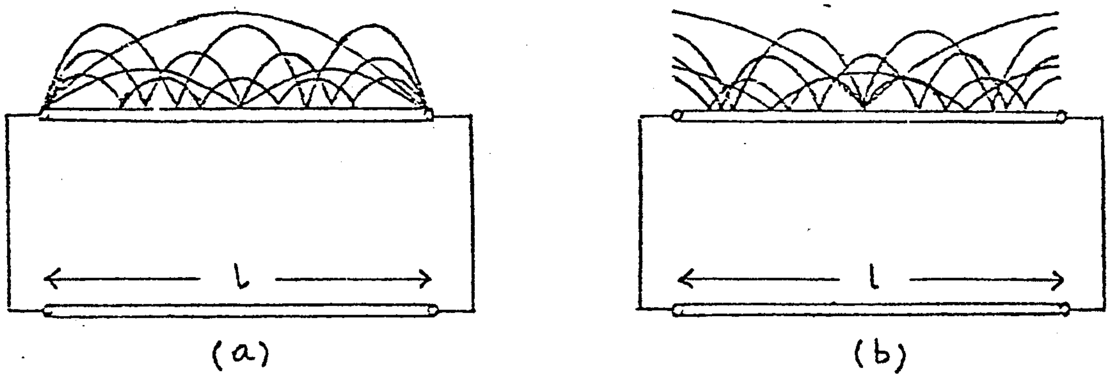
$$l = \frac{\pi}{2\beta}, \frac{3\pi}{2\beta}, \frac{5\pi}{2\beta}, \dots$$

$$= \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

$$= \frac{n\lambda}{4}, \quad n = 1, 3, 5, \dots$$

$$\lambda_n = \frac{4l}{n}, \quad n = 1, 3, 5, \dots$$

Line short-circuited at both ends:

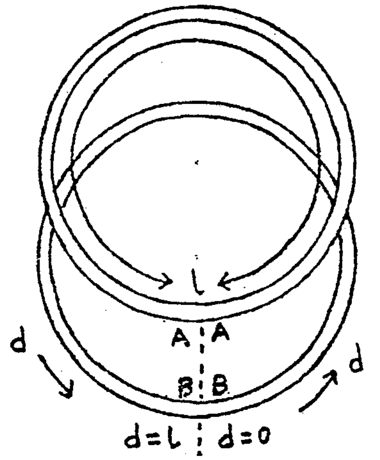


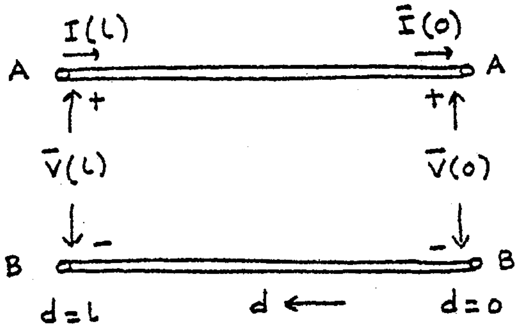
4.2.5. Standing wave patterns for (a) voltage, and (b) current, corresponding to the natural modes of oscillation for a line short-circuited at both ends.

$$l = \frac{n\lambda_n}{2}, \quad n = 1, 2, 3, \ldots$$

$$\therefore \lambda_n = \frac{2l}{n}, \quad n = 1, 2, 3, \ldots$$

Ring Transmission Line





$$\boxed{\left.\begin{array}{l}\bar{V}(0) = \bar{V}(l)\\ \bar{I}(0) = \bar{I}(l)\end{array}\right\} \text{B.C.}}$$

$$\bar{V}(d) = \bar{V}^+ e^{j\beta d} + \bar{V}^- e^{-j\beta d}$$

$$\bar{I}(d) = \frac{1}{Z_0}\left(\bar{V}^+ e^{j\beta d} - \bar{V}^- e^{-j\beta d}\right)$$

$$\bar{V}^+ e^{j\beta(0)} + \bar{V}^- e^{-j\beta(0)} = \bar{V}^+ e^{j\beta(l)} + \bar{V}^- e^{-j\beta(l)}$$

$$\frac{1}{Z_0}\left[\bar{V}^+ e^{j\beta(0)} - \bar{V}^- e^{-j\beta(0)}\right] = \frac{1}{Z_0}\left[\bar{V}^+ e^{j\beta(l)} - \bar{V}^- e^{-j\beta(l)}\right]$$

$$\bar{V}^+(1 - e^{j\beta l}) = \bar{V}^-(e^{-j\beta l} - 1)$$

$$\bar{V}^+(1 - e^{j\beta l}) = \bar{V}^-(1 - e^{-j\beta l})$$

$$e^{-j\beta l} - 1 = 1 - e^{-j\beta l}$$

$$2e^{-j\beta l} = 2$$

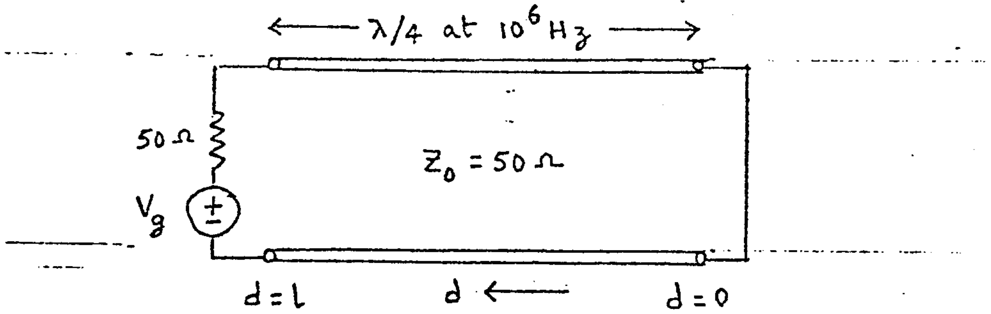
$$e^{-j\beta l} = 1$$

$$\beta l = 2n\pi, \quad n = 1,2,3,\ldots$$

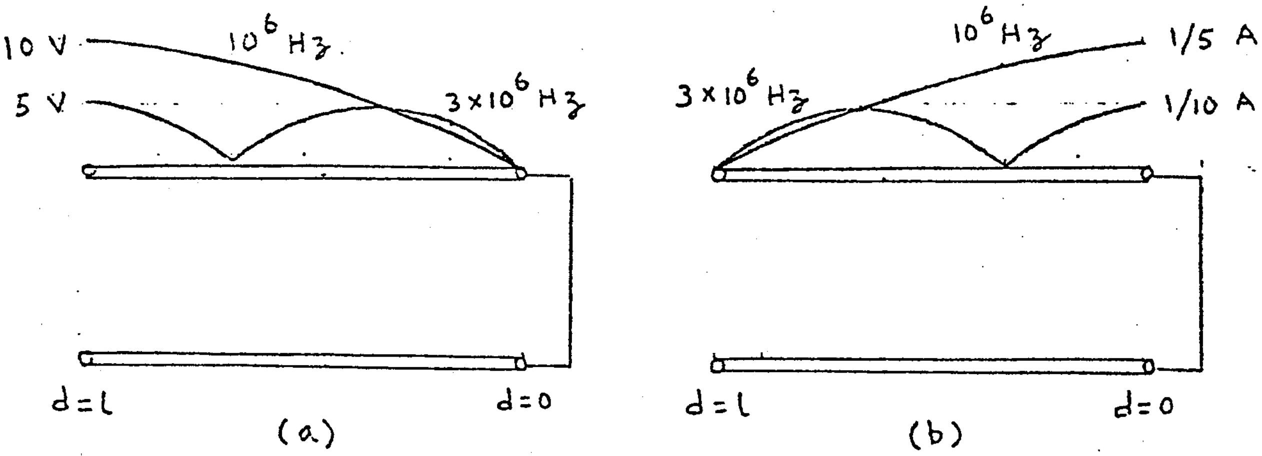
$$l = n\lambda, \quad n = 1,2,3,\ldots$$

$$\boxed{\lambda_n = \frac{l}{n}, \quad n = 1,2,3,\ldots}$$

Example involving source with two harmonically related frequencies:



4.2.6 A short-circuited line of length $l = \lambda/4$ at 10^6 Hz and driven by a voltage source $V_g = 10 \cos 2\pi \times 10^6 t + 5 \cos 6\pi \times 10^6 t$ V.



(a) (b)

4.2.7. Standing wave patterns for (a) voltage, and (b) current, at the two frequencies 10^6 Hz and 3×10^6 Hz for the system of Fig. 4.2.6.

For $f = 10^6$ Hz, $l = \frac{\lambda}{4}$

$$\text{rms line voltage} = \frac{10}{\sqrt{2}} \sin \frac{\pi d}{2l}$$

$$\text{rms line current} = \frac{1}{5\sqrt{2}} \cos \frac{\pi d}{2l}$$

For $f = 3 \times 10^6$ Hz, $l = \frac{3\lambda}{4}$

$$\text{rms line voltage} = \frac{5}{\sqrt{2}} \left| \sin \frac{3\pi d}{2l} \right|$$

$$\text{rms line current} = \frac{1}{10\sqrt{2}} \left| \cos \frac{3\pi d}{2l} \right|$$

∴ For the superposition of the two frequencies,

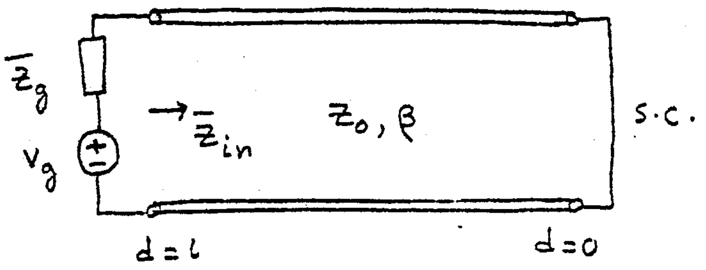
✱

$$\text{rms line voltage} = \left[\left(\frac{10}{\sqrt{2}} \sin \frac{\pi d}{2l} \right)^2 + \left(\frac{5}{\sqrt{2}} \sin \frac{3\pi d}{2l} \right)^2 \right]^{1/2}$$

$$\text{rms line current} = \left[\left(\frac{1}{5\sqrt{2}} \cos \frac{\pi d}{2l} \right)^2 + \left(\frac{1}{10\sqrt{2}} \cos \frac{3\pi d}{2l} \right)^2 \right]^{1/2}$$

d	RMS voltage, V	RMS current, A
0	0	0.158
$l/3$	5	0.122
$l/2$	5.59	0.112
$2l/3$	6.12	0.1
l	7.91	0

Input Impedance of Short-Circuited Line

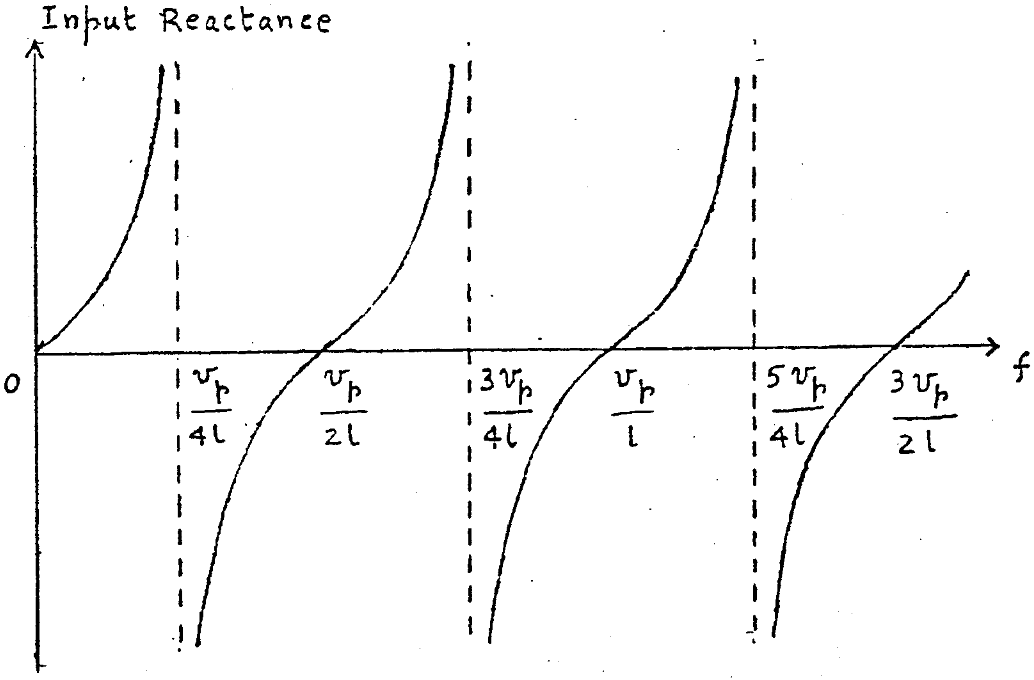


Define Line Impedance $\bar{Z}(d) = \dfrac{\bar{V}(d)}{\bar{I}(d)}$.

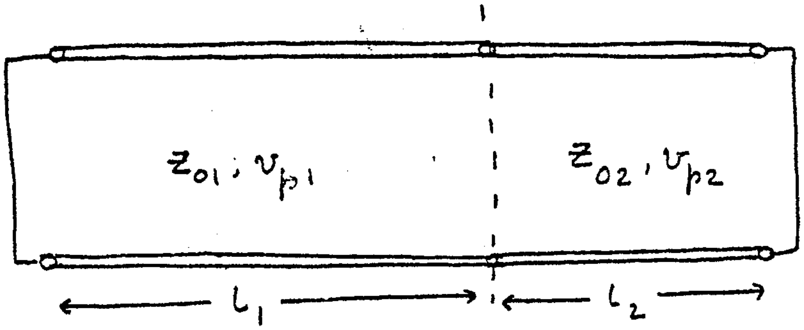
$$\bar{Z}(d) = \frac{2j\bar{V}^+ \sin\beta d}{\frac{2\bar{V}^+}{Z_0}\cos\beta d} = j Z_0 \tan\beta d$$

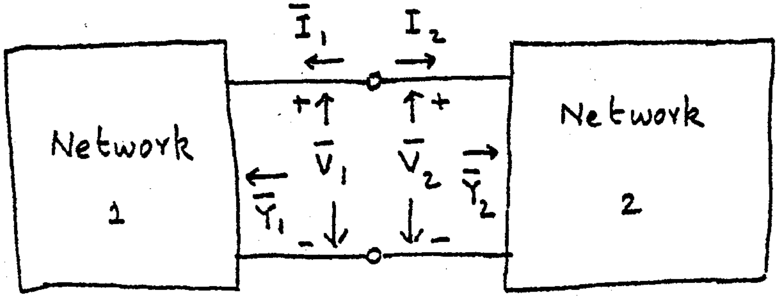
Then

$$\boxed{\begin{aligned}\bar{Z}_{in} &= \bar{Z}(l) = j Z_0 \tan\beta l \\ &= j Z_0 \tan \frac{2\pi f}{v_p} l\end{aligned}}$$



A Resonant System





$$\bar{V}_1 = \bar{V}_2$$

$$\bar{I}_1 + \bar{I}_2 = 0$$

$$\bar{I}_1 = -\bar{I}_2$$

$$\frac{\bar{I}_1}{\bar{V}_1} = -\frac{\bar{I}_2}{V_2}$$

$$\bar{Y}_1 = -\bar{Y}_2$$

$$\boxed{\bar{Y}_1 + \bar{Y}_2 = 0}$$

characteristic Equation for Resonance

$$\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} = 0$$

$$\frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 \bar{Z}_2} = 0 \longrightarrow \bar{Z}_1 + \bar{Z}_2 = 0$$

$$\boxed{j Z_{01} \tan \frac{2\pi f}{v_{p1}} l_1 + j Z_{02} \tan \frac{2\pi f}{v_{p2}} l_2 = 0}$$

<u>Example</u>: $Z_{01} = 2 Z_{02}$, $l_1 = 2 l_2$, $v_{p1} = v_{p2} = v_p$

$$\tan \frac{2\pi f}{v_p} l_1 + \frac{1}{2} \tan \frac{\pi f}{v_p} l_1 = 0$$

$$\tan \frac{2\pi f}{v_p} l_1 = -\frac{1}{2} \tan \frac{\pi f}{v_p} l_1$$

$$\tan 2x = -\frac{1}{2} \tan x \quad \text{where } x = \frac{\pi f}{v_p} l_1$$

$$\frac{2 \cancel{\tan} x}{1 - \tan^2 x} = -\frac{1}{2} \cancel{\tan} x$$

$$\frac{1}{1-\tan^2 x} = \frac{-1}{4}$$

$$\tan^2 x - 1 = 4$$

$$\tan^2 x = 5$$

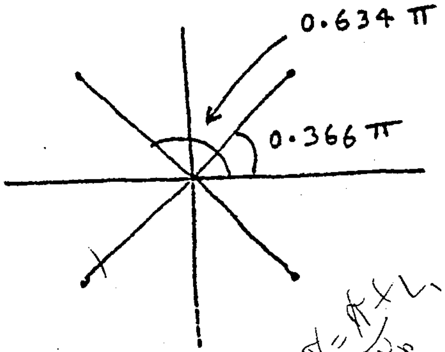
$$\tan x = 0 \quad \text{or} \quad 1 - \tan^2 x = -4$$

$$\tan x = \sqrt{5}$$

$$\tan x = 0, \quad \text{or} \quad \pm\sqrt{5}$$

$$x = n\pi, (0.366 + n)\pi, (0.634 + n)\pi,$$

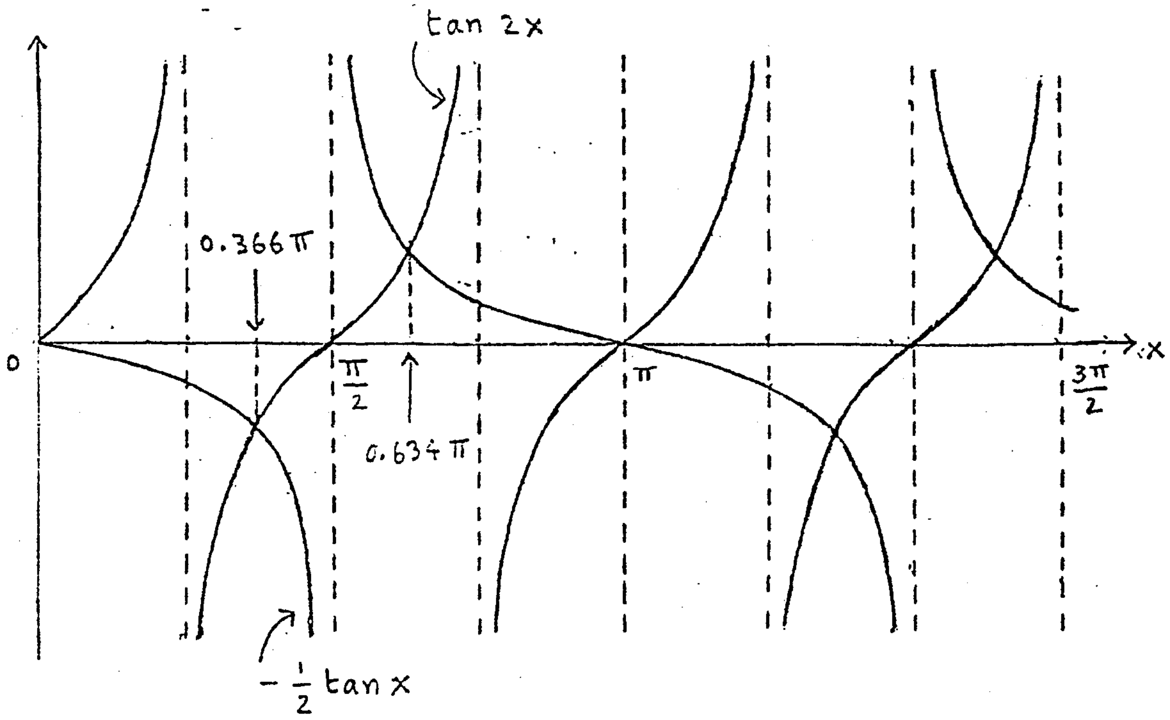
$$n = 0, 1, 2, 3, \ldots$$



$$f = \frac{n v_p}{l_1}, \frac{(0.366 + n) v_p}{l_1}, \frac{(0.634 + n) v_p}{l_1},$$

$$n = 0, 1, 2, 3, \ldots$$

$$.634\cancel{\pi} = \frac{\cancel{\pi} f \times L_1}{v_p}$$



4.2.10. Graphical solution of the equation tan 2x = $-\frac{1}{2}$ tan x.

Line Terminated by Complex Load

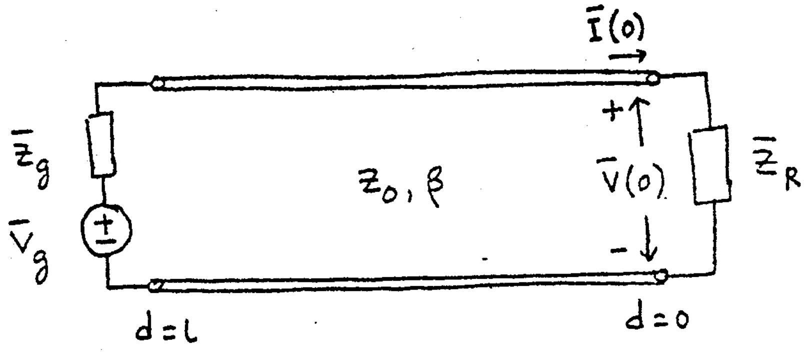


Fig. 4.4.1

$$\bar V(d) = \bar V^+ e^{j\beta d} + \bar V^- e^{-j\beta d}$$

$$\bar I(d) = \frac{1}{Z_0}(\bar V^+ e^{j\beta d} - \bar V^- e^{-j\beta d})$$

$$\bar V(0) = \bar Z_R \cdot \bar I(0)$$ B.C. at load

$$\bar V^+ + \bar V^- = \frac{\bar Z_R}{Z_0}(\bar V^+ - \bar V^-)$$

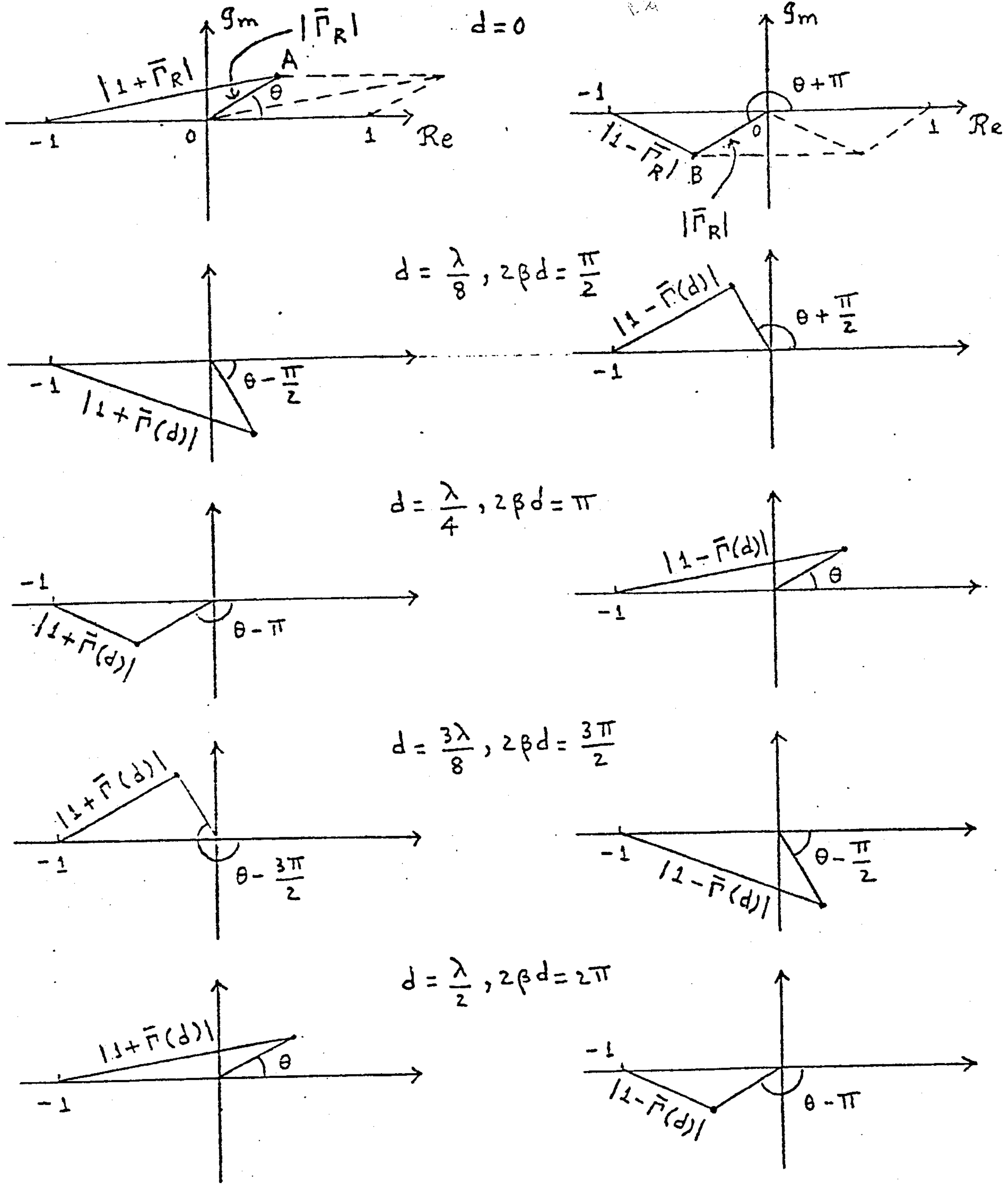
$$\boxed{\bar \Gamma_R = \frac{\bar V^-}{\bar V^+} = \frac{\bar Z_R - Z_0}{\bar Z_R + Z_0}}$$ Ref. coeff. at load

$$\boxed{\bar \Gamma(d) = \frac{\bar V^- e^{-j\beta d}}{\bar V^+ e^{j\beta d}} = \bar\Gamma_R e^{-j2\beta d} = |\bar\Gamma_R| e^{j\theta} e^{-j2\beta d}}$$ Ref. coeff at any d

$$\bar V(d) = \bar V^+ e^{j\beta d}(1 + \bar\Gamma_R e^{-j2\beta d}) = \bar V^+ e^{j\beta d}[1 + \bar\Gamma(d)]$$

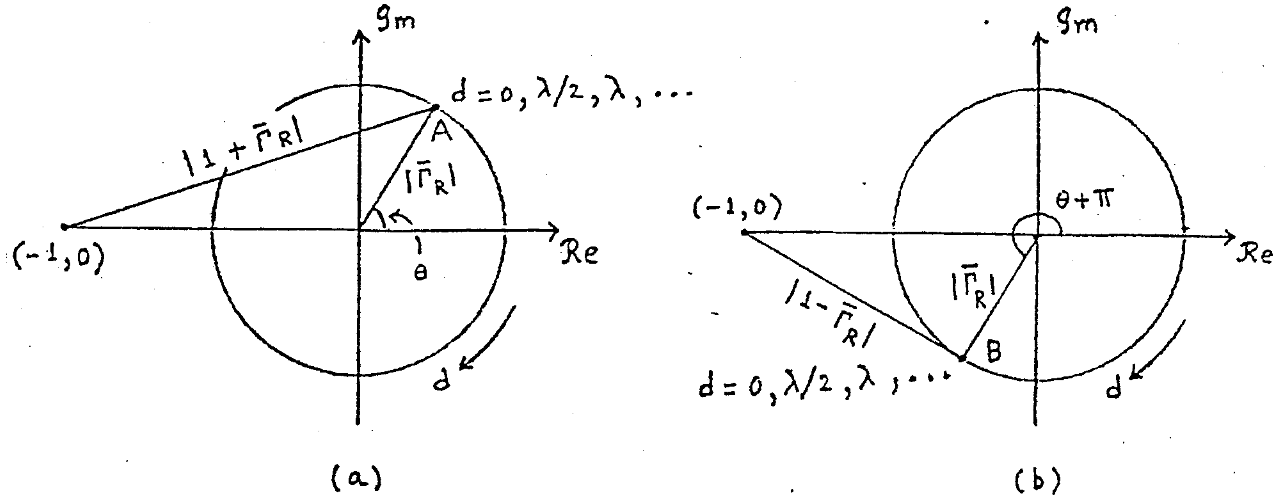
$$\bar I(d) = \frac{\bar V^+}{Z_0} e^{j\beta d}(1 - \bar\Gamma_R e^{-j2\beta d}) = \frac{\bar V^+}{Z_0} e^{j\beta d}[1 - \bar\Gamma(d)]$$

$$\boxed{\begin{aligned} |\bar V(d)| &= |\bar V^+|\,|1 + \bar\Gamma_R e^{-j2\beta d}| \\ |\bar I(d)| &= \frac{|\bar V^+|}{Z_0}\,|1 - \bar\Gamma_R e^{-j2\beta d}| \end{aligned}}$$ Standing Wave Patterns

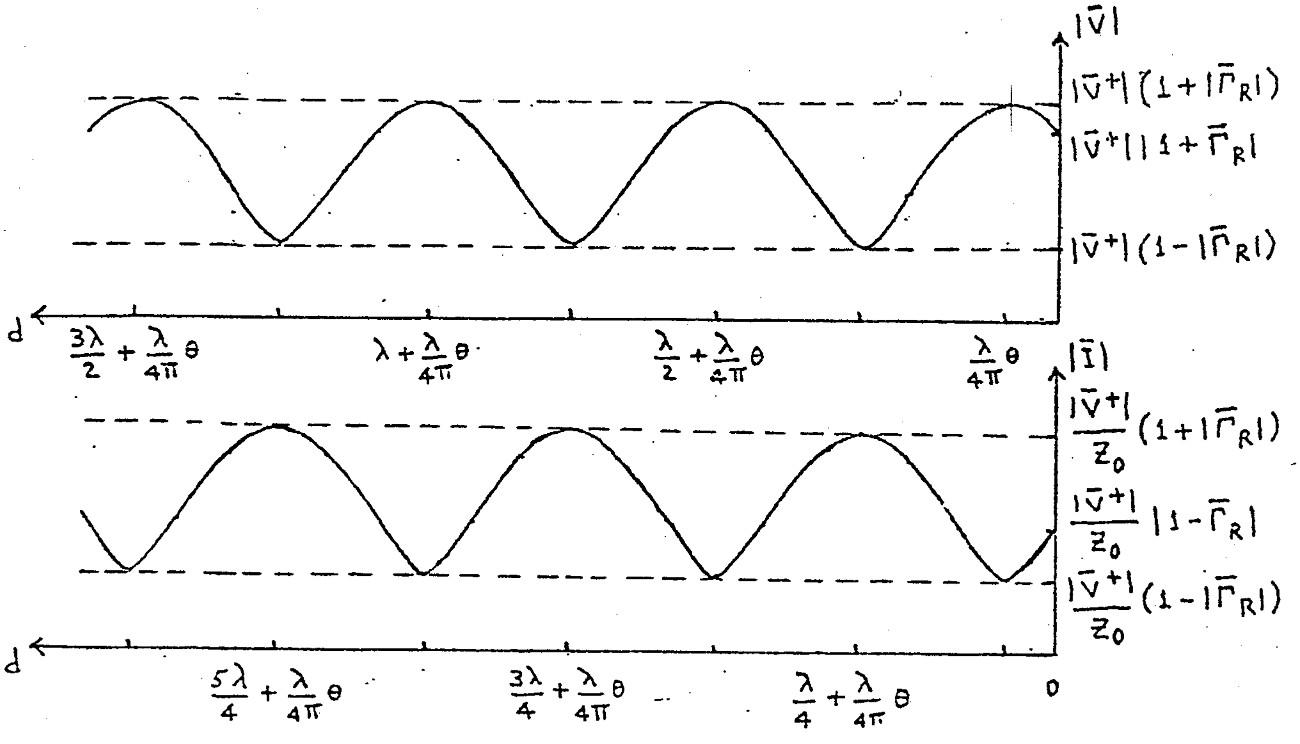


(a) (b)

4.4.2. Constructions in the complex $\bar{\Gamma}$-plane for evaluating (a) $|1 + \bar{\Gamma}(d)|$, and (b) $|1 - \bar{\Gamma}(d)|$, for several values of d, where $\bar{\Gamma}(d) = \bar{\Gamma}_R e^{-j2\beta d} = |\bar{\Gamma}_R| e^{j\theta} e^{-j(4\pi/\lambda)d}$ is the generalized voltage reflection coefficient for the line of Fig. 4.4.1.



4.4.3. Constructions in the complex $\bar{\Gamma}$-plane for evaluating (a) $|1 + \bar{\Gamma}(d)|$, and (b) $|1 - \bar{\Gamma}(d)|$, continuously with d, combining those in (a) and (b), respectively, of Fig. 4.4.2.



4.4.4. Standing wave patterns for voltage ($|\bar{V}|$) and current ($|\bar{I}|$) for the line of Fig. 4.4.1.

Standing Wave Parameters

1) Standing Wave Ratio (SWR) $= \dfrac{V_{max}}{V_{min}} = \boxed{\dfrac{I_{max}}{I_{min}}}$

$$\boxed{SWR = \frac{1 + |\bar{\Gamma}_R|}{1 - |\bar{\Gamma}_R|}}$$

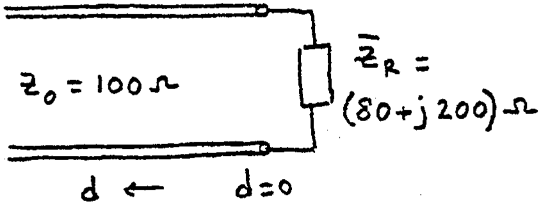
2) Location of the first voltage minimum, d_{min}

$$\theta - 2\beta d_{min} = -\pi \qquad \text{where } e^{j(\theta - 2\beta d_{min})} = e^{-j\pi} = -1$$

$$\boxed{d_{min} = \frac{\lambda}{4\pi}(\theta + \pi)}$$

3) Distance between successive voltage minima $= \dfrac{\lambda}{2}$.

Example



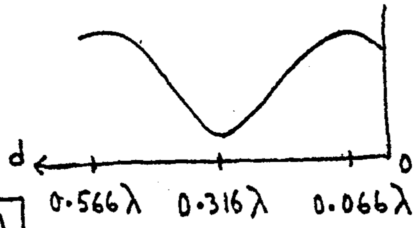
$$\bar{\Gamma}_R = \frac{\bar{Z}_R - Z_0}{\bar{Z}_R + Z_0}$$

$$= \frac{80+j200-100}{80+j200+100} = \frac{-20+j200}{180+j200}$$

$$= \frac{-1+j10}{9+j10} = \frac{(-1+j10)(9-j10)}{81+100} = \frac{91+j100}{181}$$

$$= 0.747 \angle 0.265\pi$$

$$SWR = \frac{1+0.747}{1-0.747} = \boxed{6.905}$$



$$d_{min} = \frac{\lambda}{4\pi}(0.265\pi + \pi) = \boxed{0.316\lambda}$$

Determination of Unknown Load Impedance from Standing Wave

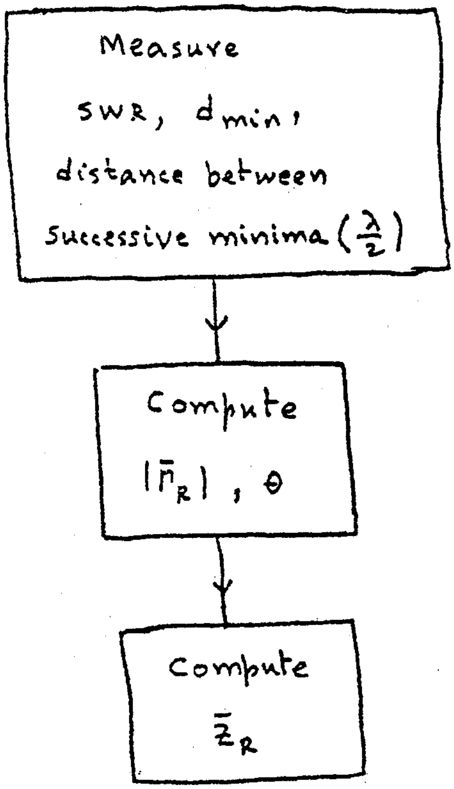
Measurements

$$SWR = \frac{1 + |\bar{\Gamma}_R|}{1 - |\bar{\Gamma}_R|} \longrightarrow |\bar{\Gamma}_R| = \frac{SWR - 1}{SWR + 1}$$

$$\theta - 2\beta d_{min} = -\pi \longrightarrow \theta = \frac{4\pi d_{min}}{\lambda} - \pi$$

$$\bar{\Gamma}_R = |\bar{\Gamma}_R| e^{j\theta} \qquad |\bar{\Gamma}_R| \angle \theta$$

$$\bar{\Gamma}_R = \frac{\bar{Z}_R - Z_0}{\bar{Z}_R + Z_0} \longrightarrow \bar{Z}_R = Z_0 \frac{1 + \bar{\Gamma}_R}{1 - \bar{\Gamma}_R}$$



Example

$Z_0 = 50\ \Omega$

$SWR = 3.0$

$d_{min} = 14.2$ cm

$\frac{\lambda}{2} = 20$ cm

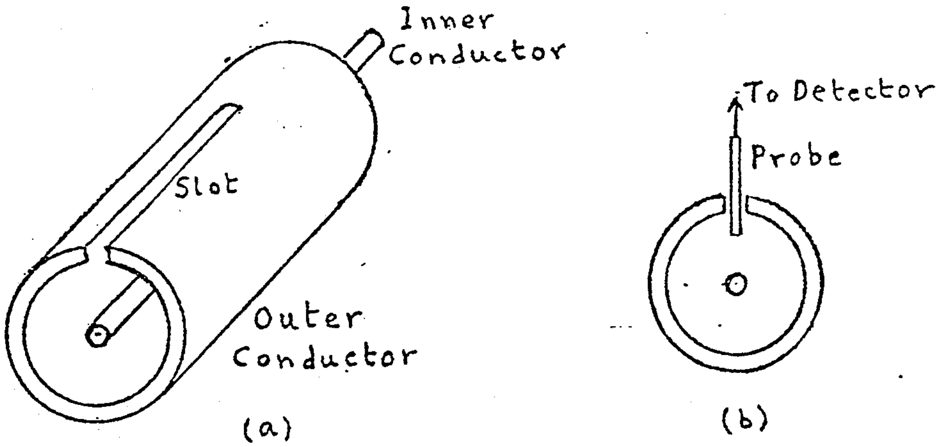
$$|\bar{\Gamma}_R| = \frac{3-1}{3+1} = 0.5$$

$$\theta = \frac{4\pi \times 14.2}{40} - \pi = 0.42\pi$$

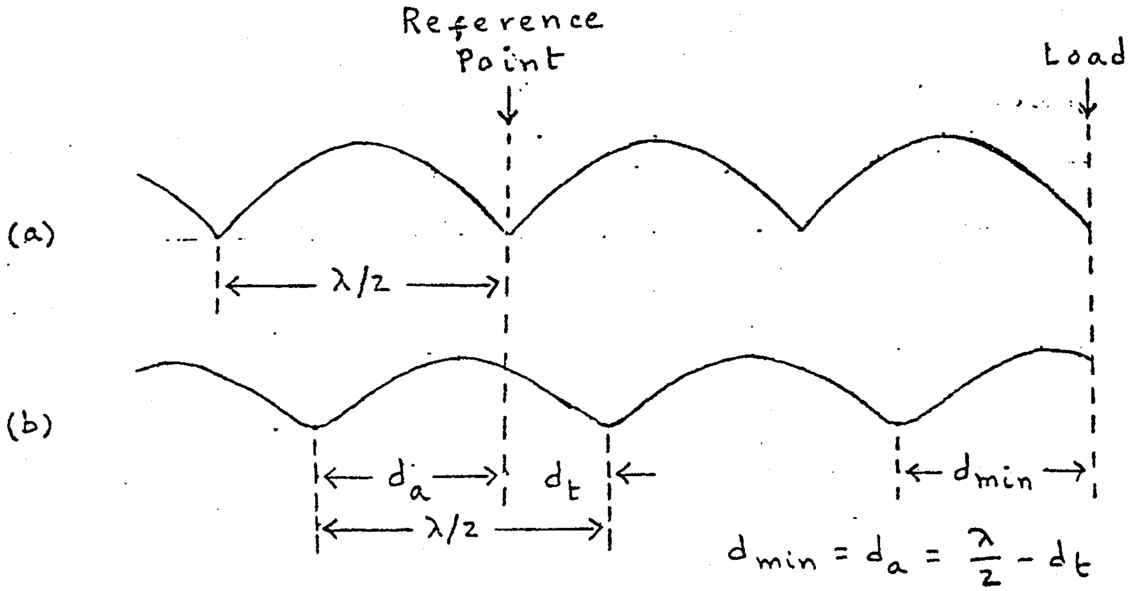
$$\bar{Z}_R = 50 \frac{1 + 0.5 e^{j0.42\pi}}{1 - 0.5 e^{j0.42\pi}}$$

$$= (37.45 + j48.365)\ \Omega$$

Slotted Line:

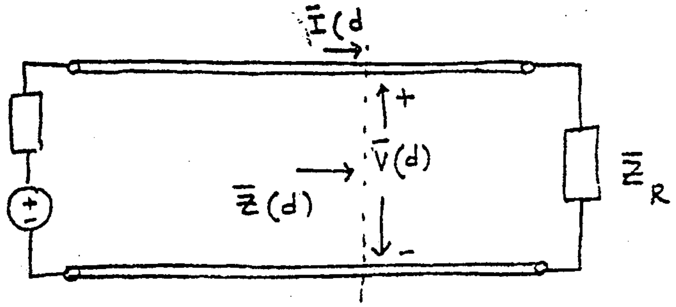


4.4.5. (a) A slotted line. (b) Cross-sectional view of the slotted line illustrating the probe arrangement.



4.4.6. For illustrating the procedure employed for the determination of d_{min}, the distance of the first voltage minimum of the standing wave pattern from the load, by making measurements away from the load.

Line Impedance



$$\bar{Z}(d) = \frac{\bar{V}(d)}{\bar{I}(d)} = \frac{\bar{V}^+ e^{j\beta d}\,[1 + \bar{\Gamma}(d)]}{\frac{\bar{V}^+}{Z_0} e^{j\beta d}\,[1 - \bar{\Gamma}(d)]}$$

$$\boxed{\bar{Z}(d) = Z_0 \frac{1 + \bar{\Gamma}(d)}{1 - \bar{\Gamma}(d)}}$$

important

(a) At locations of V_{max}, $\bar{Z}(d)$ is real and max, say, R_{max}

$$R_{max} = Z_0 \frac{1 + |\bar{\Gamma}_R|}{1 - |\bar{\Gamma}_R|} = Z_0 (SWR)$$

(b) At locations of V_{min}, $\bar{Z}(d)$ is real and min, say, R_{min}

$$R_{min} = Z_0 \frac{1 - |\bar{\Gamma}_R|}{1 + |\bar{\Gamma}_R|} = \frac{Z_0}{SWR}$$

(c) Between locations of V_{max} and V_{min}, $\bar{Z}(d)$ is complex.

(d) $\bar{Z}(d)$ repeats at intervals of $\frac{\lambda}{2}$.

(e) $\bar{Z}(d) \cdot \bar{Z}\left(d + \frac{\lambda}{4}\right) = Z_0^2$ (Quarter-Wave Transformer)

Power Flow

$$\langle P \rangle = \frac{1}{2} \mathrm{Re} \left[\bar{V}(d) \; \bar{I}^*(d) \right]$$

$$= \frac{1}{2} \mathrm{Re} \left\{ \bar{V}^+ e^{j\beta d} \left[1 + \bar{\Gamma}(d)\right] \frac{(\bar{V}^+)^*}{Z_0} e^{-j\beta d} \left[1 - \bar{\Gamma}^*(d)\right] \right\}$$

$$= \frac{1}{2} \mathrm{Re} \left\{ \frac{|\bar{V}^+|^2}{Z_0} \left[1 - |\bar{\Gamma}(d)|^2 + \bar{\Gamma}(d) - \bar{\Gamma}^*(d)\right] \right\}$$

$$= \frac{|\bar{V}^+|^2}{2 Z_0} \left[1 - |\bar{\Gamma}(d)|^2\right]$$

$$\boxed{\langle P \rangle = \frac{|\bar{V}^+|^2}{2 Z_0} \left[1 - |\bar{\Gamma}_R|^2\right]}$$

$$= \frac{1}{2 Z_0} \left[|\bar{V}^+| \left(1 + |\bar{\Gamma}_R|\right)\right] \left[|\bar{V}^+| \left(1 - |\bar{\Gamma}_R|\right)\right]$$

$$= \frac{V_{max} V_{min}}{2 Z_0}$$

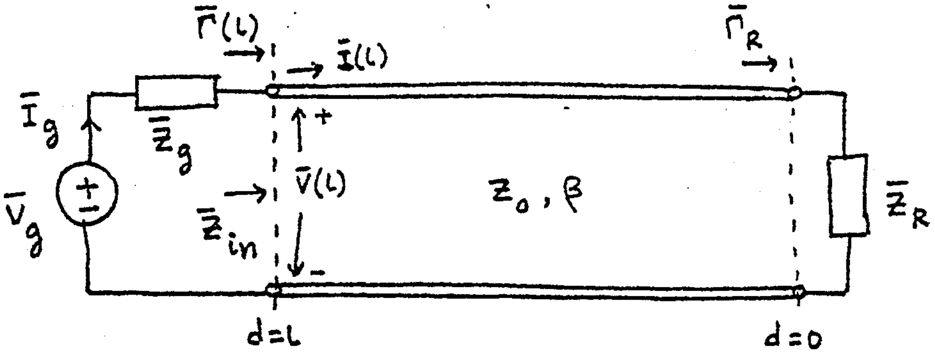
$$\boxed{\langle P \rangle = \frac{V_{max}^2}{2 (SWR) Z_0}}$$

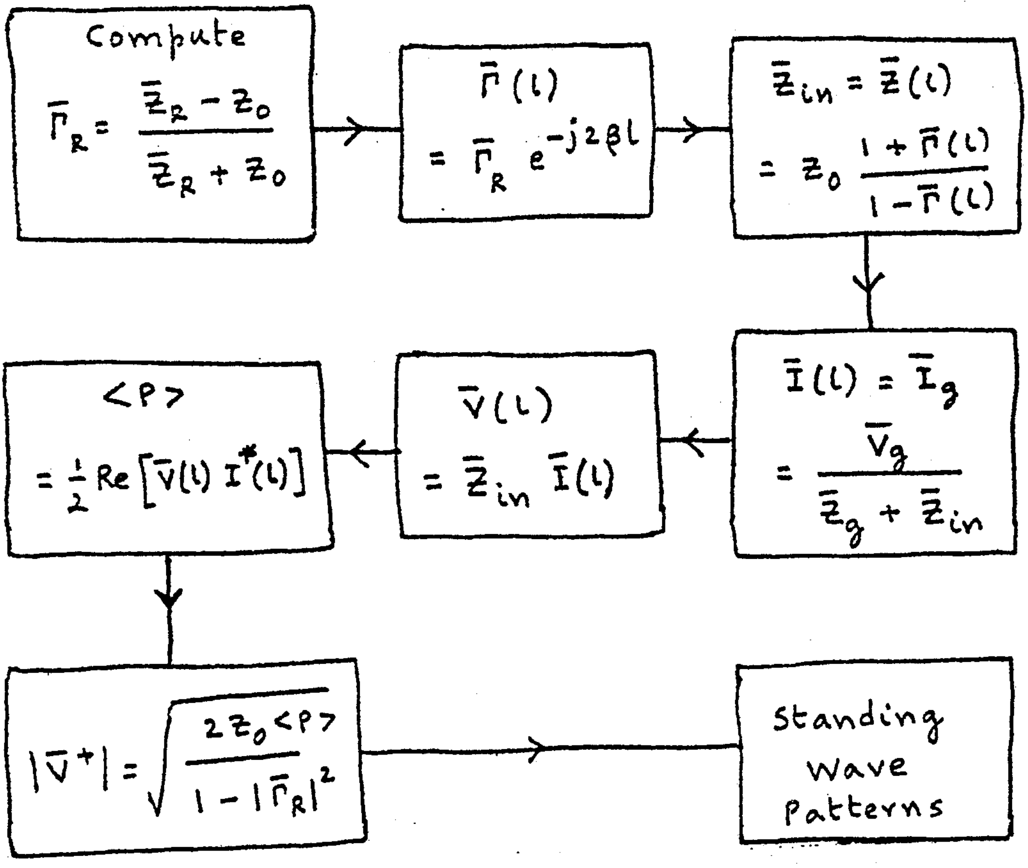
$$= \frac{1}{2} Z_0 \left[\frac{|\bar{V}^+|}{Z_0} \left(1 + |\bar{\Gamma}_R|\right)\right] \left[\frac{|\bar{V}^+|}{Z_0} \left(1 - |\bar{\Gamma}_R|\right)\right]$$

$$= \frac{1}{2} Z_0 \, I_{max} \, I_{min}$$

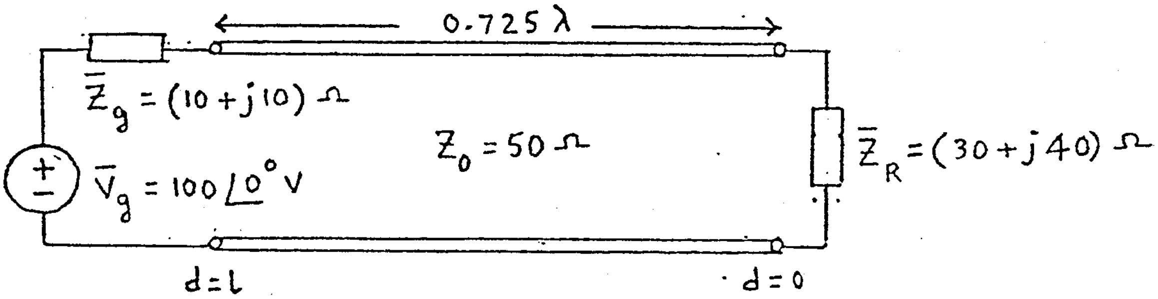
$$\boxed{\langle P \rangle = \frac{I_{max}^2 Z_0}{2 \, (SWR)}}$$

Computation of power flow from input impedance considerations:





Example



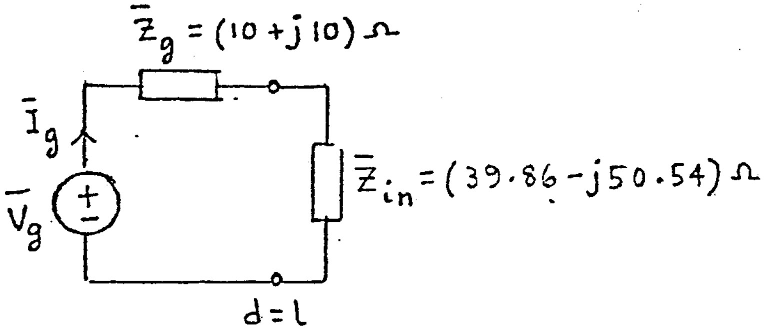
4.4.7. A transmission line system for illustrating the computation of power flow from input impedance considerations.

$$\bar{\Gamma}_R = \frac{\bar{Z}_R - Z_0}{\bar{Z}_R + Z_0} = \frac{(30+j40)-50}{(30+j40)+50} = 0.5\angle 90^\circ$$

$$\bar{\Gamma}(l) = \bar{\Gamma}_R\, e^{-j2\beta l} = 0.5\angle 90^\circ \times e^{-j\frac{4\pi}{\lambda}\times 0.725\lambda} = 0.5\angle 72^\circ$$

$$\bar{Z}_{in} = \bar{Z}(l) = Z_0 \frac{1+\bar{\Gamma}(l)}{1-\bar{\Gamma}(l)} = 50\,\frac{1+0.5\angle 72^\circ}{1-0.5\angle 72^\circ}$$

$$= 64.361\angle -51.738^\circ = (39.86 - j50.54)\,\Omega$$



4.4.8. Equivalent circuit at the input end d = l for the system of Fig. 4.4.7.

$$\bar{I}(l) = \bar{I}_g = \frac{\bar{V}_g}{\bar{Z}_g + \bar{Z}_{in}} = \frac{100 \angle 0^\circ}{(10+j10) + (39.86 - j50.54)}$$

$$= \frac{100 \angle 0^\circ}{64.261 \angle -39.114^\circ}$$

$$= 1.5562 \angle 39.114^\circ \text{ A}$$

$$\bar{V}(l) = \bar{Z}_{in}\, \bar{I}(l) = 64.361 \angle -51.738^\circ \times 1.5562 \angle 39.114^\circ$$

$$= 100.159 \angle -12.624^\circ \text{ V}$$

$$\langle P \rangle = \frac{1}{2} \mathcal{R}e \left[\bar{V}(l)\ \bar{I}^*(l) \right]$$

$$= \frac{1}{2} \mathcal{R}e \left[100.159 \angle -12.624^\circ \times 1.5562 \angle -39.114^\circ \right]$$

$$155 \angle -51.7380$$

$$= 48.26 \text{ W}$$

Proceeding further,

$$|\bar{V}^+| = \sqrt{\frac{2 Z_0 \langle P \rangle}{1 - |\bar{\Gamma}_R|^2}}$$

$$= \sqrt{\frac{100 \times 48.26}{1 - 0.25}}$$

$$= 80.22 \text{ V}$$

Standing wave patterns:

Voltage across load $= |\bar{V}^+|\,|1+\bar{\Gamma}_R| = 80.22\,|1+j0.5| = 89.69$ V

Maximum voltage $= |\bar{V}^+|\,(1+|\bar{\Gamma}_R|) = 80.22\,(1+0.5) = 120.33$ V

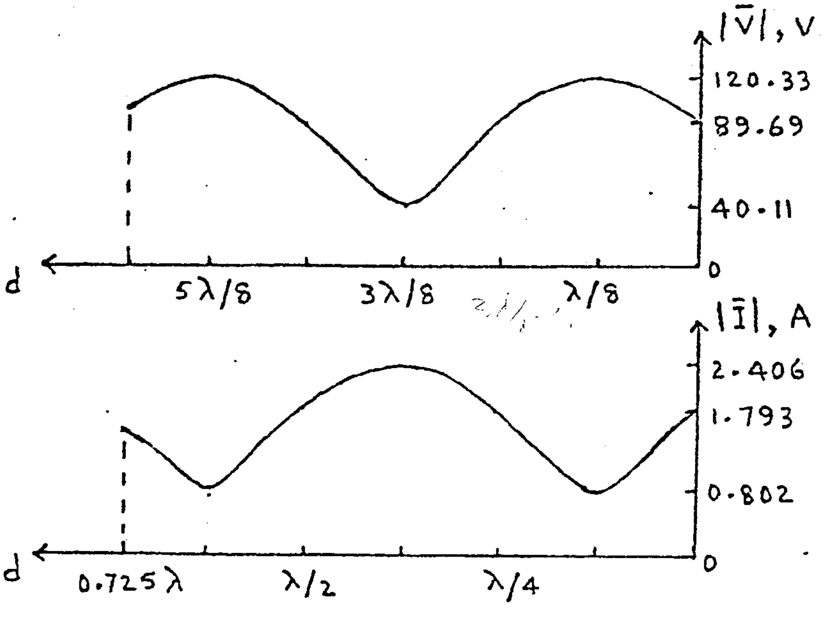
Minimum voltage $= |\bar{V}^+|\,(1-|\bar{\Gamma}_R|) = 80.22\,(1-0.5) = 40.11$ V

$$d_{min} = \frac{\lambda}{4\pi}(\theta+\pi) = \frac{\lambda}{4\pi}\left(\frac{\pi}{2}+\pi\right) = \frac{3\lambda}{8}$$

Current through load $= \dfrac{|\bar{V}^+|}{Z_0}\,|1-\bar{\Gamma}_R| = 1.604\,|1-j0.5| = 1.793$ A

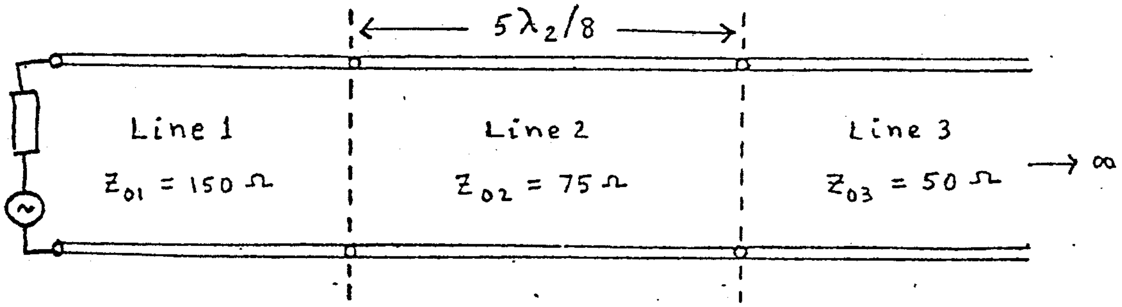
Maximum Current $= \dfrac{|\bar{V}^+|}{Z_0}\,(1+|\bar{\Gamma}_R|) = 1.604\,(1+0.5) = 2.406$ A

Minimum Current $= \dfrac{|\bar{V}^+|}{Z_0}\,(1-|\bar{\Gamma}_R|) = 1.604\,(1-0.5) = 0.802$ A

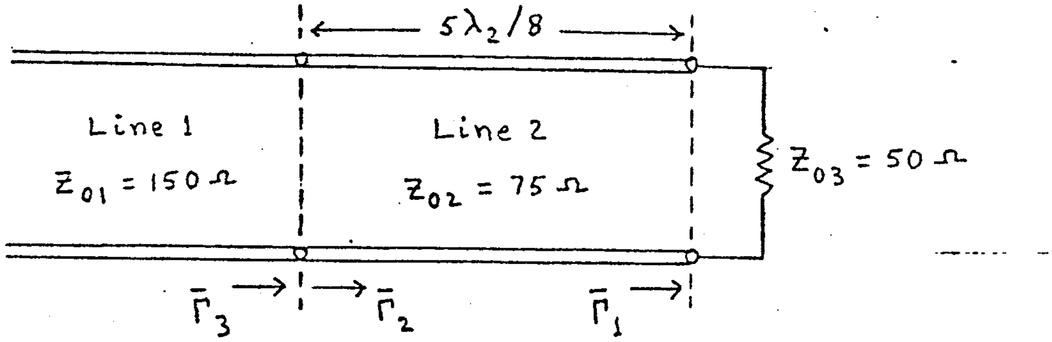


4.4.9. Standing wave patterns for voltage ($|\bar{V}|$) and current ($|\bar{I}|$) for the system of Fig. 4.4.7.

Lines in Cascade:



4.5.1. A system of three lines in cascade.



4.5.2. For showing that the input impedance of line 3 in the system of Fig. 4.5.1 is the load impedance for line 2.

$$\bar{\Gamma}_1 = \frac{50 - Z_{02}}{50 + Z_{02}} = \frac{50-75}{50+75} = \frac{1}{5} e^{j\pi}$$

$$\text{SWR in line 2} = \frac{1 + |\bar{\Gamma}_1|}{1 - |\bar{\Gamma}_1|} = \frac{1 + 1/5}{1 - 1/5} = 1.5$$

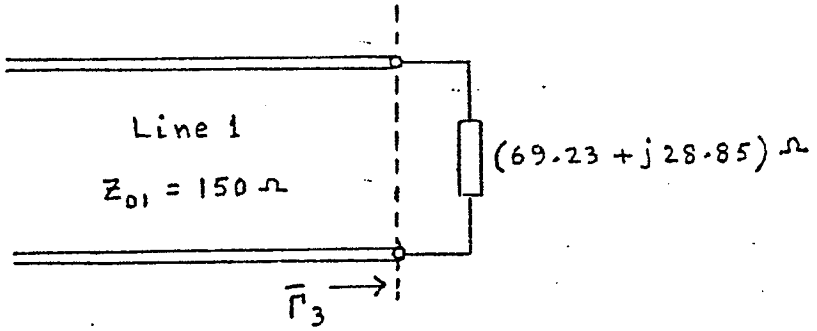
$\Gamma_l(d) = \Gamma_R e^{-j2\beta d}$

$$\bar{\Gamma}_2 = \bar{\Gamma}_1 e^{-j2\beta_2(5\lambda_2/8)} = \frac{1}{5} e^{j\pi} e^{-j\frac{5\pi}{2}}$$

$$= \frac{1}{5} e^{j\frac{\pi}{2}} = j\frac{1}{5}$$

$$\bar{Z}_{i2} = Z_{02}\,\frac{1+\bar{\Gamma}_2}{1-\bar{\Gamma}_2} = 75\,\frac{1+j\,{}^{1}/_{5}}{1-j\,{}^{1}/_{5}}$$

$$= 69.23 + j\,28.55$$



4.5.4. For showing that the input impedance of line 2 in the system of Fig. 4.5.1 is the load impedance for line 1.

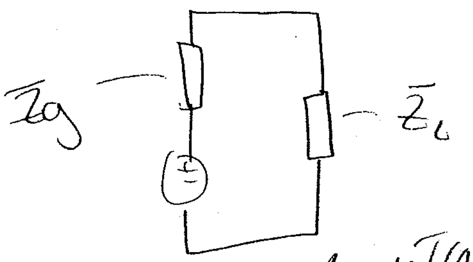
$$\bar{\Gamma}_3 = \frac{\bar{Z}_{i2} - Z_{01}}{\bar{Z}_{i2} + Z_{01}} = \frac{(69.23 + j\,28.55) - 150}{(69.23 + j\,28.55) + 150}$$

$$= 0.39\,e^{j\,0.85\pi}$$

$$\text{SWR in line 1} = \frac{1+|\bar{\Gamma}_3|}{1-|\bar{\Gamma}_3|} = \frac{1+0.39}{1-0.39}$$

$$= 2.28$$

Transmission Line Matching



for Max Power Transfer
$\bar{Z}_L = \bar{Z}_g^*$

Reasons for Matching

(a) $$\langle P \rangle = \frac{V_{max}^2}{2(SWR)Z_0}$$

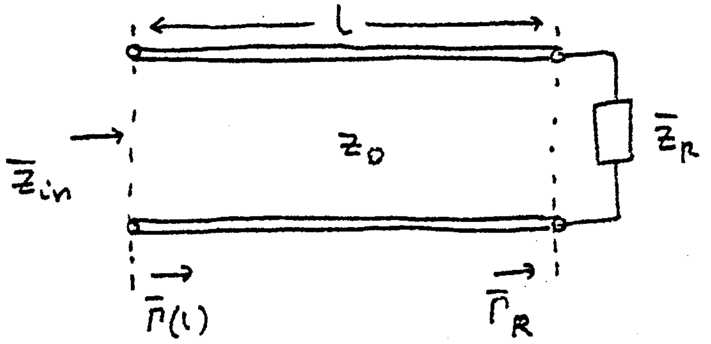
Danger of dielectric breakdown

(b) $$\langle P \rangle = \frac{I_{max}^2}{2(SWR)} Z_0$$

Danger of conductor overheating

(c) Minimize power loss in imperfect dielectric and imperfect conductors (lossy line)

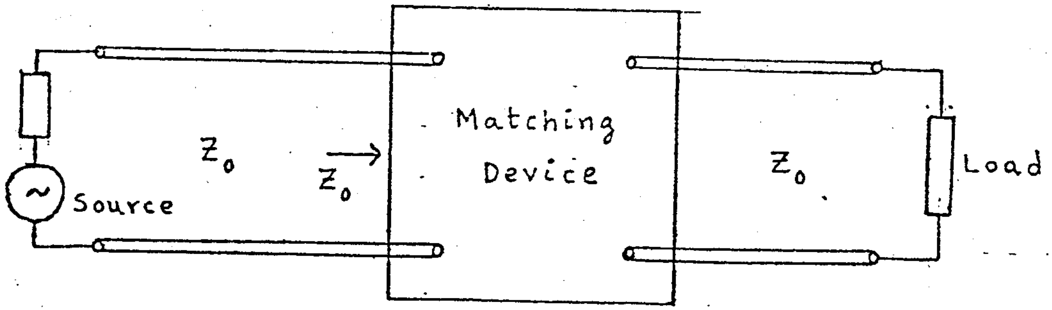
(d) Reduce sensitivity to frequency variations, that is, increase bandwidth.



$$\bar{\Gamma}(l) = \bar{\Gamma}_R \, e^{-j2\beta l}$$

$$\bar{Z}_{in} = Z_0 \frac{1 + \bar{\Gamma}(l)}{1 - \bar{\Gamma}(l)}$$

Principle behind matching

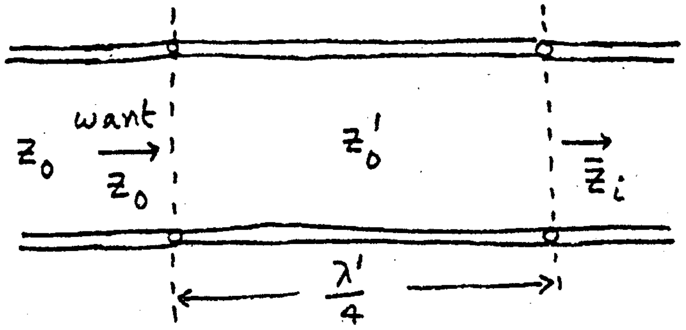


4.5.7. For illustrating the principle behind transmission-line "matching."

Quarter-wave $\left(\frac{\lambda}{4}\right)$ transformer matching

Based upon the property of line impedance that

$$\bar{z}(d) \cdot \bar{z}\left(d + \frac{\lambda}{4}\right) = z_o^2$$

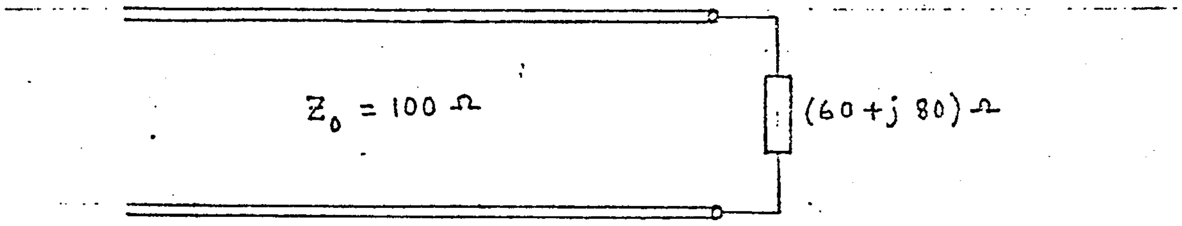


$$z_0 \bar{z}_i = (z_0')^2$$

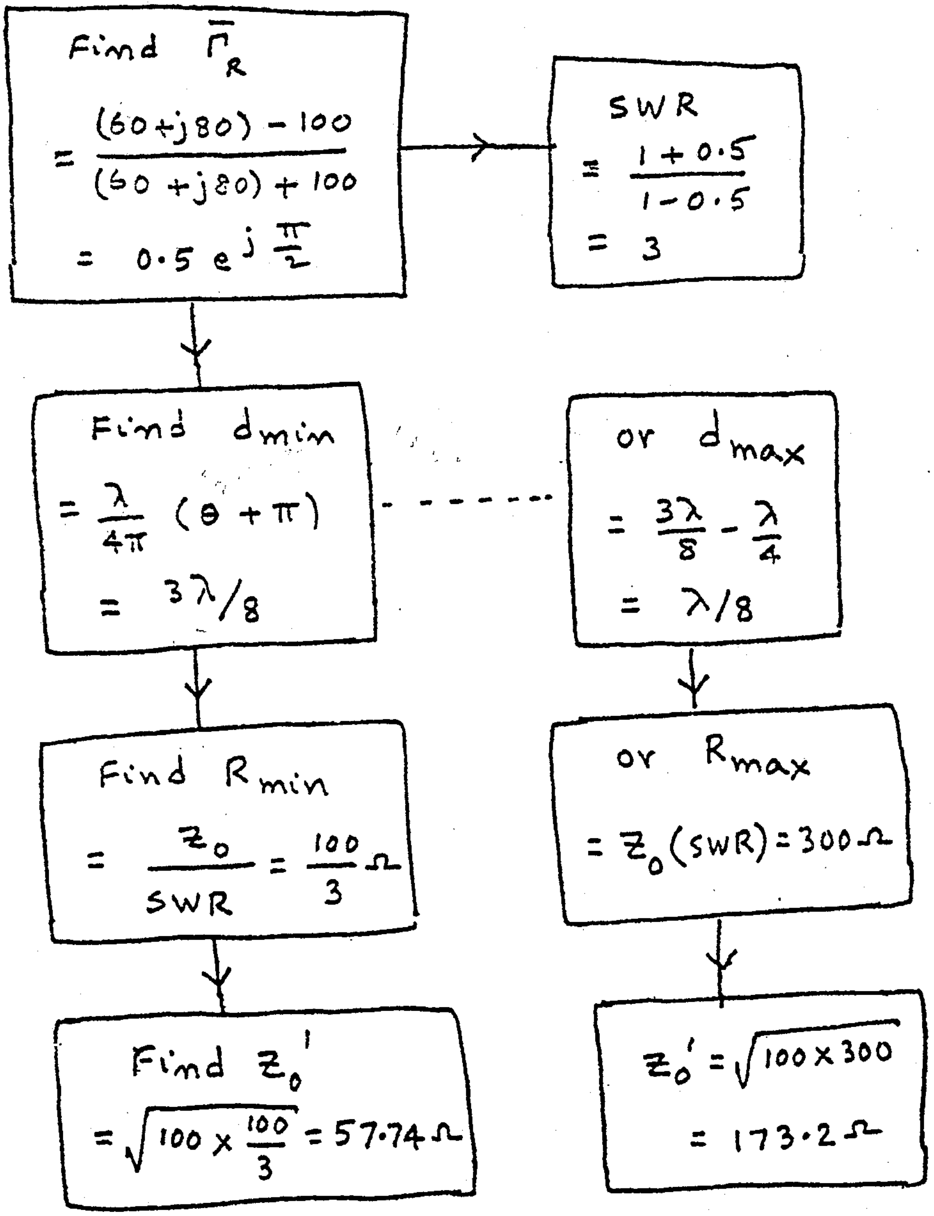
$\therefore$ For real z_0 and z_0', $\bar{z}_i$ must be real.

Then $z_0' = \sqrt{z_0 \bar{z}_i}$

Example of $\lambda/4$ transformer matching



4.5.9. A line terminated by a complex load impedance.



Find $\overline{\Gamma}_R$

$$= \frac{(60+j80)-100}{(60+j80)+100}$$

$$= 0.5\, e^{j\frac{\pi}{2}}$$

SWR

$$= \frac{1+0.5}{1-0.5}$$

$$= 3$$

Find d_{min}

$$= \frac{\lambda}{4\pi}(\theta+\pi)$$

$$= 3\lambda/8$$

or d_{max}

$$= \frac{3\lambda}{8} - \frac{\lambda}{4}$$

$$= \lambda/8$$

Find R_{min}

$$= \frac{Z_0}{SWR} = \frac{100}{3}\,\Omega$$

or R_{max}

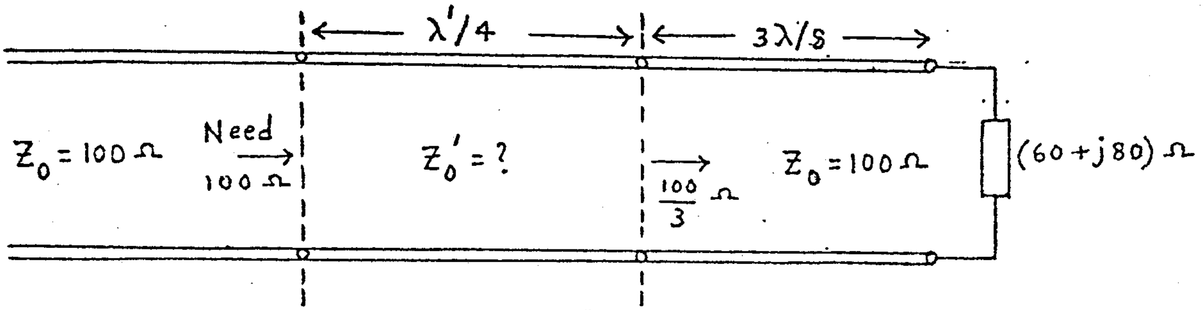
$$= Z_0(SWR) = 300\,\Omega$$

Find Z_0'

$$= \sqrt{100 \times \frac{100}{3}} = 57.74\,\Omega$$

$$Z_0' = \sqrt{100 \times 300}$$

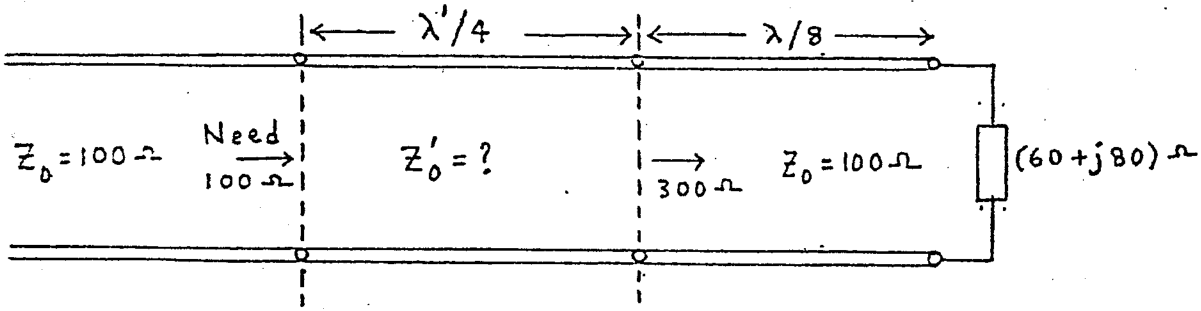
$$= 173.2\,\Omega$$



4.5.10. For the determination of the characteristic impedance Z_0' of a quarter-wave transformer, inserted at the location of a voltage minimum of the standing wave pattern, required to achieve a match, for the system of Fig. 4.5.9.

$$\text{Location} = \frac{3\lambda}{8} \text{ from load}$$

$$Z_0' = \sqrt{100 \times \frac{100}{3}} = 57.74\,\Omega$$

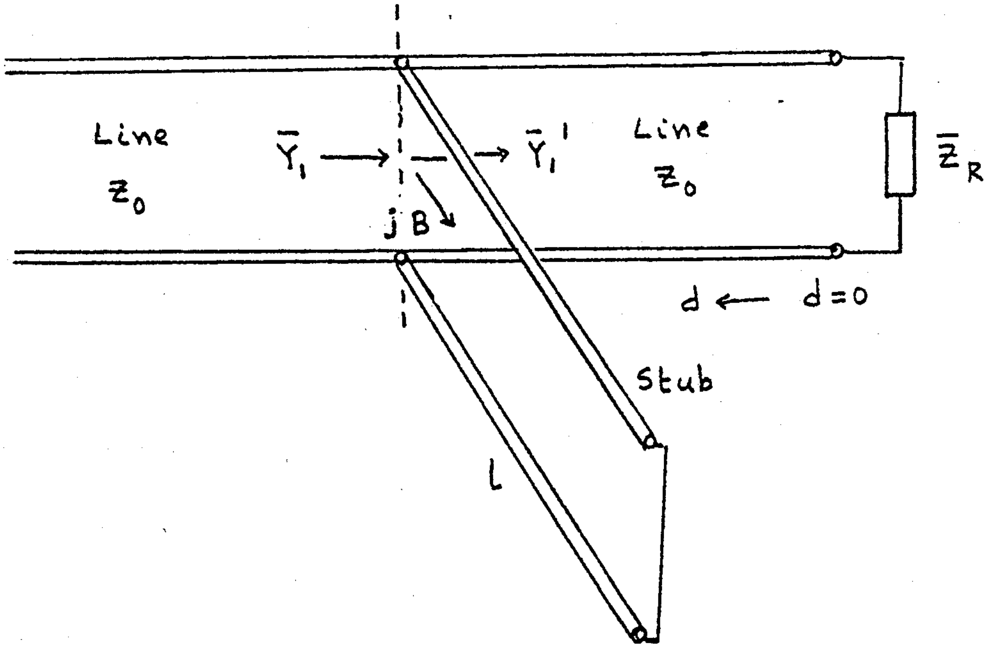


4.5.11. For the determination of the characteristic impedance Z_0' of a quarter-wave transformer, inserted at the location of a voltage maximum of the standing wave pattern, required to achieve a match, for the system of Fig. 4.5.9.

$$\text{Location} = \frac{\lambda}{8} \text{ from load}$$

$$Z_0' = \sqrt{100 \times 300} = 173.2\,\Omega$$

Stub Matching



To achieve a match, $\bar{Y}_1$ must be equal to $\frac{1}{Z_0} = Y_0$.

Since $\bar{Y}_1 = \bar{Y}_1' + jB$,

$$\bar{Y}_1' = \bar{Y}_1 - jB = Y_0 - jB$$

susceptance

G = conductance

$$\therefore \boxed{Re[\bar{Y}_1'] = Y_0}$$ ← determines stub location

$$Im[\bar{Y}_1'] = -B$$

$$\boxed{B = -Im\,\bar{Y}_1'}$$ ← determines stub length

$\bar{Z} = R + jX$ — admittance

$\bar{Y} = G + jB \quad G \neq 1/R, \; B \neq 1/X$

Example: $\bar{Z}_R = (30 - j40)\,\Omega$ $\quad Z_0 = 50\,\Omega$ (both line and stub)

Find $\bar{\Gamma}_R = \dfrac{\bar{Z}_R - Z_0}{\bar{Z}_R + Z_0}$

$$\frac{(30-j40)-50}{(30-j40)+50} = 0.5\,e^{-j\pi/2}$$

Find $\bar{\Gamma}(d) = \bar{\Gamma}_R\, e^{-j2\beta d}$

$$0.5\,e^{-j\frac{\pi}{2}} e^{-j2\beta d} = 0.5\,e^{-j(2\beta d + \frac{\pi}{2})}$$

$$\bar{Z}(d) = Z_0 \frac{1+\bar{\Gamma}(d)}{1-\bar{\Gamma}(d)}$$

$$\bar{Y}(d) = \frac{1}{\bar{Z}(d)} \cdot \frac{1-\bar{\Gamma}(d)}{1+\bar{\Gamma}(d)}$$

Find $\bar{Y}(d) = Y_0 \dfrac{1-\bar{\Gamma}(d)}{1+\bar{\Gamma}(d)}$

$$\frac{1}{50}\,\frac{1-0.5\,e^{-j(2\beta d + \pi/2)}}{1+0.5\,e^{-j(2\beta d+\pi/2)}}$$

$$= 0.02\,\frac{0.75 + j\sin(2\beta d + \pi/2)}{1.25 + \cos(2\beta d + \pi/2)}$$ [illegible] ⇒

Set $\mathrm{Re}\,\bar{Y}(d) = Y_0$ and solve for d, stub location

$$0.02\,\frac{0.75}{1.25 + \cos(2\beta d + \pi/2)} = 0.02$$

$$\cos(2\beta d + \pi/2) = -0.5$$

$$d = \frac{\lambda}{24} \text{ or } \frac{5\lambda}{24}$$ (stub location)

Find $\mathrm{Im}\,\bar{Y}(d)$ at d found above

$$0.02\,\frac{\sin(2\beta d + \pi/2)}{1.25 + \cos(2\beta d + \pi/2)} = \begin{cases} 0.02 \times 1.15 & \text{for } d = \lambda/24 \\ 0.02 \times (-1.15) & \text{for } d = 5\lambda/24 \end{cases}$$

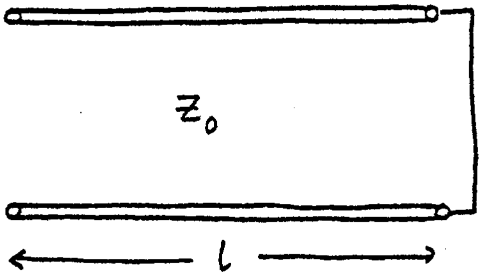
Set $B = -Y_0 \cot \beta l = -\mathrm{Im}[\bar{Y}(d)]$ found above and compute l, stub length

$$-0.02 \cot \beta l = \begin{cases} -0.02 \times 1.15 & \text{for } d = \frac{\lambda}{24} \\ 0.02 \times 1.15 & \text{for } d = \frac{5\lambda}{24} \end{cases}$$

$$\cot \beta l = \begin{cases} 1.15 & \text{for } d = \lambda/24 \\ -1.15 & \text{for } d = 5\lambda/24 \end{cases}$$

$$l = \begin{cases} 0.114\lambda & \text{for } d = \lambda/24 \\ 0.386\lambda & \text{for } d = 5\lambda/24 \end{cases}$$

Finding stub length



Input impedance of short-circuited line of length l $= j Z_0 \tan \beta l$

$\therefore$ Input admittance $= \dfrac{1}{j Z_0 \tan \beta l} = -j Y_0 \cot \beta l$

$$j B = -j Y_0 \cot \beta l$$

$$\boxed{B = -Y_0 \cot \beta l}$$

Note that Z_0 and Y_0 are those of the stub.

Thus, set $-Y_0 \cot \beta l = -\mathrm{Im}[\bar{Y}(d)]$ at stub location and solve for l.

SMITH CHART

(18 - 23)

Smith Chart

Basis for the development of Smith chart

$$\bar{Z}(d) = Z_0 \frac{1+\bar{\Gamma}(d)}{1-\bar{\Gamma}(d)}$$

Define Normalized Line Impedance, $\bar{z}(d)$, to be

$$\boxed{\bar{z}(d) = \frac{\bar{Z}(d)}{Z_0} = \frac{1+\bar{\Gamma}(d)}{1-\bar{\Gamma}(d)}}$$

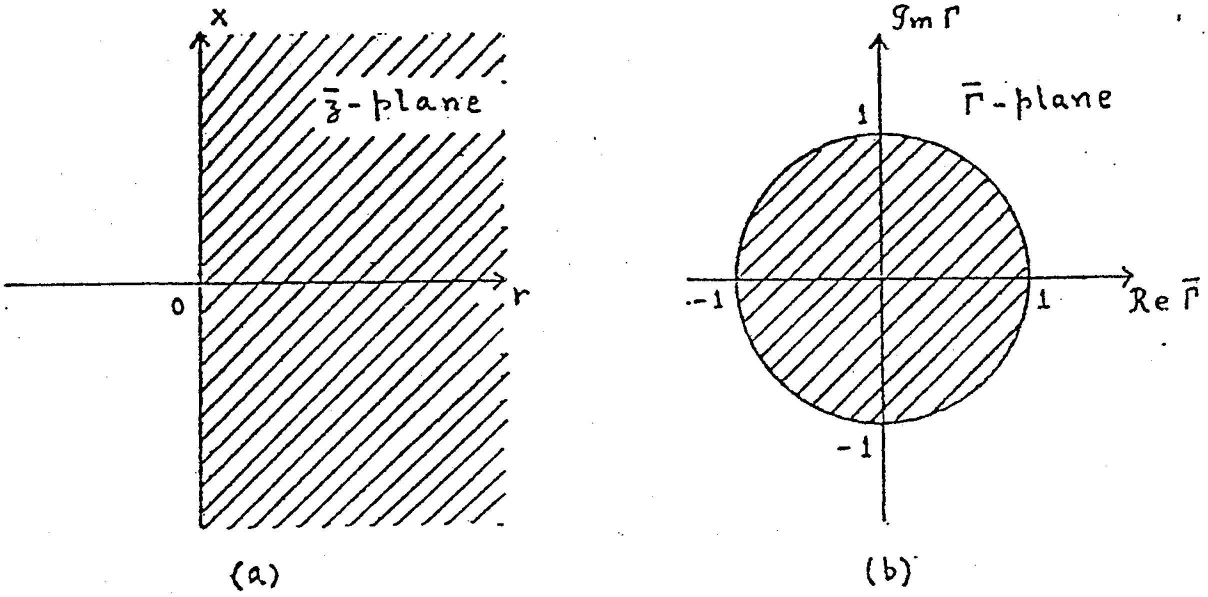
Then

$$\boxed{\bar{\Gamma}(d) = \frac{\bar{z}(d) - 1}{\bar{z}(d) + 1}}$$

Letting $\bar{z} = r + jx$, we find that

$$|\bar{\Gamma}| = \frac{|r+jx-1|}{|r+jx+1|} = \frac{\sqrt{(r-1)^2+x^2}}{\sqrt{(r+1)^2+x^2}} \leq 1 \quad \text{for } r \geq 0.$$

∴ All passive normalized line impedances are mapped on to the region in the $\bar{\Gamma}$-plane bounded by the circle of radius unity and centered at the origin. We call this circle the "unit circle."



5.1.1 (a) Region in the complex $\bar{z}$-plane corresponding to $r \geq 0$. (b) The corresponding region in the complex $\bar{\Gamma}$-plane, based upon the relationship between $\bar{\Gamma}$ and $\bar{z}$ given by Equation (5.1.3).

Construction of Smith Chart

$$\bar{\Gamma} = \frac{\bar{z} - 1}{\bar{z} + 1} = \frac{r + jx - 1}{r + jx + 1}$$

$$= \frac{(r-1) + jx}{(r+1) + jx} = \frac{[(r-1) + jx]\,[(r+1) - jx]}{(r+1)^2 + x^2}$$

$$= \frac{r^2 - 1 + x^2 + j\,2x}{(r+1)^2 + x^2}$$

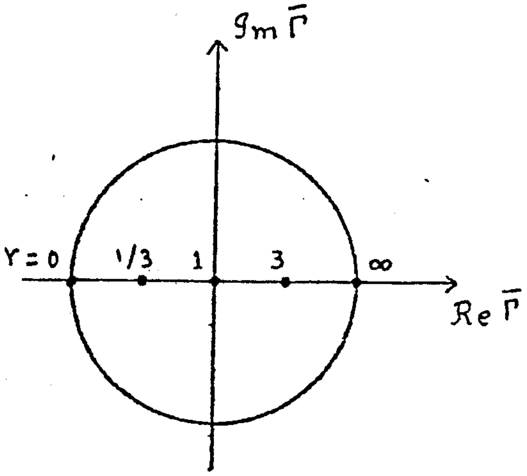
$$\therefore \boxed{\begin{aligned} \text{Re}\,\bar{\Gamma} &= \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \\ \text{Im}\,\bar{\Gamma} &= \frac{2x}{(r+1)^2 + x^2} \end{aligned}}$$

(a) $\bar{z}$ is purely real ($x = 0$)

$$\operatorname{Re} \bar{\Gamma} = \frac{r^2 - 1}{(r+1)^2} = \frac{r-1}{r+1} \quad ; \quad \operatorname{Im} \bar{\Gamma} = 0$$

$\therefore$ $\bar{\Gamma}$ is purely real.

r	0	1/3	1	3	∞
$\bar{\Gamma}$	-1	$-1/2$	0	1/2	1



(b) $\bar{z}$ is purely imaginary $(r = 0)$

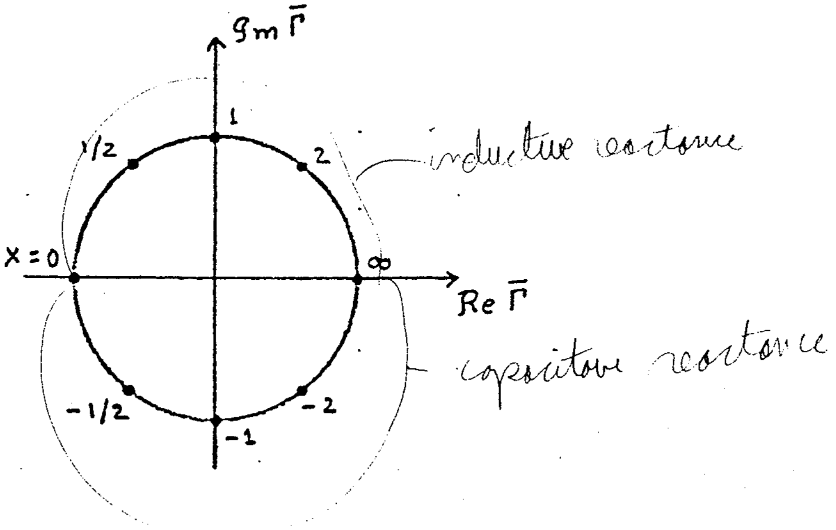
$$\mathrm{Re}\,\bar{\Gamma} = \frac{x^2-1}{x^2+1} \quad ; \quad \mathrm{Im}\,\bar{\Gamma} = \frac{2x}{x^2+1}$$

$$|\bar{\Gamma}| = \left[\left(\frac{x^2-1}{x^2+1}\right)^2 + \left(\frac{2x}{x^2+1}\right)^2\right]^{1/2} = 1$$

$$\angle\bar{\Gamma} = \tan^{-1}\frac{2x}{x^2-1}$$

$\therefore$ $\bar{\Gamma}$ lies on the unit circle

x	0	$\pm 1/2$	± 1	± 2	$\pm\infty$
$\bar{\Gamma}$	$1\angle\pi$	$1\angle\pm 0.705\pi$	$1\angle\pm\pi/2$	$1\angle\pm 0.295\pi$	$1\angle 0^\circ$



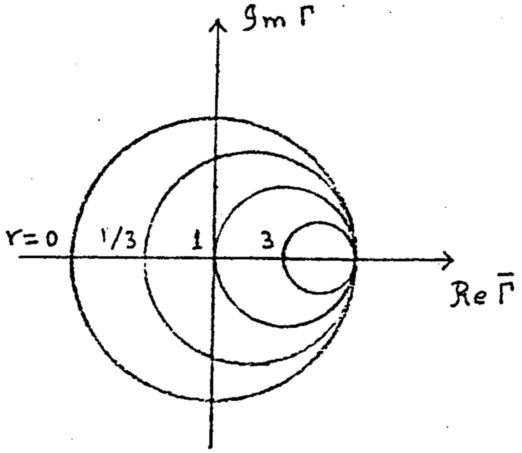
(c) $\bar{z}$ complex but with constant real part.

$$\left[\mathrm{Re}\,\bar{\Gamma} - \frac{r}{r+1}\right]^2 + \left[\mathrm{Im}\,\bar{\Gamma}\right]^2 = \left(\frac{1}{r+1}\right)^2$$

Locii are circles with centers at

$\mathrm{Re}\,\bar{\Gamma} = \frac{r}{r+1}$ and $\mathrm{Im}\,\bar{\Gamma} = 0$, and radii $\frac{1}{r+1}$.

r	0	1/3	1	3	∞
center	(0,0)	(1/4,0)	(1/2,0)	(3/4,0)	(1,0)
radius	1	3/4	1/2	1/4	0



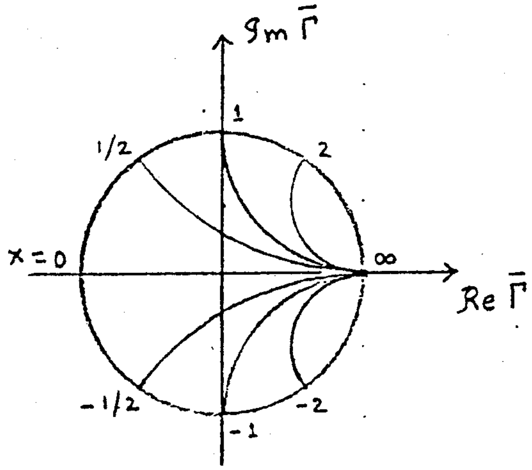
(d) $\bar{z}$ is complex but with constant imaginary part

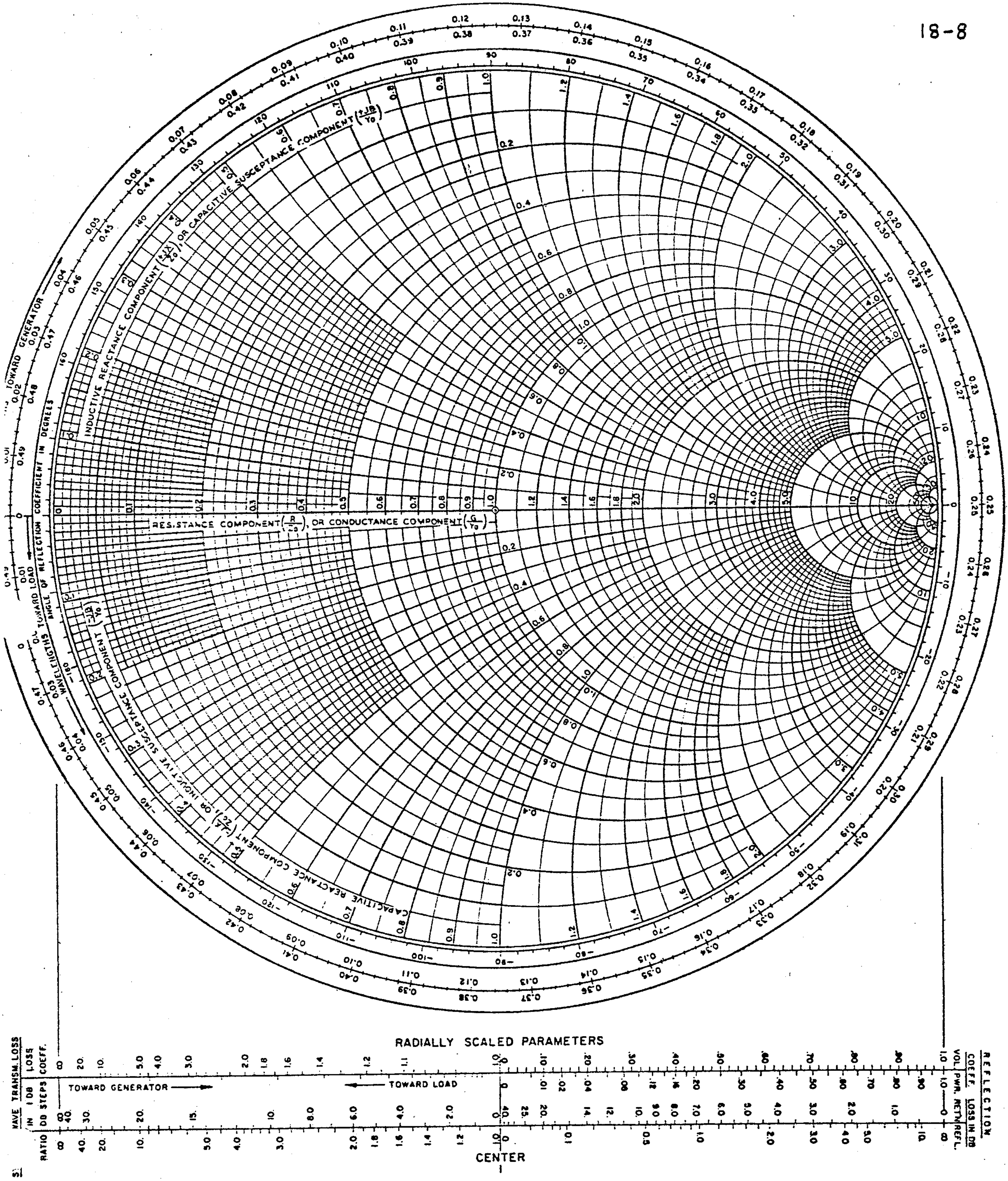
$$\left[\mathrm{Re}\,\bar{\Gamma} - 1\right]^2 + \left[\mathrm{Im}\,\bar{\Gamma} - \frac{1}{x}\right]^2 = \left(\frac{1}{x}\right)^2$$

Locii are circles with centers at

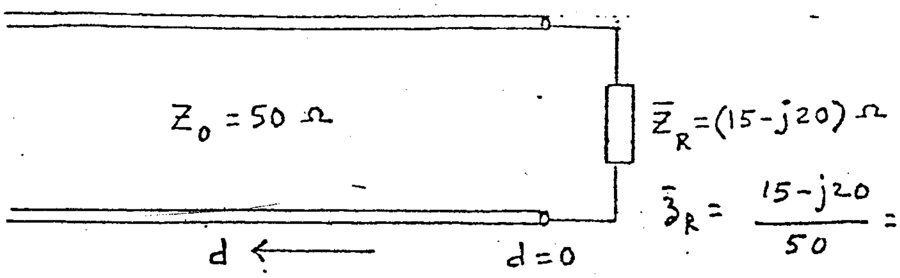
$\mathrm{Re}\,\bar{\Gamma} = 1$ and $\mathrm{Im}\,\bar{\Gamma} = \frac{1}{x}$, and radii $\frac{1}{|x|}$.

x	0	± ½	± 1	± 2	± ∞
center	(1, ∞)	(1, ± 2)	(1, ± 1)	(1, ± ½)	(1, 0)
radius	∞	2	1	½	0




WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE COMPONENT (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
SWR
WAVE RATIO
TRANSM. LOSS IN 1 DB STEPS
TRANSM. LOSS COEFF.
REFLECTION COEFF. VOL.
REFLECTION COEFF. PWR.
RETN. LOSS IN DB
REFL.
CENTER

Determination of standing wave Parameters ($\bar{\Gamma}_R$, SWR, d_{min})



$$\bar{z}_R = \frac{15-j20}{50} = 0.3-j0.4$$

5.2.1 A transmission line system for illustrating the determination of standing wave parameters by using the Smith chart.

complete revision one λ/2

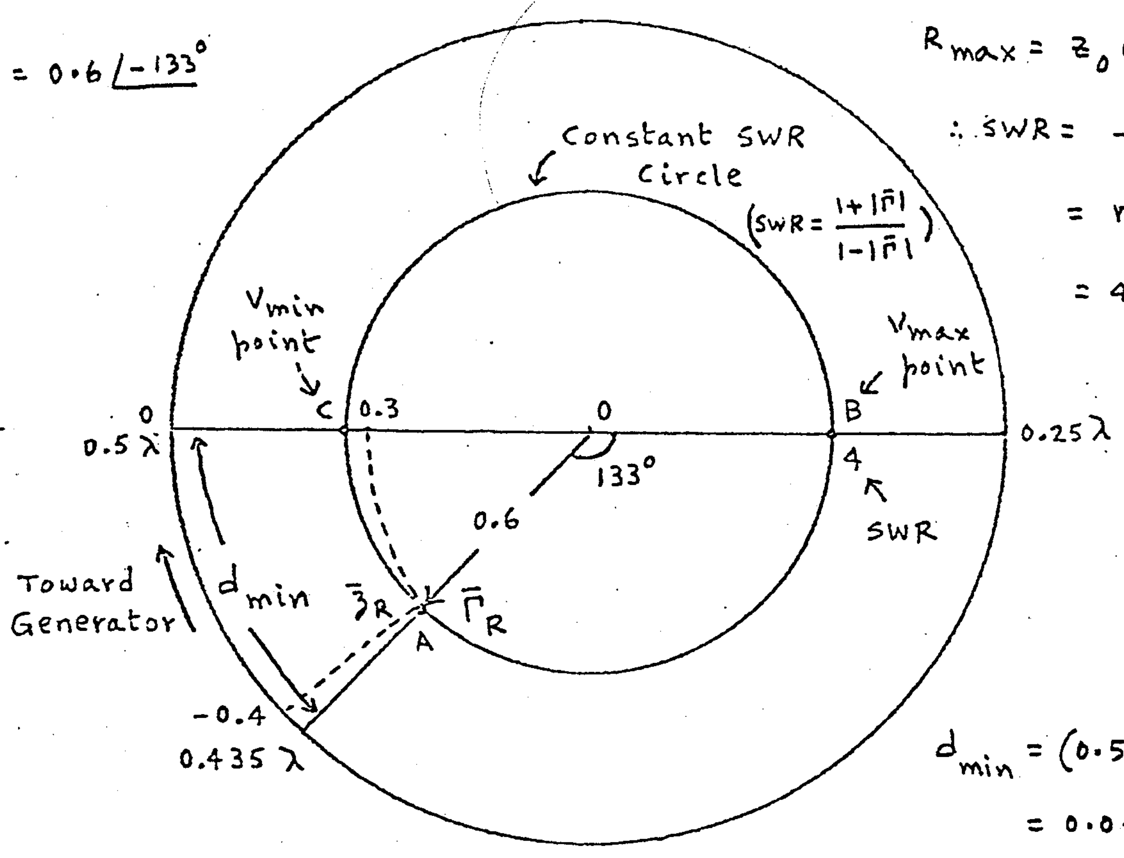
$\bar{\Gamma}_R = 0.6\underline{/-133^\circ}$

$R_{max} = z_0(SWR)$

$\therefore SWR = \frac{R_{max}}{z_0}$

$= r_{max}$

$= 4.0$



$d_{min} = (0.5 - 0.435)\lambda$

$= 0.065\lambda$

5.2.2 Determination of standing wave parameters for the system of Figure 5.2.1 by using the Smith chart.

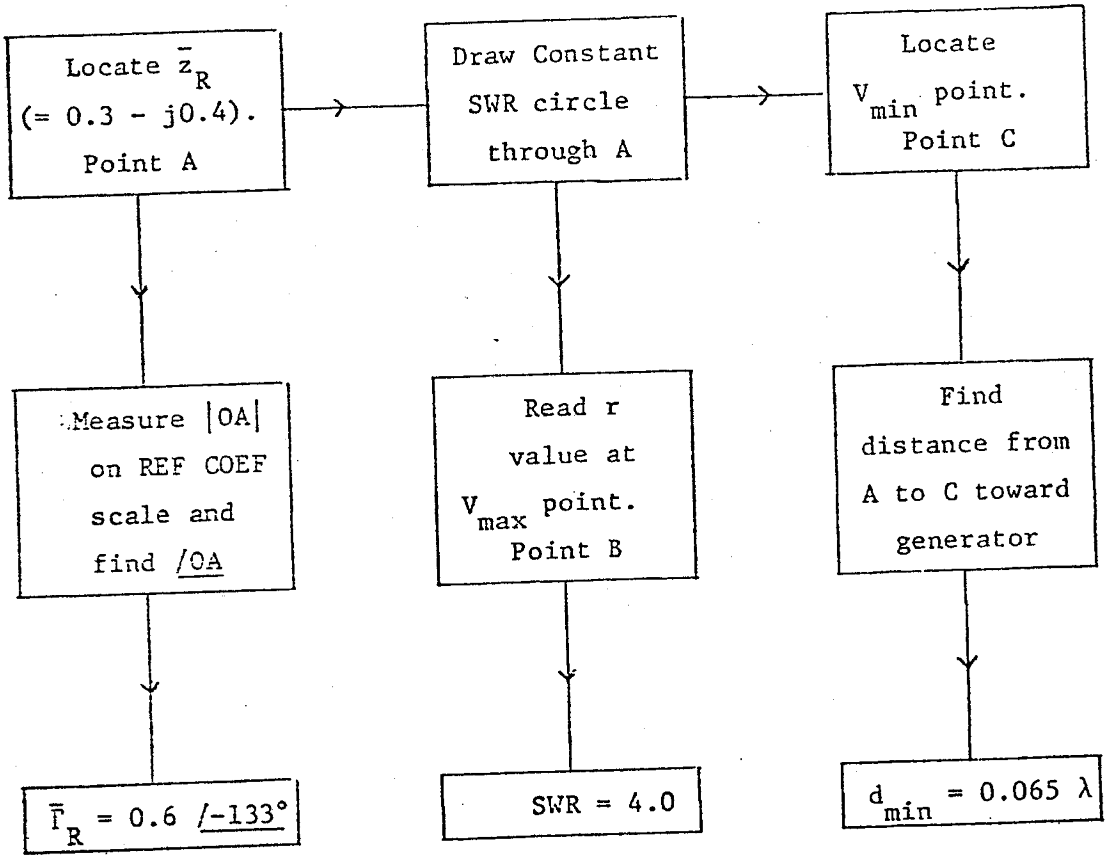
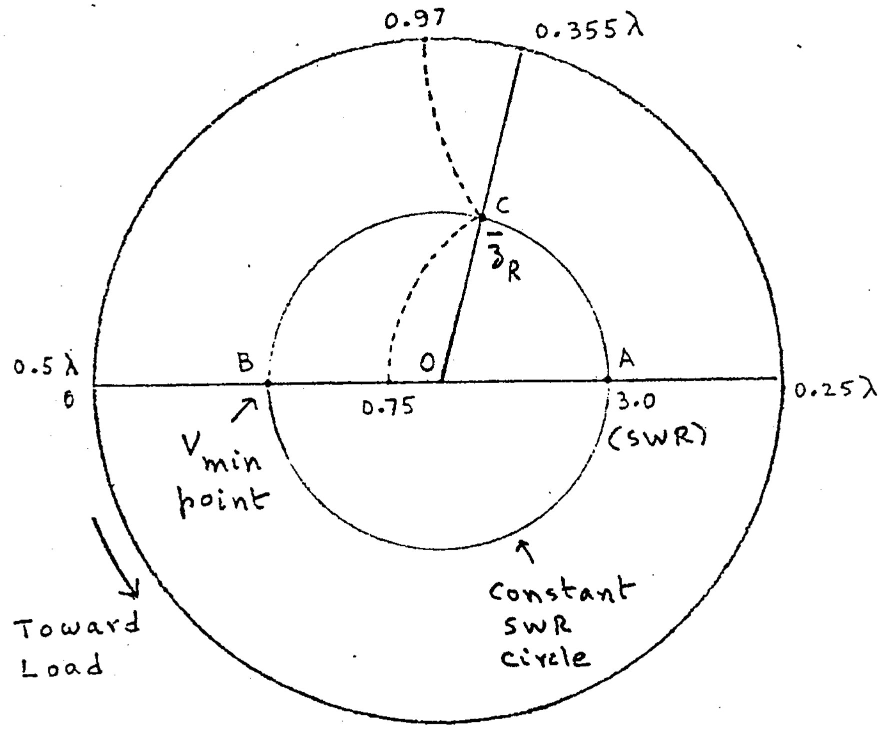


Figure 5.2.3. Flow diagram summarizing the procedures employed in Figure 5.2.2 for finding the standing wave parameters for the system of Figure 5.2.1.

Determination of Unknown Load Impedance ($\bar{Z}_R$) from standing Wave Data (SWR, λ, d_{min})



5.2.4 Determination of an unknown load impedance from standing wave data by using the Smith chart.

Data:

$Z_0 = 50\,\Omega$

$SWR = 3.0$

$\lambda = 40$ cm

$d_{min} = 14.2$ cm $= 0.355\lambda$

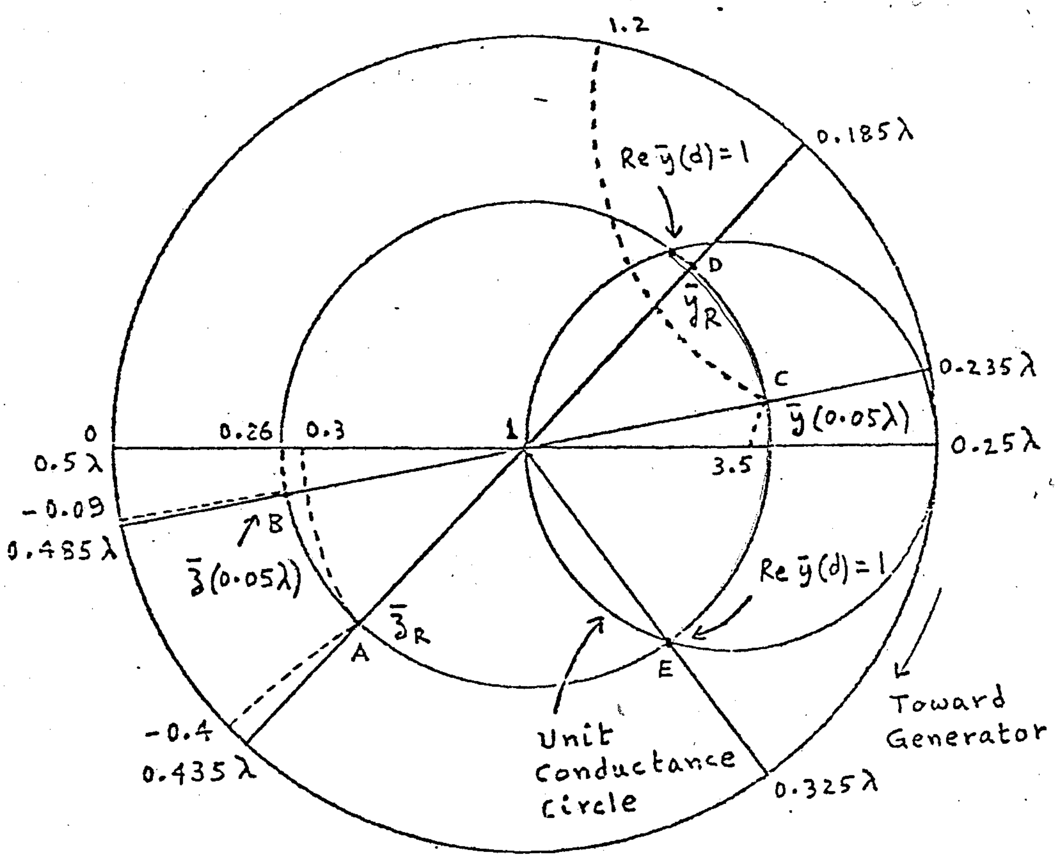
$\bar{z}_R = 0.75 + j0.97$

$\bar{Z}_R = Z_0\,\bar{z}_R$

$= 50(0.75 + j\,0.97)$

$= (37.5 + j\,48.5)\,\Omega$

Determination of Line Impedance and Admittance (at $d = 0.05\lambda$) and Location at which $\mathrm{Re}\,\bar{Y}(d) = Y_0$



5.2.5 Determination of certain quantities pertinent to the line impedance and admittance behaviors for the system of Figure 5.2.1 by using the Smith chart.

$$\bar{Z}(0.05\lambda) = Z_0 \times \bar{z}(0.05\lambda) = 50(0.26 - j0.09) = (13 - j4.5)\,\Omega$$

$$\bar{Z}(d) \cdot \bar{Z}(d + \lambda/4) = Z_0^2$$

$$\therefore \bar{z}(d) \cdot \bar{z}(d + \lambda/4) = 1 \longrightarrow \bar{y}(d) = \bar{z}(d + \lambda/4)$$

$$\bar{y}(0.05\lambda) = \bar{z}(0.05\lambda + \lambda/4) = (3.5 + j1.2)$$

$$\bar{Y}(0.05\lambda) = Y_0 \times \bar{y}(0.05\lambda) = \frac{1}{50}(3.5 + j1.2) = (0.07 + j0.024)\,\mho$$

Nearest location from load at which $\mathrm{Re}\,\bar{Y}(d) = Y_0$ or $\mathrm{Re}\,\bar{y}(d) = 1$ is distance from D to E toward generator $= (0.325 - 0.185)\lambda = 0.14\lambda$

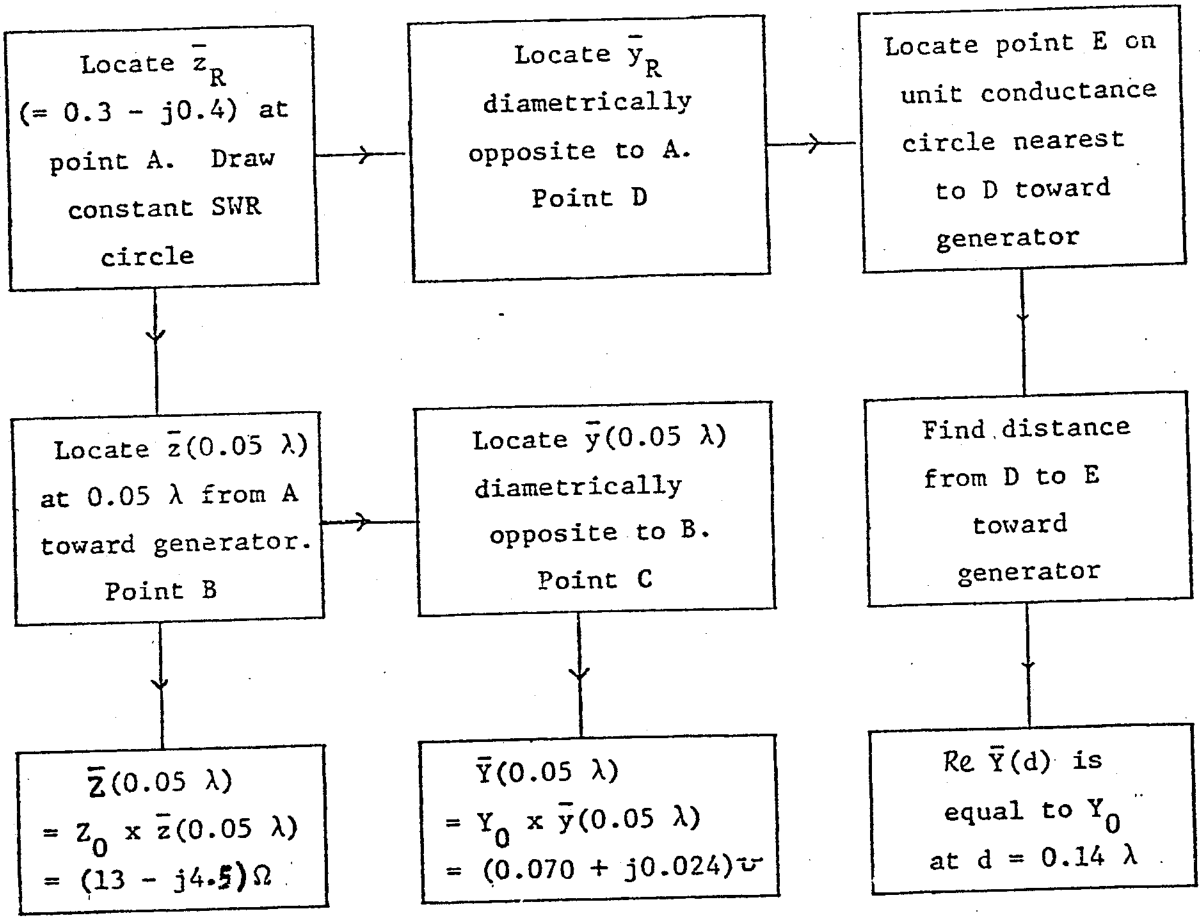
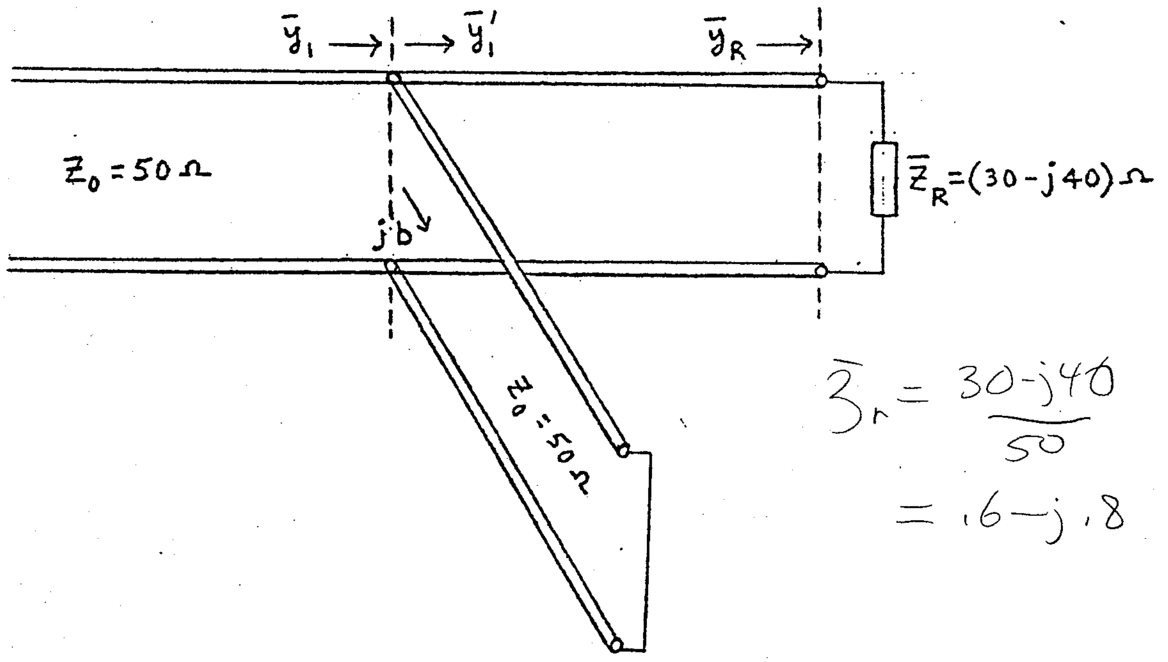


Figure 5.2.6. Flow diagram summarizing the procedures employed in Figure 5.2.5 for finding certain quantities pertinent to the line impedance and admittance behaviors for the system of Figure 5.2.1.

Single stub Matching

For match, $\bar{y}_1$ must be equal to $(1+j0)$.



$$\bar{z}_r = \frac{30-j40}{50} = .6-j.8$$

5.3.1 A transmission line system for illustrating the solution of the "single stub matching" problem by using the Smith chart.

$$y_1 = \bar{y}_1' + jb$$

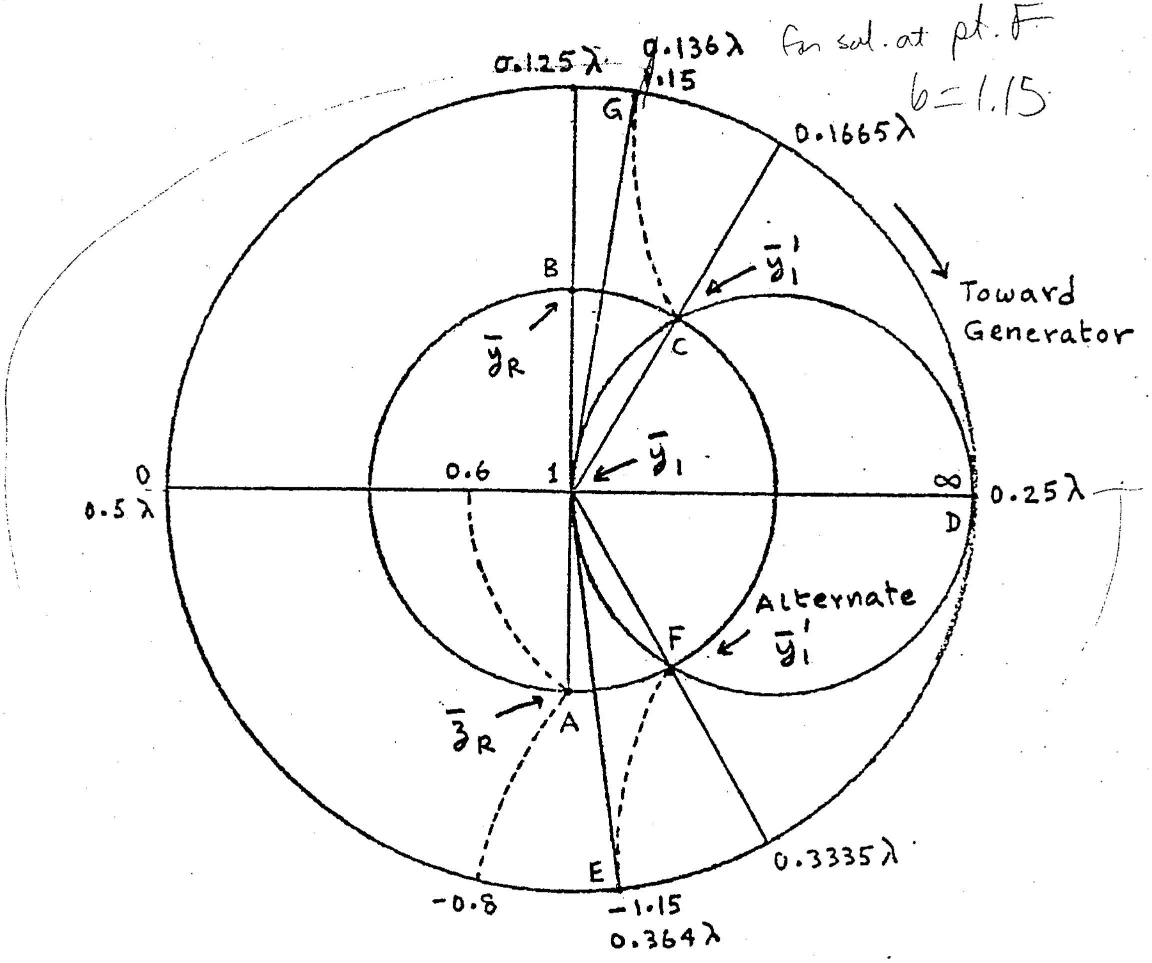
$$\bar{y}_1' = y_1 - jb = 1 - jb$$

$\therefore \bar{y}_1'$ must fall on the unit conductance circle.

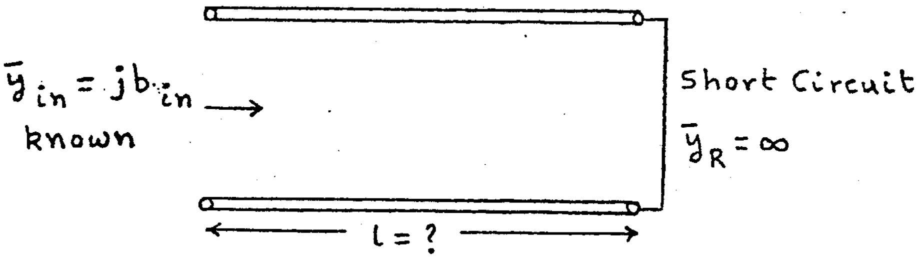
$\mathrm{Im}\,\bar{y}_1' = -b$

For sol. corresponding to pt. C,

$\mathrm{Im}\,\bar{y}_1 = 1.15 \quad \therefore \quad b = -1.15$



5.3.2 Solution of the single stub matching problem for the system of Figure 5.3.1 by using the Smith chart.



5.3.3 Formulation of the problem of finding the length l of a short-circuited stub of known normalized input susceptance b_{in}.

Stub Location		stub Length
$(0.1665 - 0.125)\lambda = 0.0415\lambda$	$\longrightarrow$	$(0.364 - 0.25)\lambda = 0.114\lambda$
$(0.3335 - 0.125)\lambda = 0.2085\lambda$	$\longrightarrow$	$(0.136 + 0.25)\lambda = 0.386\lambda$

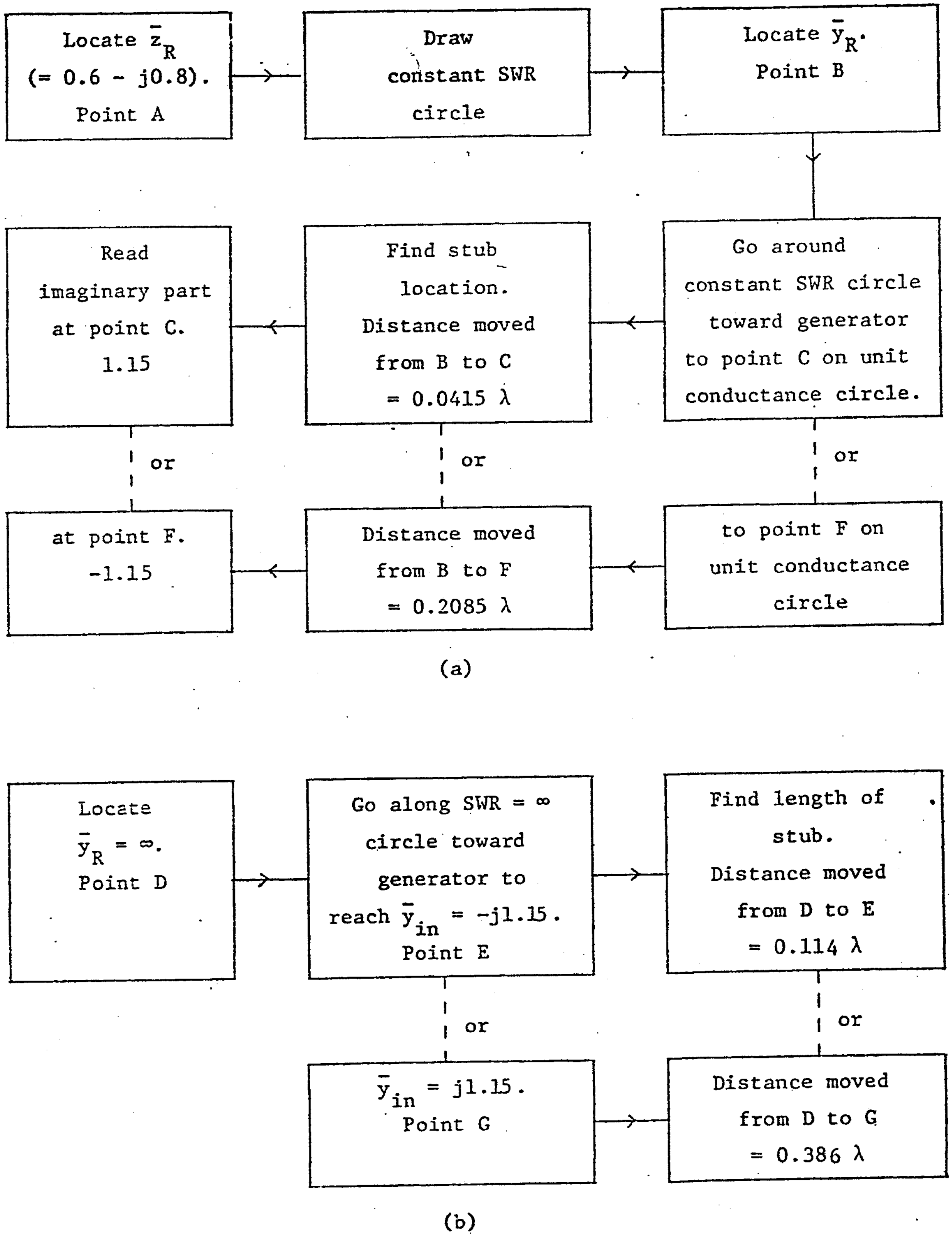
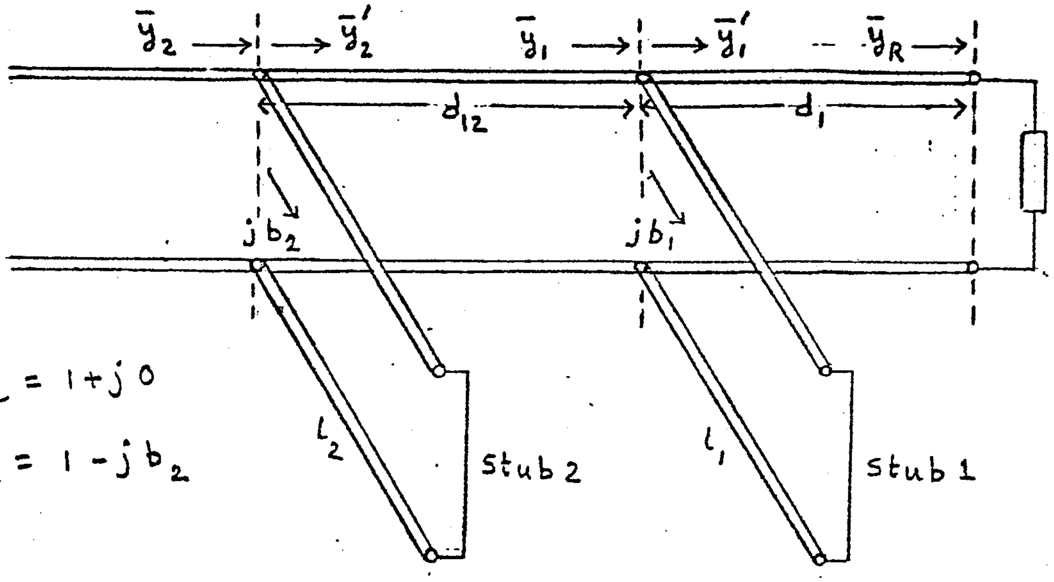


Figure 5.3.4. Flow diagrams summarizing the procedures employed in Figure 5.3.2 for finding (a) the location, and (b) the length of the stub required to achieve a match in the system of Figure 5.3.1.

Double stub Matching

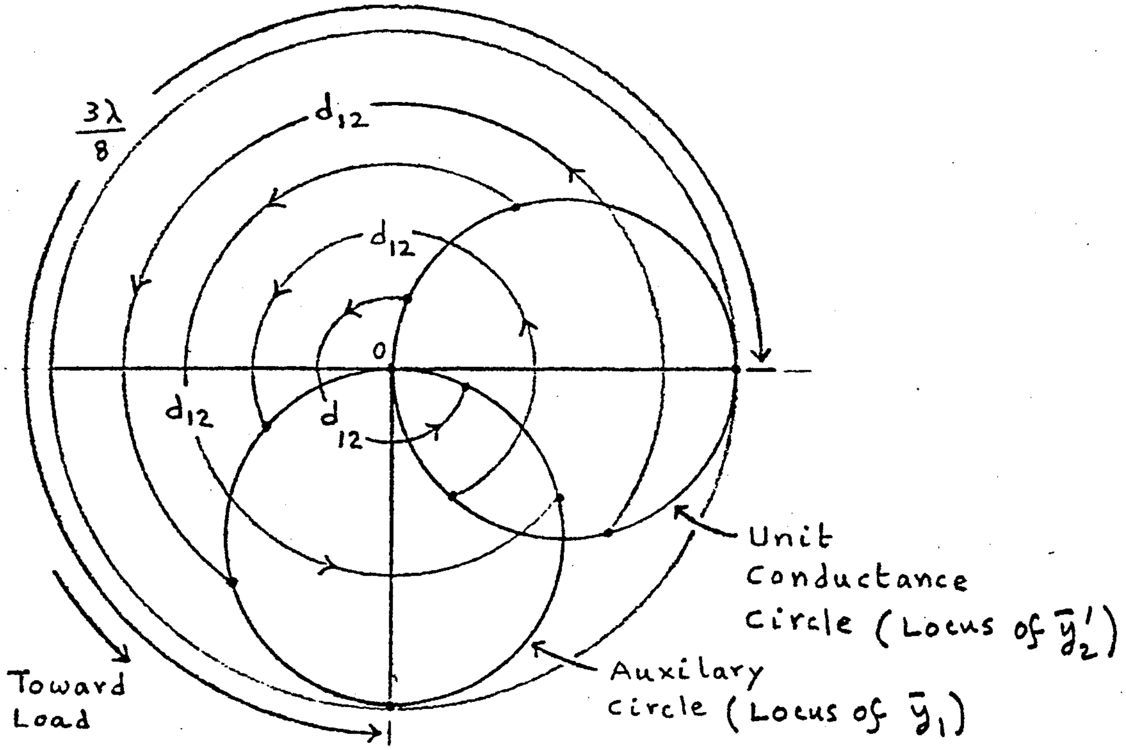
$\bar{y}_2 = \bar{y}_2' + jb_2 \ni \bar{y}_2' = y_2 - jb_2 = 1 - jb_2$



$\bar{y}_2 = 1 + j0$

$\bar{y}_2' = 1 - jb_2$

5.3.5 For illustrating the principle behind the "double stub matching" technique.



5.3.6 Rotation of the unit conductance circle by d_{12}(= 3 λ/8) toward the load about 0 for illustrating the construction of the auxilary circle, that is, the locus of $\bar{y}_1$ for possible match for the double stub matching arrangement of Figure 5.3.5.

Solution to Double stub Matching Problem

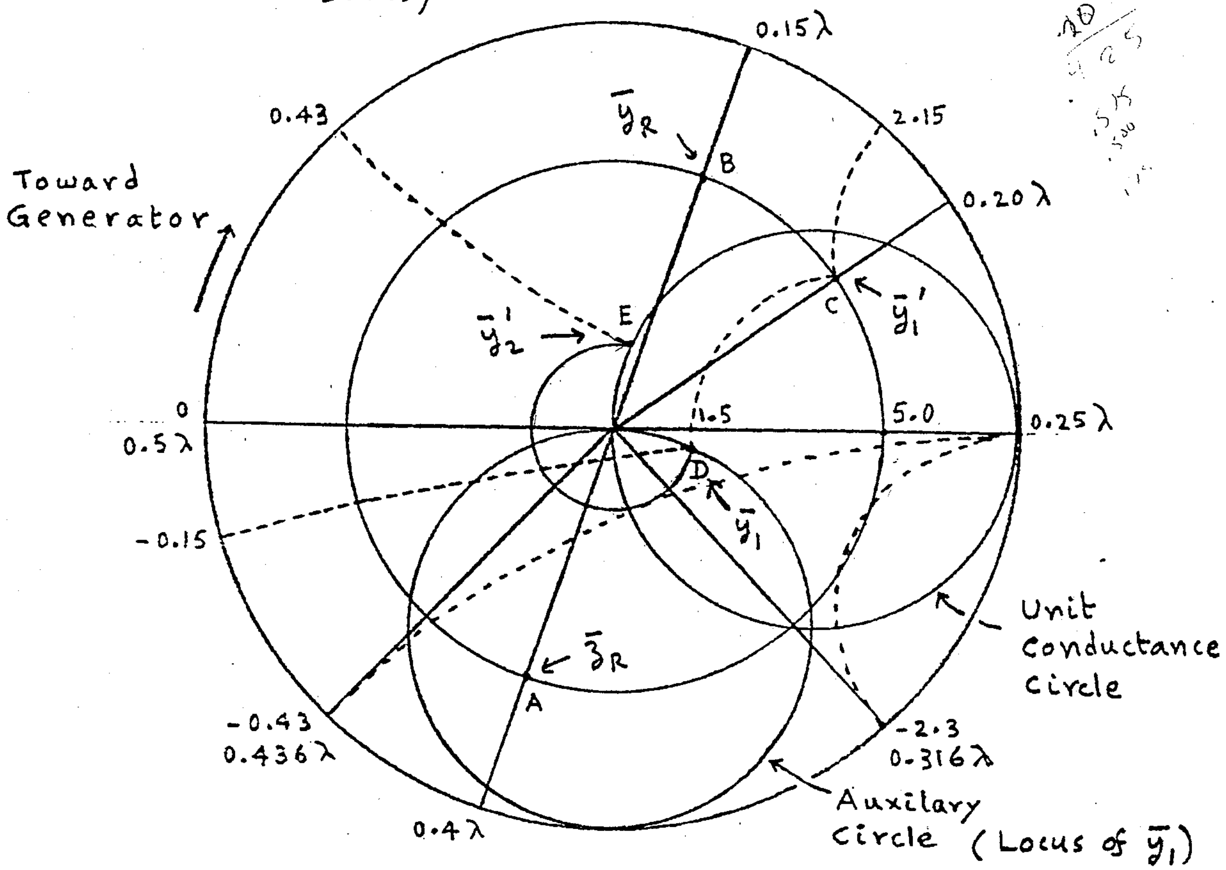
SWR = 5.0

$d_{min} = 0.1\lambda$

$Z_0 = 50\Omega$ (both line and stubs)

$d_1 = 0.05\lambda$

$d_{12} = 0.375\lambda = \frac{3}{8}\lambda$



5.3.7 Solution of a double stub matching problem by using the Smith chart.

$$\bar{y}_1 = \bar{y}_1' + jb_1$$

$$jb_1 = \bar{y}_1 - \bar{y}_1'$$

$$= (1.5 - j0.15) - (1.5 + j2.15)$$

$$= -j2.30$$

$$l_1 = (0.316 - 0.25)\lambda$$

$$= 0.066\lambda$$

$$\bar{y}_2 = \bar{y}_2' + jb_2$$

$$jb_2 = \bar{y}_2 - \bar{y}_2'$$

$$= -\operatorname{Im}\bar{y}_2'$$

$$= -j0.43$$

$$l_2 = (0.436 - 0.25)\lambda$$

$$= 0.186\lambda$$

Alternate Solution to Double Stub Matching Problem

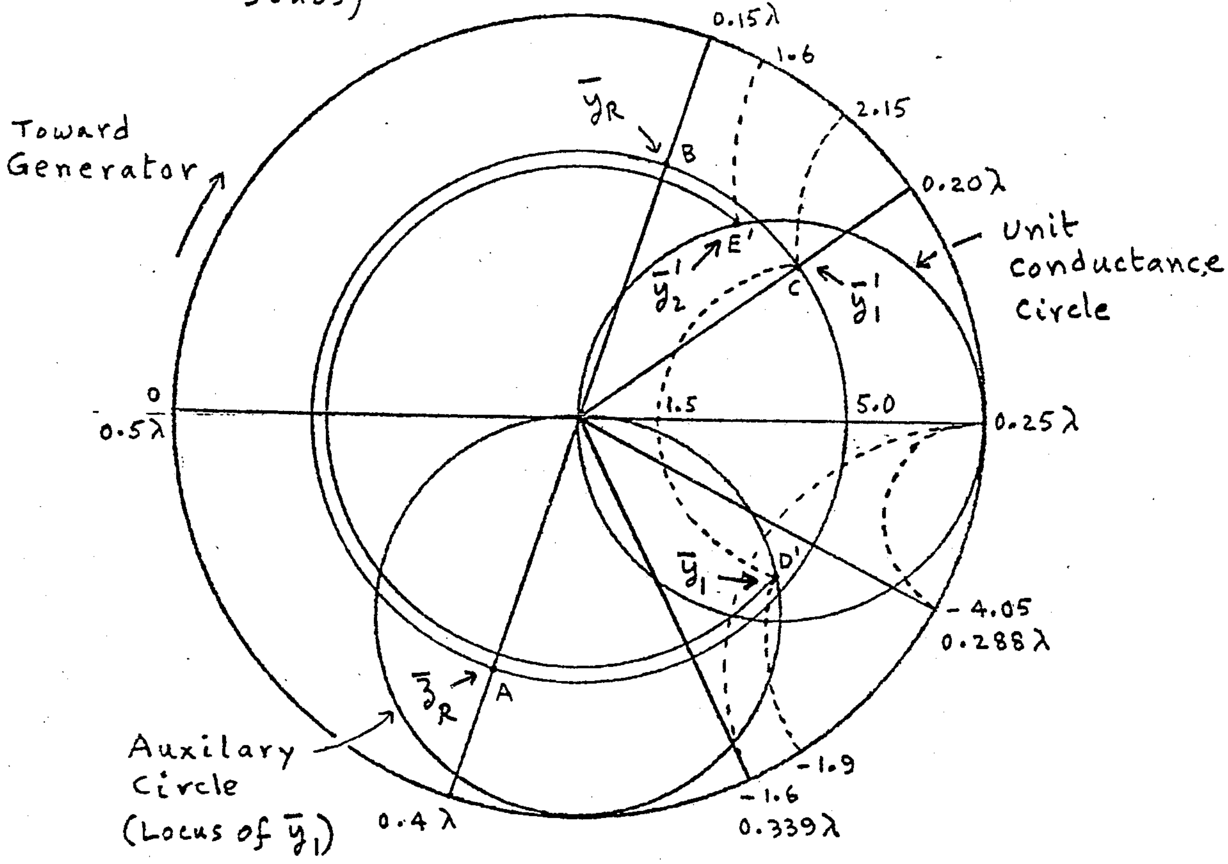
$SWR = 5{\cdot}0$

$d_{min} = 0{\cdot}1\lambda$

$z_0 = 50\Omega$ (both line and stubs)

$d_1 = 0{\cdot}05\lambda$

$d_{12} = 0{\cdot}375\lambda = \frac{3}{8}\lambda$



5.3.8 Solution to the double stub matching problem,alternate to that in Figure 5.3.7.

$$\bar{y}_1 = \bar{y}_1' + jb_1$$

$$jb_1 = \bar{y}_1 - \bar{y}_1'$$

$$= (1{\cdot}5 - j1{\cdot}9) - (1{\cdot}5 + j2{\cdot}15)$$

$$= -j\,4{\cdot}05$$

$$l_1 = (0{\cdot}288 - 0{\cdot}25)\lambda$$

$$= 0{\cdot}038\lambda$$

$$\bar{y}_2 = \bar{y}_2' + jb_2$$

$$jb_2 = \bar{y}_2 - \bar{y}_2'$$

$$= -\operatorname{Im}\bar{y}_2'$$

$$= -j\,1{\cdot}6$$

$$l_2 = (0{\cdot}339 - 0{\cdot}25)\lambda$$

$$= 0{\cdot}089\lambda$$

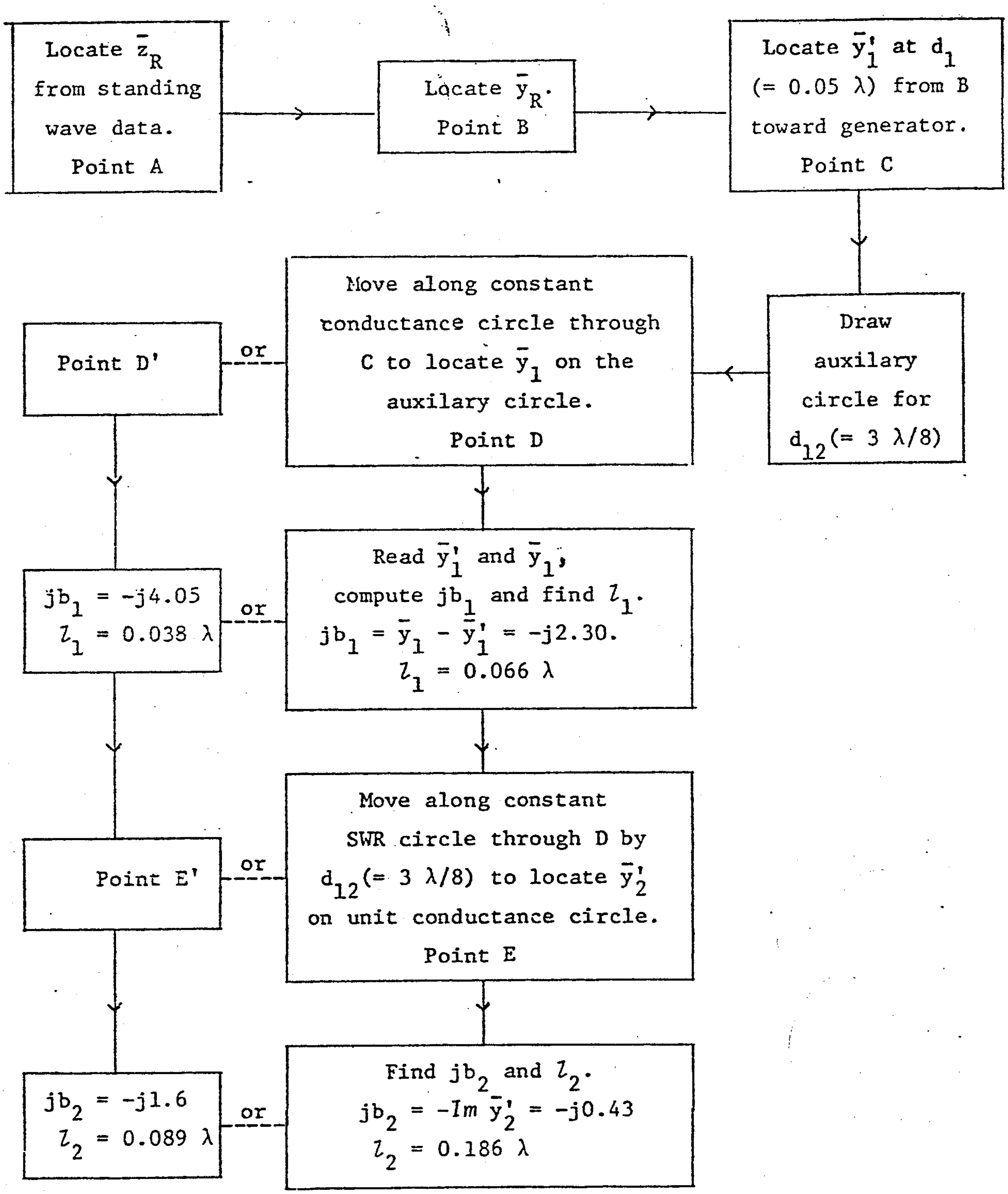
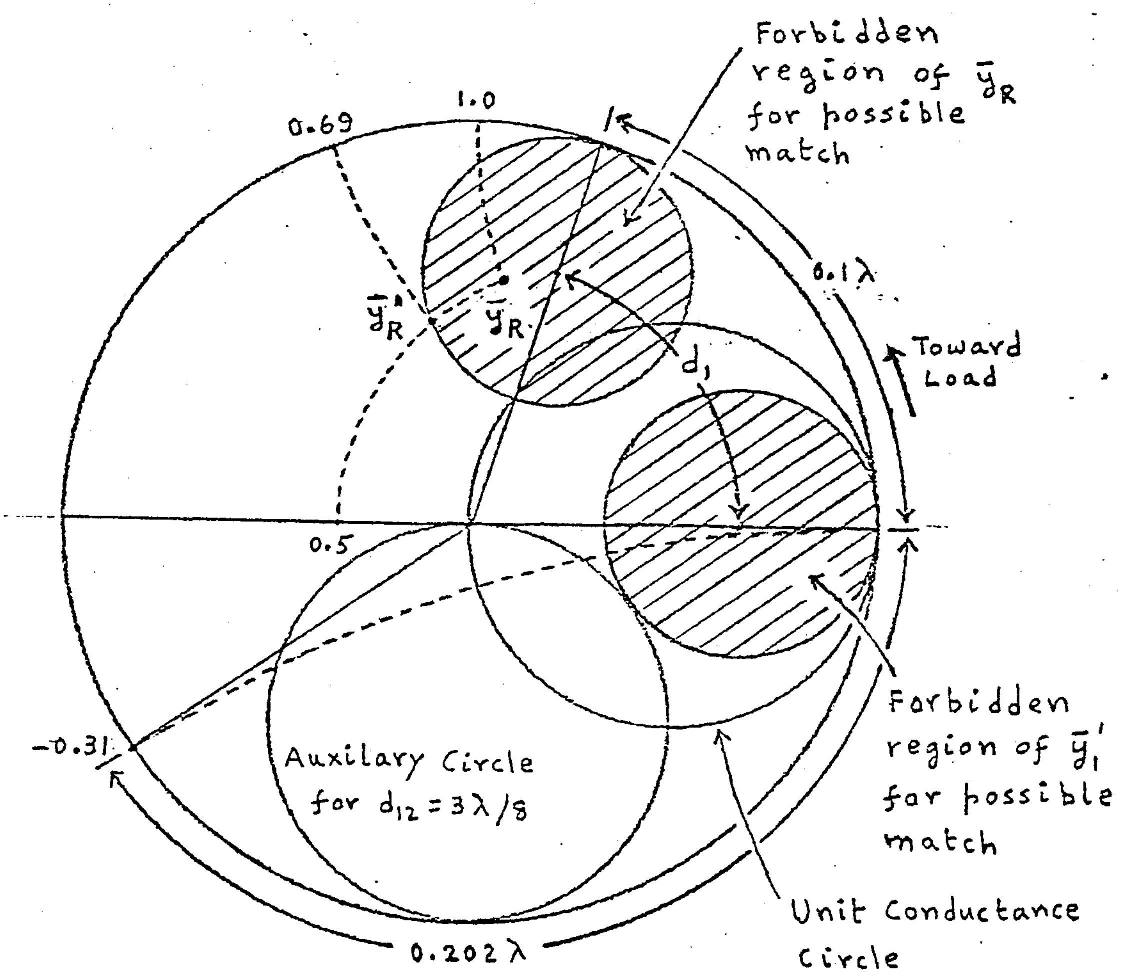
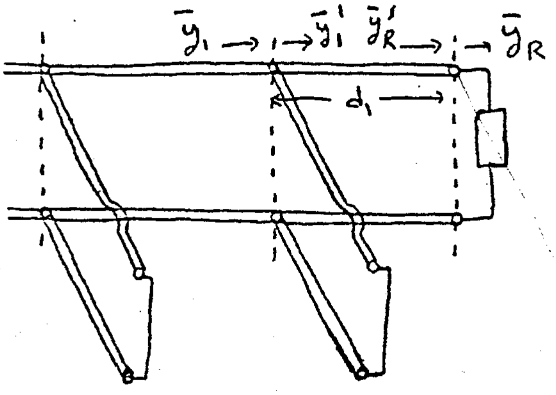


Figure 5.3.9. Flow diagram summarizing the procedures employed in Figures 5.3.7 and 5.3.8 for the solution of the double stub matching problem.

Forbidden Region in Double Stub Matching



5.3.10 Construction of the forbidden regions of $\bar{y}_1'$ and $\bar{y}_R$ for possible match in the double stub matching problem for $d_{12} = 3\lambda/8$ and $d_1 = 0.1\lambda$, and determination of the length of a third stub at the load to achieve a match for a load that originally falls in the forbidden region.



Let input susceptance of third stub at load be jb_3. Then

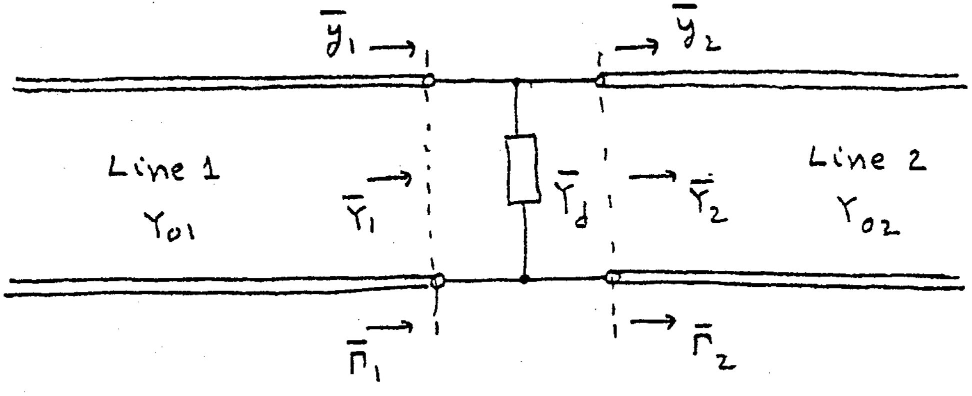
$$\bar{y}_R' = \bar{y}_R + jb_3$$

$$jb_3 = \bar{y}_R' - \bar{y}_R = j(0.69 - 1.0)$$

$$= -j0.31$$

∴ Length of third stub = 0.202 λ.

Transformation Across a Discontinuity



$$\bar{Y}_1 = \bar{Y}_2 + \bar{Y}_d$$

$$\bar{y}_1 = \frac{\bar{Y}_1}{Y_{01}} = \frac{\bar{Y}_2}{Y_{01}} + \frac{\bar{Y}_d}{Y_{01}}$$

$$= \frac{Y_{02}}{Y_{01}} \left(\frac{\bar{Y}_2}{Y_{02}} + \frac{\bar{Y}_d}{Y_{02}} \right)$$

$$= a \left(\bar{y}_2 + \bar{y}_d \right)$$

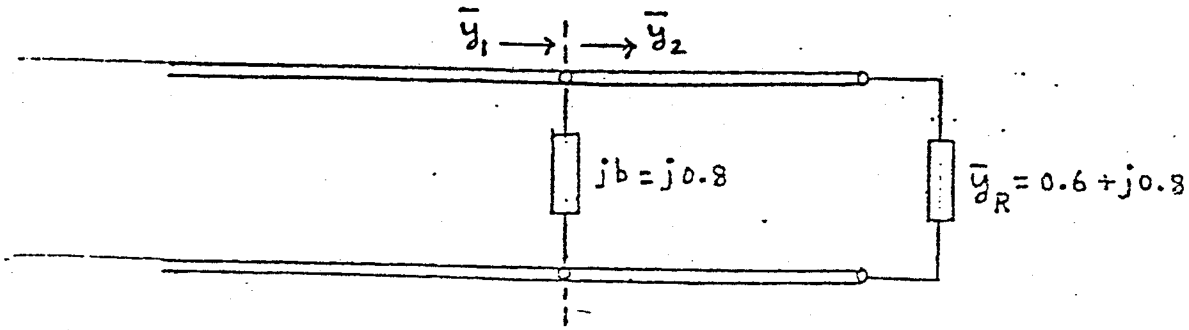
$$\frac{1 - \bar{\Gamma}_1}{1 + \bar{\Gamma}_1} = a \left(\frac{1 - \bar{\Gamma}_2}{1 + \bar{\Gamma}_2} + \bar{y}_d \right)$$

$$\boxed{\bar{\Gamma}_1 = \frac{(1 + a - a\bar{y}_d)\bar{\Gamma}_2 + (1 - a - a\bar{y}_d)}{(1 - a + a\bar{y}_d)\bar{\Gamma}_2 + (1 + a + a\bar{y}_d)}}$$

Bilinear tranformation

Maps circles onto circles.

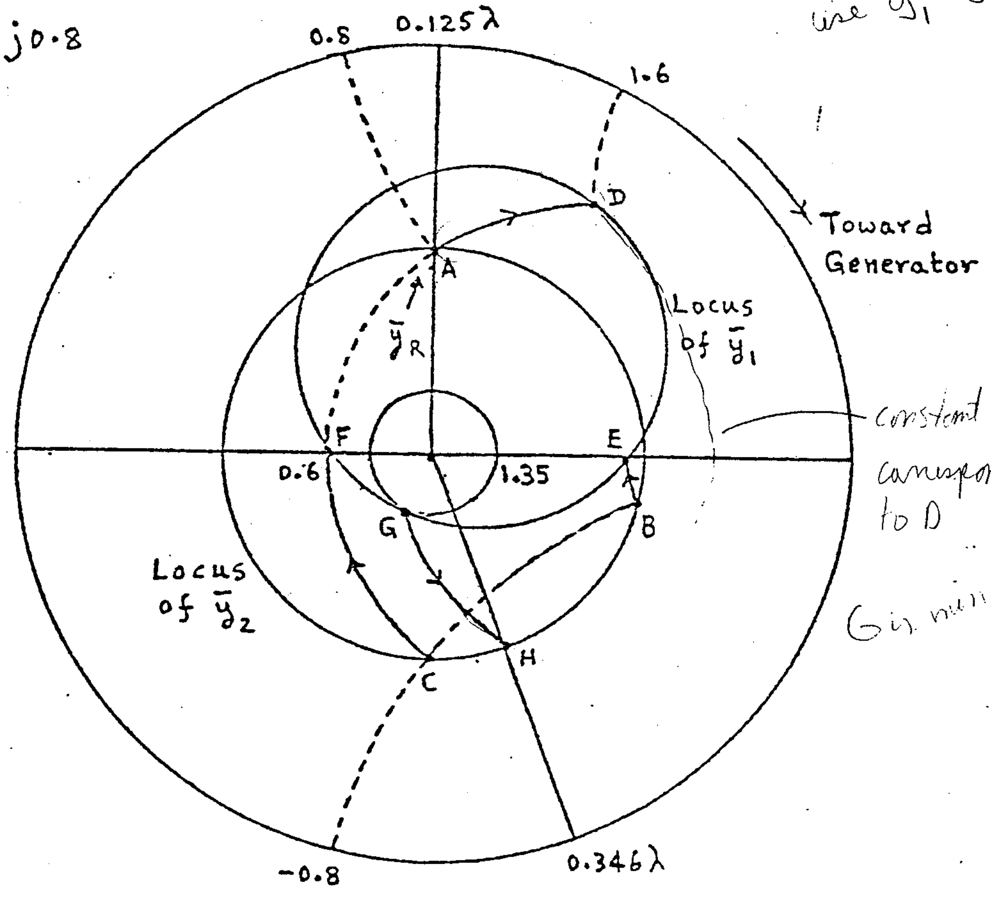
EXAMPLE of minimizing SWR with a fixed element



5.4.2 A transmission line system in which a susceptance of fixed value sliding along the line forms a discontinuity.

To get locus of $\bar{y}_1$ use $\bar{y}_1 = \bar{y}_2 + j0.8$

$\bar{y}_1 = \bar{y}_2 + j0.8$



constant SWR corresponding to D

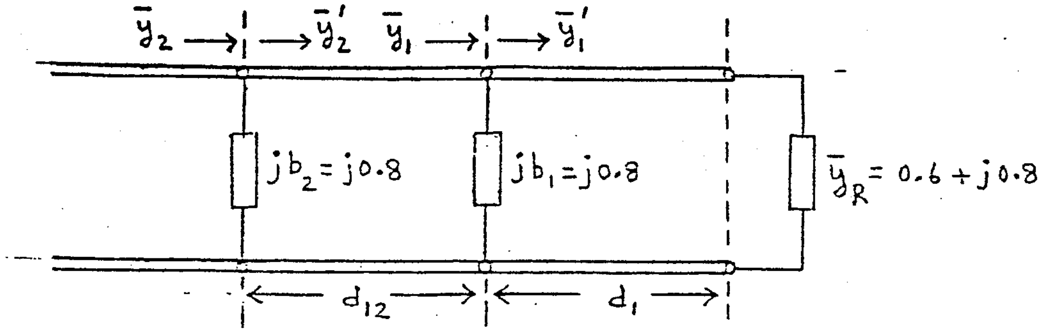
G is min SWR

5.4.3 Construction of the locus of $\bar{y}_1$ for the system of Figure 5.4.2 as the susceptance jb is slid along the line, and determination of the minimum SWR that can be achieved to the left of the susceptance and the location of the susceptance to achieve the minimum SWR.

Minimum SWR = 1.35

Location of jb from load = $(0.346 - 0.125)\lambda = 0.221\lambda$.

Matching with Two Fixed Elements

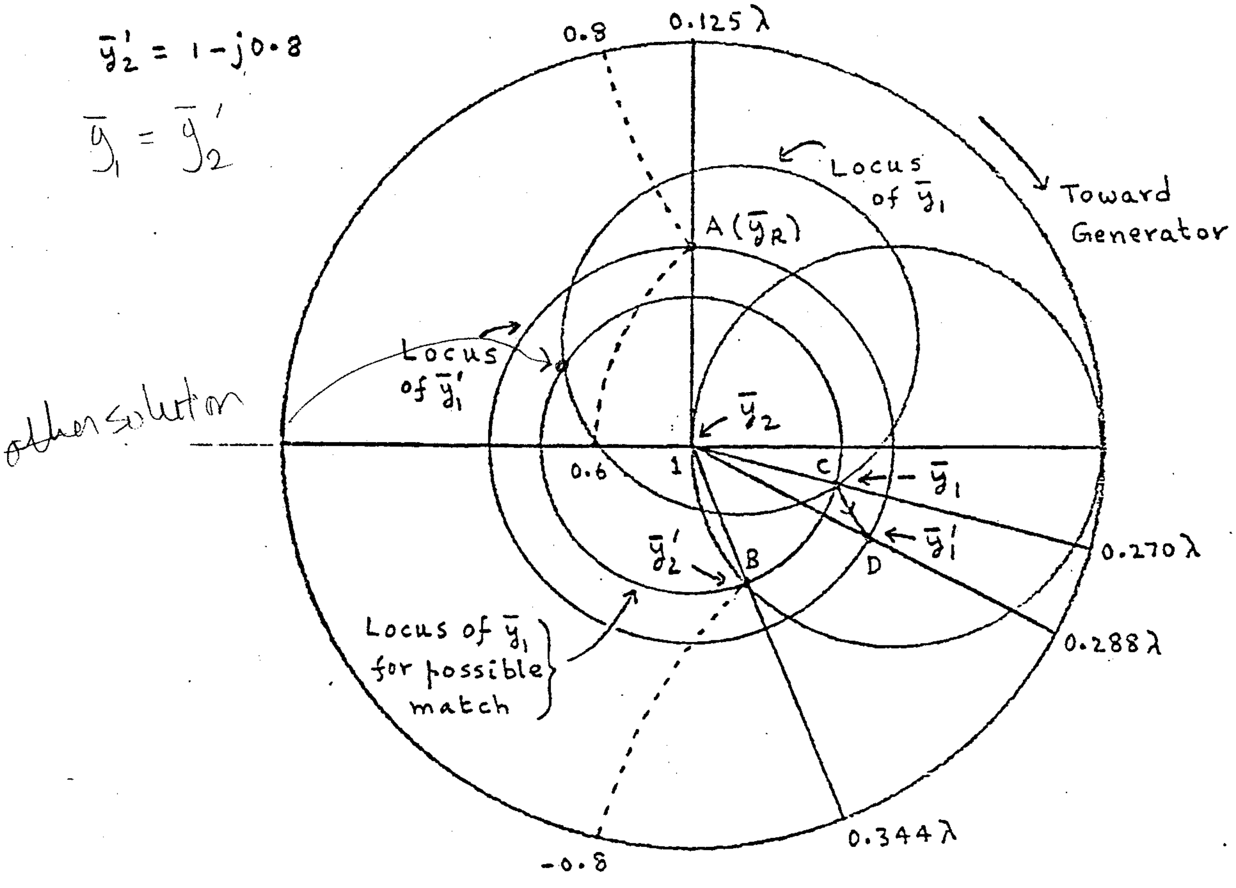


5.4.4 A transmission line system for illustrating matching with two movable susceptances of fixed values.

$\bar{y}_2 = 1 + j0$

$\bar{y}_2' = 1 - j0.8$

$\bar{y}_1 = \bar{y}_2'$



5.4.5 Solution of the matching problem for the system of Figure 5.4.4

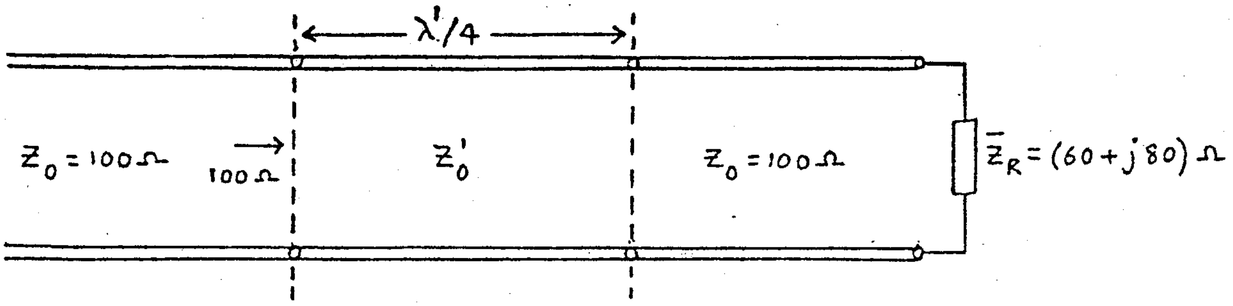
d_1 = distance from A to D toward generator

$= (0.288 - 0.125)\lambda = 0.163\lambda$

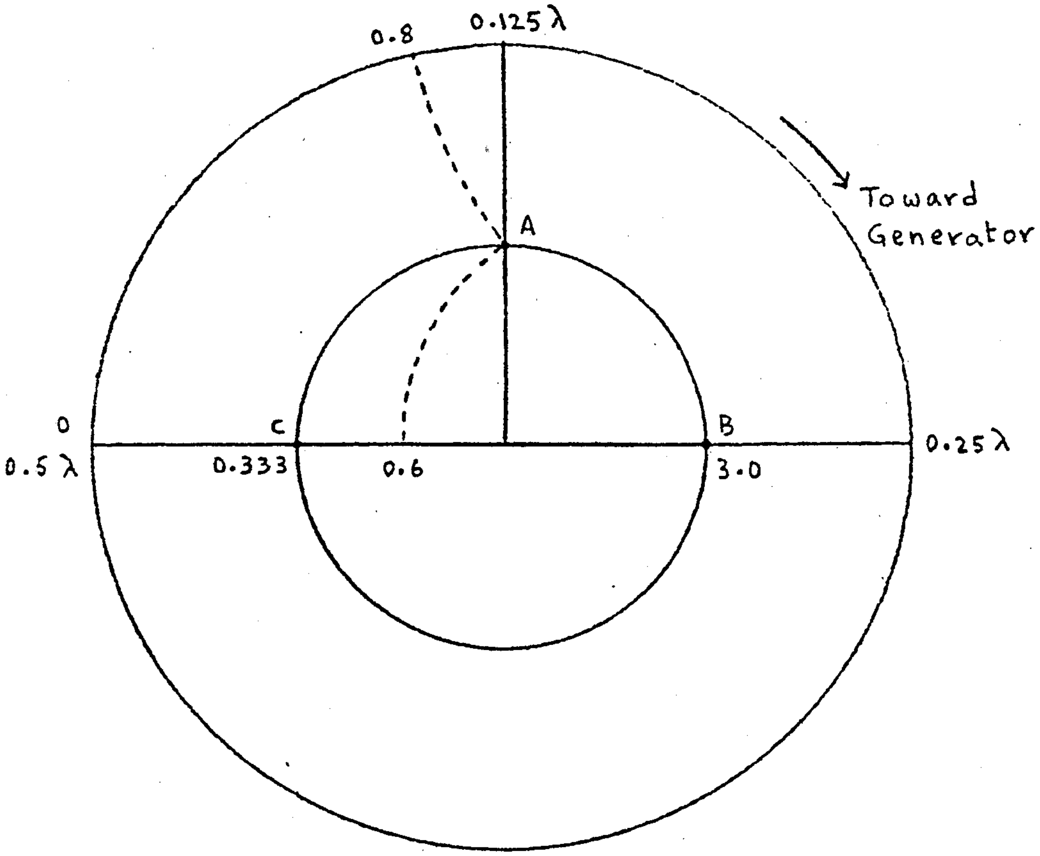
d_{12} = distance from C to B toward generator

$= (0.344 - 0.270)\lambda = 0.074\lambda$

Solution of Quarter-Wave Transformer Matching Problem



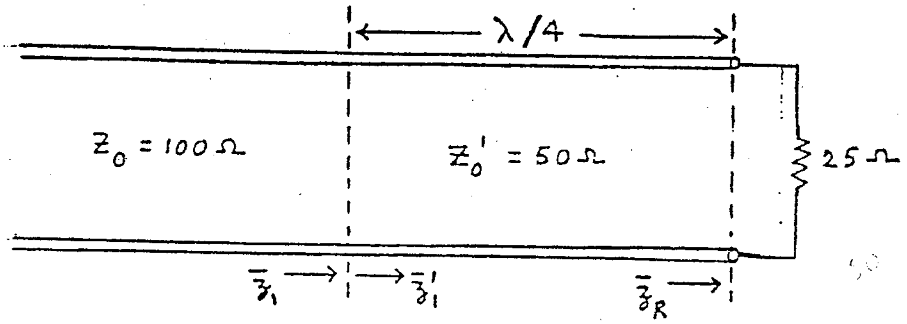
5.4.6 A transmission line system employing a quarter-wave transformer for achieving a match between the line and the load.



5.4.7 Solution of the quarter-wave transformer matching problem for the system of Figure 5.4.6.

Location	$\bar{z}(d)$	Z_0'
$(0.25 - 0.125)\lambda = 0.125\lambda$	$3.0 \times 100 = 300\Omega$	$\sqrt{100 \times 300} = 173.2\,\Omega$
$(0.50 - 0.125)\lambda = 0.375\lambda$	$0.333 \times 100 = 33.3\,\Omega$	$\sqrt{100 \times 33.3} = 57.7\,\Omega$

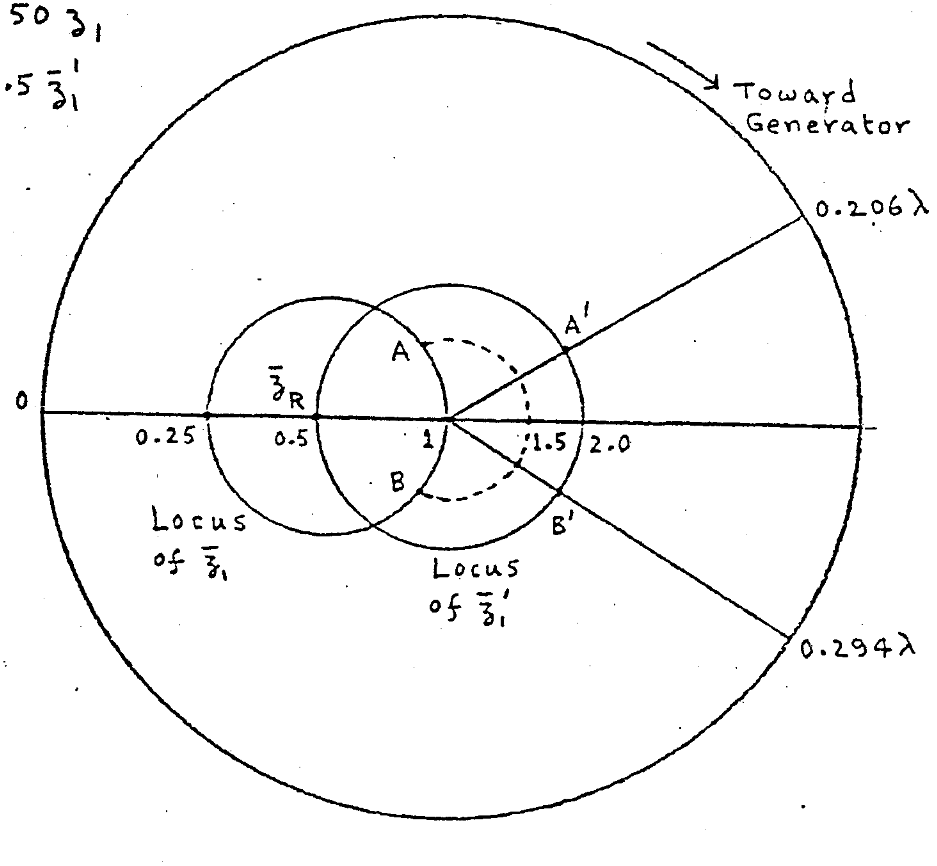
Determination of Bandwidth of a λ/4 Transformer Matched System



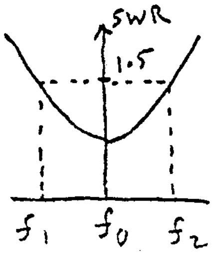
5.4.8 A quarter-wave transformer matched system.

$$100\,\bar{z}_1 = 50\,\bar{z}_1'$$

$$\bar{z}_1 = 0.5\,\bar{z}_1'$$



5.4.9 For the determination of the bandwidth between the two frequencies on either side of the design frequency for which the SWR on the main line of the system of Figure 5.4.8 is 1.5.



$$0.206\lambda \rightarrow f_1 = \frac{0.206}{0.25} f_0 = 0.824 f_0$$

$$0.294\lambda \rightarrow f_2 = \frac{0.294}{0.25} f_0 = 1.176 f_0$$

$$BW = (1.176 - 0.824) f_0 = 0.352 f_0.$$

WAVEGUIDES

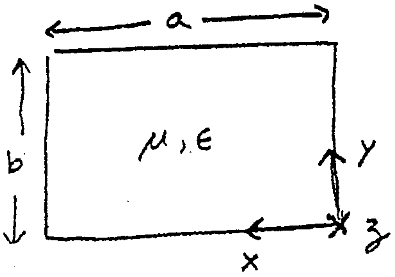
(24-27)

Rectangular Waveguides : TM and TE Waves

TEM waves : $E_z = H_z = 0$

TM waves : $H_z = 0$

TE waves : $E_z = 0$



$$\underset{\sim}{\nabla} \times \underset{\sim}{\bar{E}} = -j\omega\mu \underset{\sim}{\bar{H}} \qquad \underset{\sim}{\nabla} \times \underset{\sim}{\bar{H}} = j\omega\epsilon \underset{\sim}{\bar{E}}$$

$$\frac{\partial \bar{E}_z}{\partial y} - \frac{\partial \bar{E}_y}{\partial z} = -j\omega\mu \bar{H}_x \qquad \frac{\partial \bar{H}_z}{\partial y} - \frac{\partial \bar{H}_y}{\partial z} = j\omega\epsilon \bar{E}_x$$

$$\frac{\partial \bar{E}_x}{\partial z} - \frac{\partial \bar{E}_z}{\partial x} = -j\omega\mu \bar{H}_y \qquad \frac{\partial \bar{H}_x}{\partial z} - \frac{\partial \bar{H}_z}{\partial x} = j\omega\epsilon \bar{E}_y$$

$$\frac{\partial \bar{E}_y}{\partial x} - \frac{\partial \bar{E}_x}{\partial y} = -j\omega\mu \bar{H}_z \qquad \frac{\partial \bar{H}_y}{\partial x} - \frac{\partial \bar{H}_x}{\partial y} = j\omega\epsilon \bar{E}_z$$

phase [illegible]

$$\underset{\sim}{\bar{E}}(x,y,z) = \underset{\sim}{\bar{E}}(x,y)\, e^{\mp j\beta_z z} \; ; \; \underset{\sim}{\bar{H}}(x,y,z) = \underset{\sim}{\bar{H}}(x,y)\, e^{\mp j\beta_z z}$$

$$\frac{\partial \bar{E}_i}{\partial z} = \mp j\beta_z \bar{E}_i \; ; \; \frac{\partial \bar{H}_i}{\partial z} = \mp j\beta_z \bar{H}_i \; , \quad i = x, y, z$$

$$\frac{\partial \bar{E}_z}{\partial y} \pm j\beta_z \bar{E}_y = -j\omega\mu \bar{H}_x \qquad \frac{\partial \bar{H}_z}{\partial y} \pm j\beta_z \bar{H}_y = j\omega\epsilon \bar{E}_x$$

$$\mp j\beta_z \bar{E}_x - \frac{\partial \bar{E}_z}{\partial x} = -j\omega\mu \bar{H}_y \qquad \mp j\beta_z \bar{H}_x - \frac{\partial \bar{H}_z}{\partial x} = j\omega\epsilon \bar{E}_y$$

$$\frac{\partial \bar{E}_y}{\partial x} - \frac{\partial \bar{E}_x}{\partial y} = -j\omega\mu \bar{H}_z \qquad \frac{\partial \bar{H}_y}{\partial x} - \frac{\partial \bar{H}_x}{\partial y} = j\omega\epsilon \bar{E}_z$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} i_x & i_y & i_z \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix} \qquad \vec{\nabla} \times (\vec{\nabla} \times A) = \begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) & \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) & \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \end{vmatrix}$$

$$\boxed{\begin{aligned}
\bar{E}_x &= \frac{1}{\beta_z^2-\beta^2}\left[\pm j\beta_z\frac{\partial \bar{E}_z}{\partial x} + j\omega\mu\frac{\partial \bar{H}_z}{\partial y}\right]\\
\bar{E}_y &= \frac{1}{\beta^2-\beta_z^2}\left[\mp j\beta_z\frac{\partial \bar{E}_z}{\partial y} + j\omega\mu\frac{\partial \bar{H}_z}{\partial x}\right]\\
\bar{H}_x &= \frac{1}{\beta^2-\beta_z^2}\left[j\omega\epsilon\frac{\partial \bar{E}_z}{\partial y} \mp j\beta_z\frac{\partial \bar{H}_z}{\partial x}\right]\\
\bar{H}_y &= \frac{1}{\beta_z^2-\beta^2}\left[j\omega\epsilon\frac{\partial \bar{E}_z}{\partial x} \pm j\beta_z\frac{\partial \bar{H}_z}{\partial y}\right]
\end{aligned}}$$

Thus $\bar{E}_x$, $\bar{E}_y$, $\bar{H}_x$, and $\bar{H}_y$, that is, the transverse field components can be obtained from the solutions for $\bar{E}_z$ and $\bar{H}_z$, that is, the longitudinal field components. Hence, it is sufficient if the wave equations for $\bar{E}_z$ and $\bar{H}_z$ are solved. These wave equations are obtained as follows:

$$\underset{\sim}{\nabla}\times\underset{\sim}{\nabla}\times\underset{\sim}{\bar{E}} = -j\omega\mu\,\underset{\sim}{\nabla}\times\underset{\sim}{\bar{H}} \qquad \underset{\sim}{\nabla}\times\underset{\sim}{\nabla}\times\underset{\sim}{\bar{H}} = j\omega\epsilon\,\underset{\sim}{\nabla}\times\underset{\sim}{E}$$

$$\underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{\bar{E}}) - \nabla^2\underset{\sim}{\bar{E}} = -j\omega\mu(j\omega\epsilon\underset{\sim}{\bar{E}}) \qquad \underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{\bar{H}}) - \nabla^2\underset{\sim}{\bar{H}} = j\omega\epsilon(-j\omega\mu\underset{\sim}{\bar{H}})$$

$$\nabla^2\underset{\sim}{\bar{E}} + \beta^2\underset{\sim}{\bar{E}} = 0 \qquad \nabla^2\underset{\sim}{\bar{H}} + \beta^2\underset{\sim}{\bar{H}} = 0$$

Since $\nabla^2\underset{\sim}{A} = \underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{A}) - \underset{\sim}{\nabla}\times\underset{\sim}{\nabla}\times\underset{\sim}{A}$

$$= (\nabla^2 A_x)\,\hat{x} + (\nabla^2 A_y)\,\hat{y} + (\nabla^2 A_z)\,\hat{z}$$

The wave equations for $\bar{E}_z$ and $\bar{H}_z$ are

$$= \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A\cancel{x}}{\partial z^2}$$

$$\nabla^2 \bar{E}_z + \beta^2 \bar{E}_z = 0 \qquad \nabla^2 \bar{H}_z + \beta^2 \bar{H}_z = 0$$

$$\frac{\partial^2 \bar{E}_z}{\partial x^2} + \frac{\partial^2 \bar{E}_z}{\partial y^2} + \frac{\partial^2 \bar{E}_z}{\partial z^2} + \beta^2 \bar{E}_z = 0 \qquad \frac{\partial^2 \bar{H}_z}{\partial x^2} + \frac{\partial^2 \bar{H}_z}{\partial y^2} + \frac{\partial^2 \bar{H}_z}{\partial z^2} + \beta^2 \bar{H}_z = 0$$

$$\frac{\partial^2 \bar{E}_z}{\partial x^2} + \frac{\partial^2 \bar{E}_z}{\partial y^2} - \beta_z^2 \bar{E}_z + \beta^2 \bar{E}_z = 0 \qquad \frac{\partial^2 \bar{H}_z}{\partial x^2} + \frac{\partial^2 \bar{H}_z}{\partial y^2} - \beta_z^2 \bar{H}_z + \beta^2 \bar{H}_z = 0$$

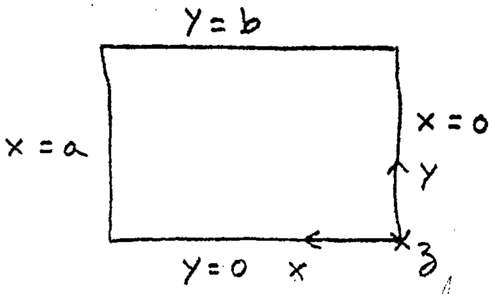
$$\boxed{\begin{aligned} \frac{\partial^2 \bar{E}_z}{\partial x^2} + \frac{\partial^2 \bar{E}_z}{\partial y^2} &= (\beta_z^2 - \beta^2)\, \bar{E}_z \qquad (1) \\ \frac{\partial^2 \bar{H}_z}{\partial x^2} + \frac{\partial^2 \bar{H}_z}{\partial y^2} &= (\beta_z^2 - \beta^2)\, \bar{H}_z \qquad (2) \end{aligned}}$$

For TM waves, $\bar{H}_z = 0$; we solve (1) subject to the boundary conditions at the walls of the guide.

For TE waves, $\bar{E}_z = 0$; we solve (2) subject to the boundary conditions at the walls of the guide.

The boundary conditions require that all tangential components of $\underset{\sim}{E}$ be zero at the walls of the guide.

TM Waves ($\bar{H}_z \equiv 0$)



For a perfect cnd surface

$E_{tang} = 0$

$$\frac{\partial^2 \bar{E}_z}{\partial x^2} + \frac{\partial^2 \bar{E}_z}{\partial y^2} = (\beta_z^2 - \beta^2)\,\bar{E}_z$$

B.C.

$\bar{E}_z = 0$ for $x = 0$, $0 < y < b$

$\bar{E}_z = 0$ for $x = a$, $0 < y < b$

$\bar{E}_z = 0$ for $y = 0$, $0 < x < a$

$\bar{E}_z = 0$ for $y = b$, $0 < x < a$

Solution of Wave Equation:

Assume $\bar{E}_z = X(x) \cdot Y(y)$.

Then $X''(x) \cdot Y(y) + X(x) \cdot Y''(y) = (\beta_z^2 - \beta^2)\, X(x) \cdot Y(y)$

$$\frac{X''}{X} + \frac{Y''}{Y} = \beta_z^2 - \beta^2$$

$$\frac{X''}{X} = -\beta_x^2, \text{ a constant}$$

$$\frac{Y''}{Y} = -\beta_y^2, \text{ a constant}$$

Note That $\boxed{\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2}$

$$X = \bar{A}_1 e^{j\beta_x x} + \bar{A}_2 e^{-j\beta_x x}$$

$$Y = \bar{B}_1 e^{j\beta_y y} + \bar{B}_2 e^{-j\beta_y y}$$

$$\bar{E}_z = (\bar{A}_1 e^{j\beta_x x} + \bar{A}_2 e^{-j\beta_x x})(\bar{B}_1 e^{j\beta_y y} + \bar{B}_2 e^{-j\beta_y y})\, e^{\mp j\beta_z z}$$

$\bar{E}_z = 0$ for $x = 0,\ 0 < y < b \longrightarrow \bar{A}_1 + \bar{A}_2 = 0 \longrightarrow \bar{A}_2 = -\bar{A}_1$

$\bar{E}_z = 0$ for $y = 0,\ 0 < x < a \longrightarrow \bar{B}_1 + \bar{B}_2 = 0 \longrightarrow \bar{B}_2 = -\bar{B}_1$

$$\bar{E}_z = \bar{A} \sin \beta_x x \sin \beta_y y\, e^{\mp j\beta_z z}$$

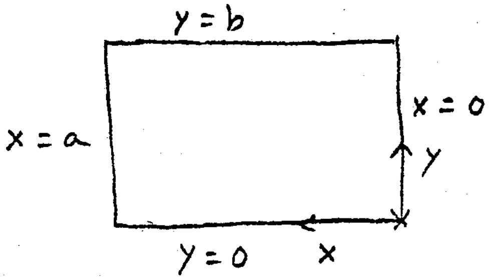
$\bar{E}_z = 0$ for $x = a,\ 0 < y < b \longrightarrow \sin \beta_x a = 0 \longrightarrow \boxed{\beta_x = \frac{m\pi}{a},\ m = 0,1,2,\ldots}$

$\bar{E}_z = 0$ for $y = b,\ 0 < x < a \longrightarrow \sin \beta_y b = 0 \longrightarrow \boxed{\beta_y = \frac{n\pi}{b},\ n = 0,1,2,\ldots}$

$$\boxed{\begin{aligned}
\bar{E}_z &= \bar{A} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, e^{\mp j\beta_z z} \\
\bar{E}_x &= \frac{1}{\beta_z^2 - \beta^2}\left(\pm j\beta_z \frac{m\pi}{a} \bar{A} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right) e^{\mp j\beta_z z} \\
\bar{E}_y &= \frac{1}{\beta^2 - \beta_z^2}\left(\mp j\beta_z \frac{n\pi}{b} \bar{A} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}\right) e^{\mp j\beta_z z} \\
\bar{H}_x &= \frac{1}{\beta^2 - \beta_z^2}\left(jw\epsilon \frac{n\pi}{b} \bar{A} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}\right) e^{\mp j\beta_z z} \\
\bar{H}_y &= \frac{1}{\beta_z^2 - \beta^2}\left(jw\epsilon \frac{m\pi}{a} \bar{A} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right) e^{\mp j\beta_z z}
\end{aligned}}$$

TE Waves $(\bar{E}_z \equiv 0)$

$$\frac{\partial^2 \bar{H}_z}{\partial x^2} + \frac{\partial^2 \bar{H}_z}{\partial y^2} = (\beta_z^2 - \beta^2)\,\bar{H}_z$$



B.C.

$\bar{E}_x = 0$ for $y=0,\ 0<x<a$ $\longrightarrow$ $\dfrac{\partial \bar{H}_z}{\partial y} = 0$ for $y=0,\ 0<x<a$

$\bar{E}_x = 0$ for $y=b,\ 0<x<a$ $\longrightarrow$ $\dfrac{\partial \bar{H}_z}{\partial y} = 0$ for $y=b,\ 0<x<a$

$\bar{E}_y = 0$ for $x=0,\ 0<y<b$ $\longrightarrow$ $\dfrac{\partial \bar{H}_z}{\partial x} = 0$ for $x=0,\ 0<y<b$

$\bar{E}_y = 0$ for $x=a,\ 0<y<b$ $\longrightarrow$ $\dfrac{\partial \bar{H}_z}{\partial x} = 0$ for $x=a,\ 0<y<b$

Solution of Wave Equation

$$\bar{H}_z = (\bar{C}_1 e^{j\beta_x x} + \bar{C}_2 e^{-j\beta_x x})(\bar{D}_1 e^{j\beta_y y} + \bar{D}_2 e^{-j\beta_y y})\, e^{\mp j\beta_z z}$$

$$\boxed{\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2}$$

$\dfrac{\partial \bar{H}_z}{\partial x} = 0$ for $x=0,\ 0<y<b \longrightarrow \bar{C}_1 - \bar{C}_2 = 0 \longrightarrow \bar{C}_2 = \bar{C}_1$

$\dfrac{\partial \bar{H}_z}{\partial y} = 0$ for $y=0,\ 0<x<a \longrightarrow \bar{D}_1 - \bar{D}_2 = 0 \longrightarrow \bar{D}_2 = \bar{D}_1$

$$\bar{H}_z = \bar{C} \cos\beta_x x \cos\beta_y y \; e^{\mp j\beta_z z}$$

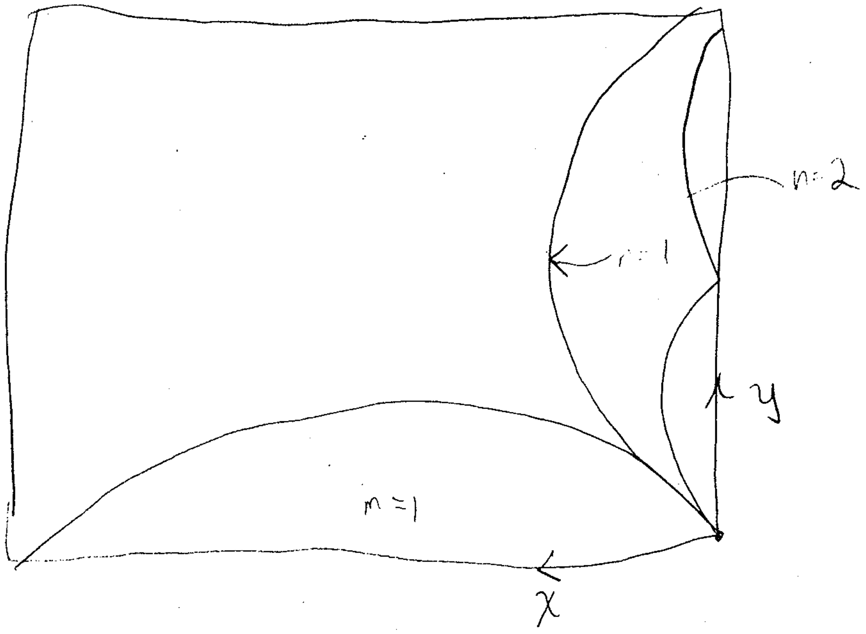
$\dfrac{\partial \bar{H}_z}{\partial x} = 0$ for $x=a,\ 0<y<b \longrightarrow \sin\beta_x a = 0 \longrightarrow \boxed{\beta_x = \dfrac{m\pi}{a},\ m=0,1,2,\ldots}$

$\dfrac{\partial \bar{H}_z}{\partial y} = 0$ for $y=b,\ 0<x<a \longrightarrow \sin\beta_y b = 0 \longrightarrow \boxed{\beta_y = \dfrac{n\pi}{b},\ n=0,1,2,\ldots}$

TM$_{m,n}$ mode

$$\boxed{\begin{aligned}
\bar{H}_z &= \bar{C} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, e^{\mp j\beta_z z} \\
\bar{E}_x &= \frac{1}{\beta^2-\beta_z^2}\left(j\omega\mu \frac{n\pi}{b}\, \bar{C} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\right) e^{\mp j\beta_z z} \\
\bar{E}_y &= \frac{1}{\beta_z^2-\beta^2}\left(j\omega\mu \frac{m\pi}{a}\, \bar{C} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\right) e^{\mp j\beta_z z} \\
\bar{H}_x &= \frac{1}{\beta_z^2-\beta^2}\left(\mp j\beta_z \frac{m\pi}{a}\, \bar{C} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\right) e^{\mp j\beta_z z} \\
\bar{H}_y &= \frac{1}{\beta^2-\beta_z^2}\left(\pm j\beta_z \frac{n\pi}{b}\, \bar{C} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\right) e^{\mp j\beta_z z}
\end{aligned}}$$

not possible to have TM$_{0,0}$
lowest $m,n=1,1$



Characteristics of TM and TE waves

From the field expressions, we note that the x and y variations of the field components correspond to complete standing waves with m half sinusoids in the x direction and n half sinusoids in the y direction. Hence they are called $TM_{m,n}$ and $TE_{m,n}$ modes.

> Note that for TM modes, both m and n must be nonzero whereas for TE modes, either m or n must be nonzero

For both types of modes,

$$\beta_z^2 = \beta^2 - \beta_x^2 - \beta_y^2$$

$$= \omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$\mathrm{Re}\left[e^{j\beta z}e^{j\omega t}\right]$

$\cos(\omega t + \beta z)$ — prop occurs

For $\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$, $\beta_z^2 > 0$, β_z is real and hence propagation occurs.

$e^{j\beta_z z} = e^{j\beta z}$

so freq matters

For $\omega^2\mu\epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$, $\beta_z^2 < 0$, β_z is imaginary and hence no propagation.

$e^{j\cdot j\beta z} = e^{-\beta z}$

$\mathrm{Re}\left[e^{-\beta z}e^{j\omega t}\right]$

$e^{-\beta z}\cos\omega t$

no prop

Thus there exists a <u>cutoff frequency</u> for each mode below which propagation does not occur. This cutoff frequency is given by

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\boxed{f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$f < f_c$, no propagation

$f > f_c$, propagation

by convention $a \geq b$

or, in terms of wavelength

$$\boxed{\lambda_c = \frac{v_p}{f_c} = \frac{1}{f_c\sqrt{\mu\epsilon}} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$$

$\lambda < \lambda_c$, propagation

$\lambda > \lambda_c$, no propagation

The mode with the lowest cutoff frequency is called the <u>dominant mode</u>. It is the $TE_{1,0}$ mode.

$$\boxed{\begin{aligned} [f_c]_{TE_{1,0}} &= \frac{1}{2a\sqrt{\mu\epsilon}} \\ [\lambda_c]_{TE_{1,0}} &= 2a \end{aligned}}$$

Dominant mode

Then

$$\frac{f_c}{[f_c]_{TE_{1,0}}} = \sqrt{m^2 + \left(n\,\frac{a}{b}\right)^2}$$

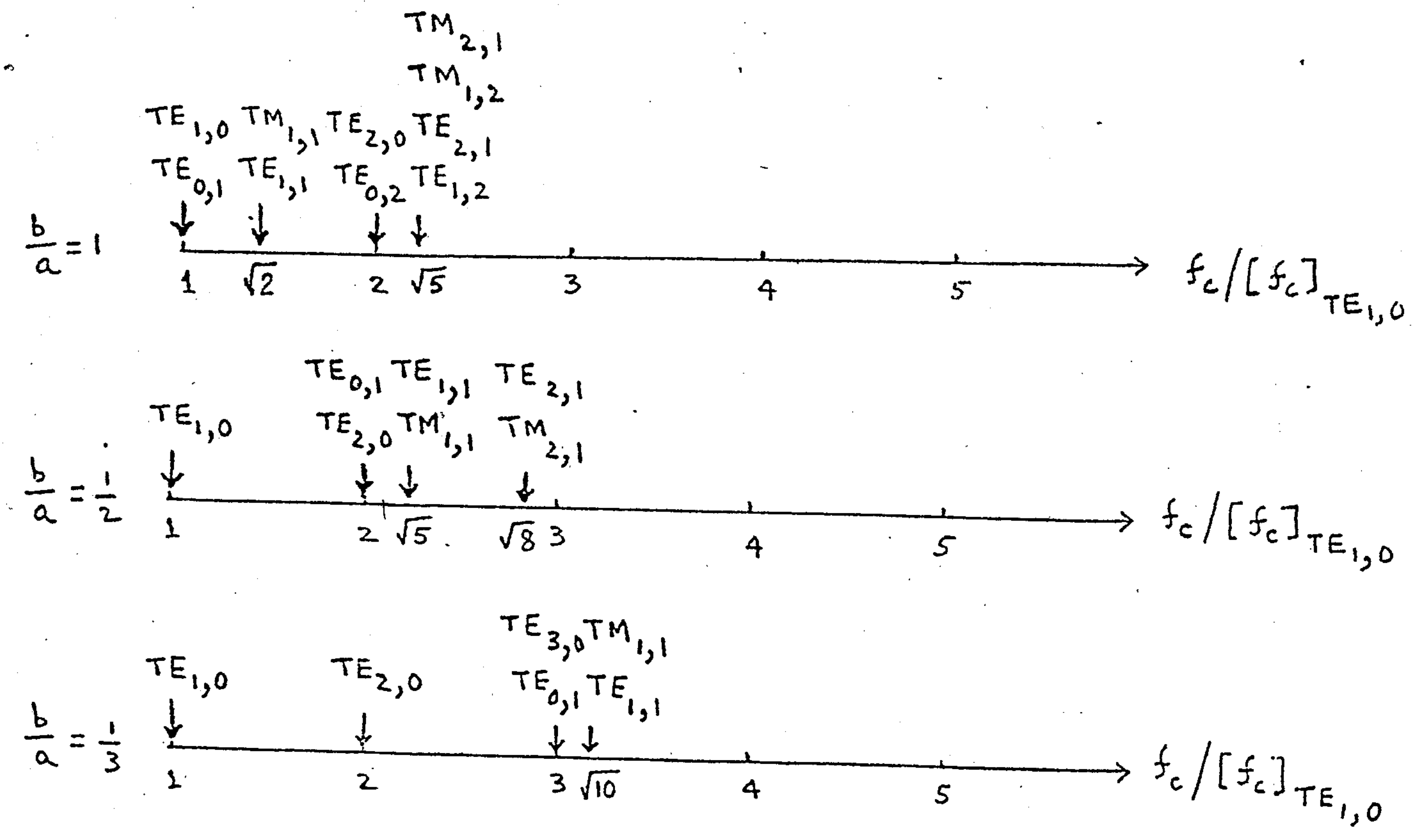
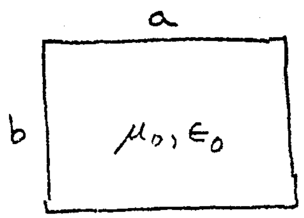


Fig. 2.2.2 Lowest four cutoff frequencies, referred to the cutoff frequency of the dominant mode, and the corresponding modes for three cases of rectangular waveguide dimensions.

Example:



$a = 3$ cm , $b = 1.5$ cm

air dielectric.

$$[\lambda_c]_{TE_{1,0}} = 2a = 6 \text{ cm}.$$

$$[f_c]_{TE_{1,0}} = \frac{v_p}{[\lambda_c]_{TE_{1,0}}} = \frac{3 \times 10^8}{0.06} = 5 \times 10^9 \text{ Hz}$$

$$= 5000 \text{ MHz}.$$

For a signal of frequency $f = 12{,}000$ MHz,

$$\frac{f}{[f_c]_{TE_{1,0}}} = \frac{12{,}000}{5000} = 2.4$$

∴ Propagating modes are

$$\underset{\substack{\downarrow \\ 5000 \text{ MHz}}}{TE_{1,0}}, \ \underbrace{TE_{0,1}, \ TE_{2,0}}_{\substack{\downarrow \\ 10{,}000 \text{ MHz}}}, \ \underbrace{TE_{1,1}, \text{ and } TM_{1,1}}_{\substack{\downarrow \\ 11{,}180 \text{ MHz}}}.$$

$f_c =$

Phase constant along the z direction

$$\beta_z^2 = \omega^2\mu\epsilon\left\{1 - \frac{1}{\omega^2\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]\right\}$$

$$= \beta^2\left[1 - \frac{\omega_c^2}{\omega^2}\right] = \beta^2\left[1 - \left(\frac{f_c}{f}\right)^2\right]$$

$$\beta_z = \beta\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \beta\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$$

Wavelength along the z direction

$$\lambda_g = \frac{2\pi}{\beta_z} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

Phase velocity in the z direction

$$v_{pz} = \frac{\omega}{\beta_z} = \frac{\omega}{\beta\sqrt{1-(f_c/f)^2}} \qquad v_{pz} > v_p$$

$$= \frac{v_p}{\sqrt{1-(f_c/f)^2}} = \frac{v_p}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

is a function of frequency.

$\therefore$ Dispersion occurs.

Guide characteristic impedance

$$[\eta_g]_{TM} = \frac{\bar{E}_x}{\pm H_y} = \frac{\bar{E}_y}{\mp \bar{H}_x} = \frac{\beta_z}{\omega\epsilon} = \frac{\beta_z}{\omega\sqrt{\mu\epsilon}} \cdot \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$= \frac{\beta_z}{\beta}\eta = \frac{\lambda}{\lambda_g}\eta$$

$$= \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \eta\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

$$[\eta_g]_{TE} = \frac{\bar{E}_x}{\pm \bar{H}_y} = \frac{\bar{E}_y}{\mp \bar{H}_x} = \frac{\omega\mu}{\beta_z}$$

$$= \frac{\beta}{\beta_z}\eta = \frac{\lambda_g}{\lambda}\eta$$

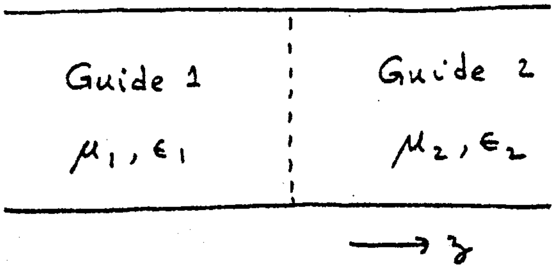
$$= \frac{\eta}{\sqrt{1-(f_c/f)^2}} = \frac{\eta}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

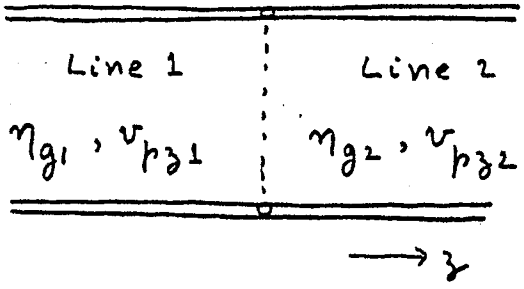
W

Table 2.2.1 Field Expressions and Associated Parameters for TM and TE Modes in a Rectangular Waveguide

Transverse Magnetic (TM) Waves	Transverse Electric (TE) Waves
Field Expressions: $(m, n = 1, 2, 3, \cdots)$ $\bar{H}_z = 0$ $\bar{E}_z = \bar{A} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, e^{\mp j(2\pi/\lambda_g)z}$ $\bar{E}_x = \mp j \frac{\lambda_c^2}{2\pi\lambda_g} \frac{m\pi}{a} \bar{A} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, e^{\mp j(2\pi/\lambda_g)z}$ $\bar{E}_y = \mp j \frac{\lambda_c^2}{2\pi\lambda_g} \frac{n\pi}{b} \bar{A} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \, e^{\mp j(2\pi/\lambda_g)z}$ $\bar{H}_x = \mp \frac{\bar{E}_y}{\eta_g}$ $\bar{H}_y = \pm \frac{\bar{E}_x}{\eta_g}$	Field Expressions: $(m, n = 0, 1, 2, \cdots$ but not both zero$)$ $\bar{E}_z = 0$ $\bar{H}_z = \bar{C} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \, e^{\mp j(2\pi/\lambda_g)z}$ $\bar{E}_x = j \frac{\lambda_c^2}{4\pi^2} \omega\mu \frac{n\pi}{b} \bar{C} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, e^{\mp j(2\pi/\lambda_g)z}$ $\bar{E}_y = -j \frac{\lambda_c^2}{4\pi^2} \omega\mu \frac{m\pi}{a} \bar{C} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \, e^{\mp j(2\pi/\lambda_g)z}$ $\bar{H}_x = \mp \frac{\bar{E}_y}{\eta_g}$ $\bar{H}_y = \pm \frac{\bar{E}_x}{\eta_g}$
$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$	$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$
$\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$	$\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$
$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}}$	$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}}$
$v_{pz} = \frac{1}{\sqrt{\mu\epsilon} \sqrt{1 - (f_c/f)^2}}$	$v_{pz} = \frac{1}{\sqrt{\mu\epsilon} \sqrt{1 - (f_c/f)^2}}$
$\eta_g = \sqrt{\frac{\mu}{\epsilon}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	$\eta_g = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - (f_c/f)^2}}$

Transmission Line Analogy





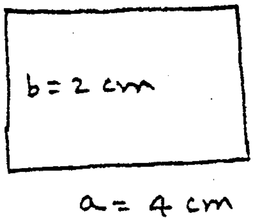
$$Z_0 \longleftrightarrow \eta_g$$

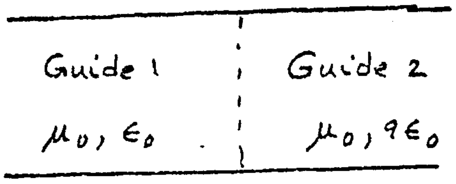
$$v_p \longleftrightarrow v_{pz}$$

$$\beta \longleftrightarrow \beta_z$$

$$\lambda \longleftrightarrow \lambda_g$$

Example





$f = 5{,}000$ MHz $\qquad TE_{1,0}$ mode

$$\lambda_c = 2a = 8 \text{ cm}.$$

$$\lambda_1 = \frac{1}{f\sqrt{\mu_0\epsilon_0}} = \frac{3\times10^8}{5\times10^9} = 0.06 \text{ m} = 6 \text{ cm} < \lambda_c$$

$$\lambda_2 = \frac{1}{f\sqrt{\mu_0 \cdot 9\epsilon_0}} = \frac{10^8}{5\times10^9} = 0.02 \text{ m} = 2 \text{ cm} < \lambda_c$$

$\therefore TE_{1,0}$ mode propagates in both sections.

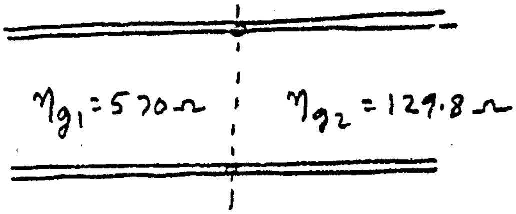
$$\eta_{g1} = \frac{\eta_1}{\sqrt{1-(\lambda_1/\lambda_c)^2}} = \frac{\sqrt{\mu_0/\epsilon_0}}{\sqrt{1-(6/8)^2}} = \frac{377}{\sqrt{1-(6/8)^2}} = 570\,\Omega$$

$$\eta_{g2} = \frac{\eta_2}{\sqrt{1-(\lambda_2/\lambda_c)^2}} = \frac{\sqrt{\mu_0/9\epsilon_0}}{\sqrt{1-(2/8)^2}} = \frac{377/3}{\sqrt{1-(2/8)^2}} = 129.8\,\Omega$$

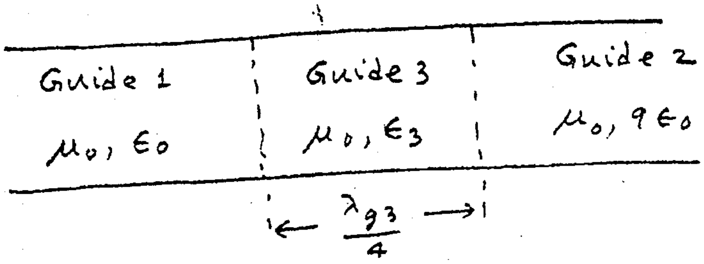
varies as f varies

$$\therefore \Gamma = \frac{\eta_{g2}-\eta_{g1}}{\eta_{g2}+\eta_{g1}}$$

$$= \frac{129.8-570}{129.8+570} = -0.629$$



Quarter-wave Transformer Matching



$$\eta_{g3} = \frac{\eta_3}{\sqrt{1-(\lambda_3/\lambda_c)^2}} = \sqrt{\eta_{g_1}\eta_{g_2}} = \sqrt{570 \times 129{\cdot}8} = 272\,\Omega$$

$$\eta_3 = \sqrt{\frac{\mu_0}{\epsilon_3}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{\frac{\epsilon_0}{\epsilon_3}} = \eta_1\sqrt{\frac{\epsilon_0}{\epsilon_3}}$$

$$\lambda_3 = \frac{1}{f\sqrt{\mu_0\epsilon_3}} = \frac{1}{f\sqrt{\mu_0\epsilon_0}} \cdot \sqrt{\frac{\epsilon_0}{\epsilon_3}} = \lambda_1\sqrt{\frac{\epsilon_0}{\epsilon_3}}$$

$$\therefore \frac{\eta_1\sqrt{\frac{\epsilon_0}{\epsilon_3}}}{\sqrt{1-\left(\frac{\lambda_1}{\lambda_c}\right)^2\frac{\epsilon_0}{\epsilon_3}}} = 272$$

$$\frac{\epsilon_0/\epsilon_3}{1-\left(\frac{6}{8}\right)^2\frac{\epsilon_0}{\epsilon_3}} = \left(\frac{272}{377}\right)^2 = 0{\cdot}5205$$

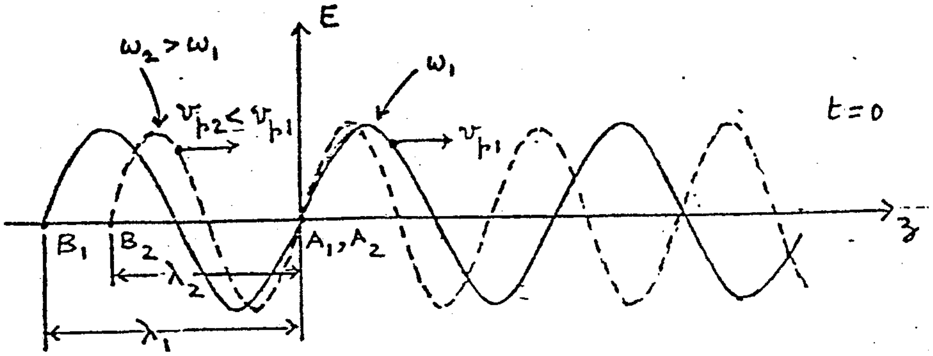
$$\frac{\epsilon_0}{\epsilon_3}\left(1 + \frac{9}{16} \times 0{\cdot}5205\right) = 0{\cdot}5205 \longrightarrow \frac{\epsilon_3}{\epsilon_0} = 2{\cdot}484$$

$$\therefore \boxed{\epsilon_3 = 2{\cdot}484\,\epsilon_0}$$

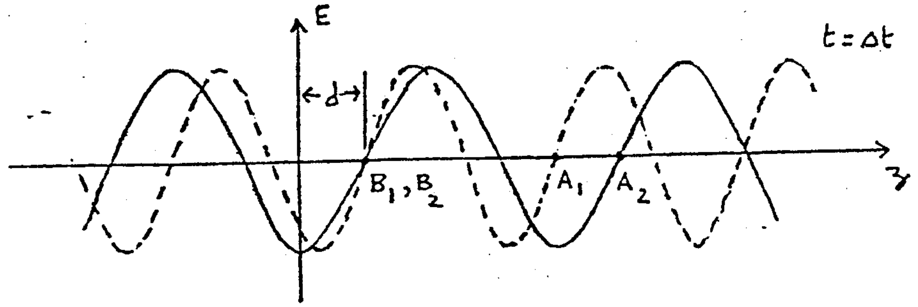
$$\lambda_{g3} = \frac{\lambda_3}{\sqrt{1-(\lambda_3/\lambda_c)^2}} = \frac{\lambda_1\sqrt{\epsilon_0/\epsilon_3}}{\sqrt{1-\left(\frac{\lambda_1}{\lambda_c}\right)^2\frac{\epsilon_0}{\epsilon_3}}} = \frac{6 \times 0{\cdot}6345}{\sqrt{1-\frac{9}{16}\times 0{\cdot}4026}} = 4{\cdot}33\text{ cm}$$

$$\therefore \boxed{\text{length of quarter-wave section} = \frac{4{\cdot}33}{4} = 1{\cdot}0825\text{ cm.}}$$

Dispersion and Group Velocity



(a)



(b)

$$v_g = \frac{d}{\Delta t}$$

$$\lambda_1 + d = v_{p1}\,\Delta t = \frac{\omega_1}{\beta_1}\Delta t$$

$$\lambda_2 + d = v_{p2}\,\Delta t = \frac{\omega_2}{\beta_2}\Delta t$$

$$d = \frac{(\omega_2/\beta_2)\lambda_1 - (\omega_1/\beta_1)\lambda_2}{(\omega_1/\beta_1) - (\omega_2/\beta_2)} = 2\pi\,\frac{\omega_2-\omega_1}{\omega_1\beta_2 - \omega_2\beta_1}$$

$$\Delta t = \frac{\lambda_1 - \lambda_2}{(\omega_1/\beta_1) - (\omega_2/\beta_2)} = 2\pi\,\frac{\beta_2-\beta_1}{\omega_1\beta_2 - \omega_2\beta_1}$$

$$\boxed{v_g = \frac{\omega_2-\omega_1}{\beta_2-\beta_1}}$$

For the waveguide,

$$\boxed{v_{gz} = \frac{\omega_2 - \omega_1}{\beta_{z2} - \beta_{z1}}}$$

For a narrowband signal

$$\boxed{v_{gz} = \frac{d\omega}{d\beta_z}}$$

For

$$\beta_z = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\frac{d\beta_z}{d\omega} = \sqrt{\mu\epsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} + \omega\sqrt{\mu\epsilon}\cdot\frac{1}{2}\left[1-\left(\frac{\omega_c}{\omega}\right)^2\right]^{-\frac{1}{2}} \frac{2\omega_c^2}{\omega^3}$$

$$= \sqrt{\mu\epsilon}\left(1 - \frac{\omega_c^2}{\omega^2} + \frac{\omega_c^2}{\omega^2}\right)\left[1-\left(\frac{\omega_c}{\omega}\right)^2\right]^{-\frac{1}{2}}$$

$$= \sqrt{\mu\epsilon}\left[1-\left(\frac{f_c}{f}\right)^2\right]^{-\frac{1}{2}}$$

$$v_{gz} = \frac{d\omega}{d\beta_z} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

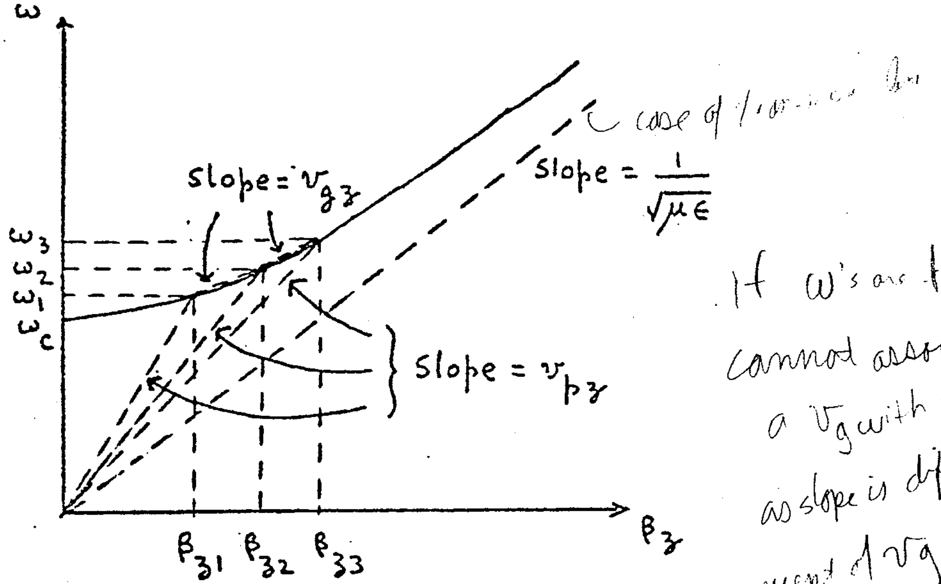
$$\boxed{v_{gz} = v_p\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \leq v_p$$

On the other hand,

$$\boxed{v_{pz} = \frac{v_p}{\sqrt{1-(f_c/f)^2}}} \geq v_p$$

Note $v_{pz}\, v_{gz} = v_p^2$.

Dispersion Diagram



If ω's are far apart cannot associated a v_g with them as slope is different ∴ only concept of v_g, for narrowband signals

Fig.2.2.4 ω versus β_z diagram for the rectangular waveguide

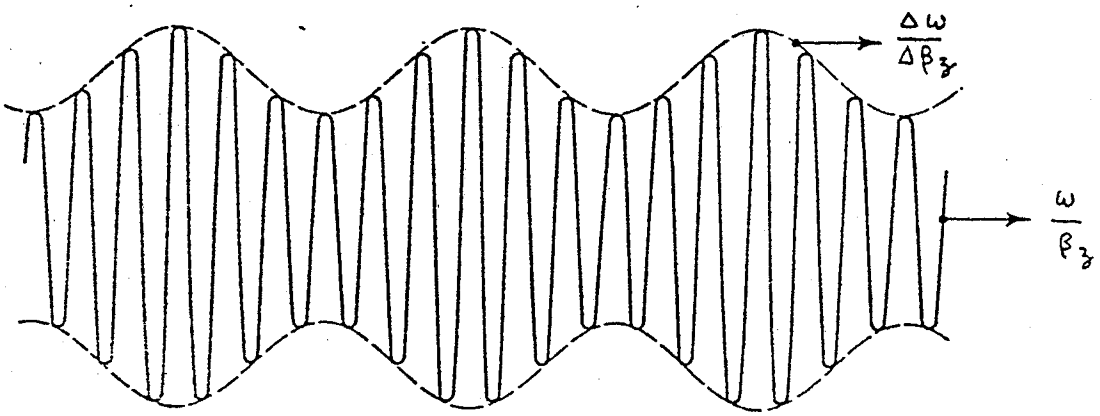
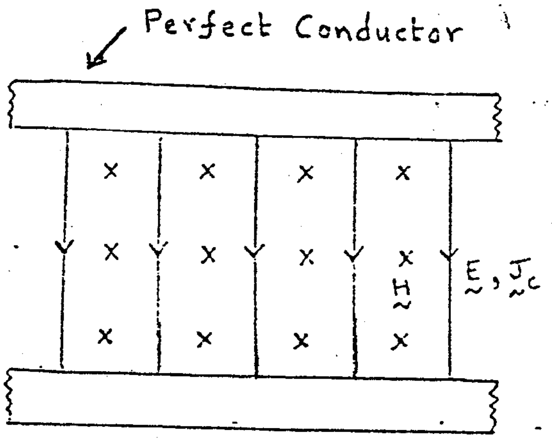


Fig. 2.2.5 For illustrating that the modulation envelope travels with the group velocity.

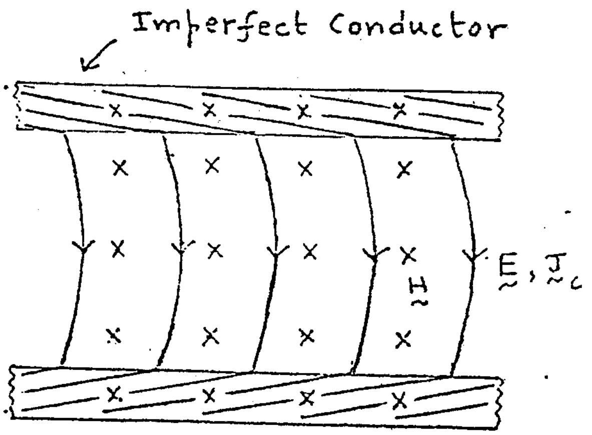
LOSSY LINES

(28-31)

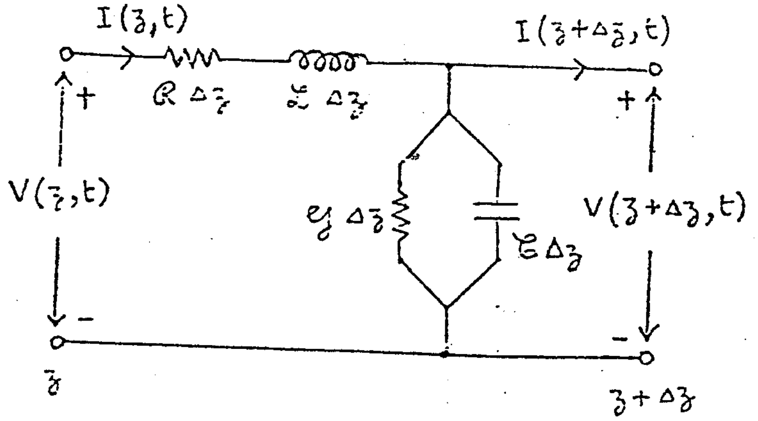
Lossy Line



(a)



(b)



Lossy Line Equations and Solution

$$\frac{\partial \bar{V}(z)}{\partial z} = -(\mathcal{R} + j\omega\mathcal{L})\,\bar{I}(z)$$

$$\frac{\partial \bar{I}(z)}{\partial z} = -(\mathcal{G} + j\omega\mathcal{C})\,\bar{V}(z)$$

$$\frac{\partial^2 \bar{V}}{\partial z^2} = -(\mathcal{R}+j\omega\mathcal{L})\frac{\partial \bar{I}}{\partial z} = (\mathcal{R}+j\omega\mathcal{L})(\mathcal{G}+j\omega\mathcal{C})\bar{V}$$

$$= \bar{\gamma}^2 \bar{V}$$

where $\boxed{\bar{\gamma} = \alpha + j\beta = \sqrt{(\mathcal{R}+j\omega\mathcal{L})(\mathcal{G}+j\omega\mathcal{C})}}$ Propagation Constant

$$\bar{V}(z) = \bar{A}\,e^{-\bar{\gamma} z} + \bar{B}\,e^{\bar{\gamma} z}$$

$$v(z,t) = \mathrm{Re}\left[A e^{j\theta} e^{-\alpha z} e^{-j\beta z} e^{j\omega t} + B e^{j\phi} e^{\alpha z} e^{j\beta z} e^{j\omega t}\right]$$

$$= A e^{-\alpha z}\cos(\omega t - \beta z + \theta) + B e^{\alpha z}\cos(\omega t + \beta z + \phi)$$

$$\therefore \quad \boxed{\bar{V}(z) = \bar{V}^+ e^{-\bar{\gamma} z} + \bar{V}^- e^{\bar{\gamma} z}}$$

$$\bar{I}(z) = -\frac{1}{\mathcal{R}+j\omega\mathcal{L}}\frac{\partial \bar{V}}{\partial z}$$

$$= -\frac{1}{\mathcal{R}+j\omega\mathcal{L}}\left(-\bar{\gamma}\bar{V}^+ e^{-\bar{\gamma} z} + \bar{\gamma}\bar{V}^- e^{\bar{\gamma} z}\right)$$

$$\boxed{\bar{I}(z) = \frac{1}{\bar{Z}_0}\left(\bar{V}^+ e^{-\bar{\gamma} z} - \bar{V}^- e^{\bar{\gamma} z}\right)} \rightarrow \bar{Z}_0 = \frac{R + j\omega\mathcal{L}}{\gamma}$$

where $\boxed{\bar{Z}_0 = \sqrt{\frac{\mathcal{R}+j\omega\mathcal{L}}{\mathcal{G}+j\omega\mathcal{C}}}}$ Characteristic Impedance

$$\bar{\gamma}^2 = (\alpha + j\beta)^2 = (\mathcal{R} + j\omega\mathcal{L})(\mathcal{G} + j\omega\mathcal{C})$$

$$\alpha^2 - \beta^2 = \mathcal{R}\mathcal{G} - \omega^2\mathcal{L}\mathcal{C}$$

$$2\alpha\beta = \omega(\mathcal{L}\mathcal{G} + \mathcal{R}\mathcal{C})$$

$$\alpha^2 + \beta^2 = \sqrt{(\mathcal{R}^2 + \omega^2\mathcal{L}^2)(\mathcal{G}^2 + \omega^2\mathcal{C}^2)}$$

$$\alpha = \left\{ \tfrac{1}{2}\left[\left\{ (\mathcal{R}^2 + \omega^2\mathcal{L}^2)(\mathcal{G}^2 + \omega^2\mathcal{C}^2) \right\}^{1/2} + (\mathcal{R}\mathcal{G} - \omega^2\mathcal{L}\mathcal{C}) \right] \right\}^{1/2}$$

$$\beta = \left\{ \tfrac{1}{2}\left[\left\{ (\mathcal{R}^2 + \omega^2\mathcal{L}^2)(\mathcal{G}^2 + \omega^2\mathcal{C}^2) \right\}^{1/2} - (\mathcal{R}\mathcal{G} - \omega^2\mathcal{L}\mathcal{C}) \right] \right\}^{1/2}$$

<u>Low Loss Line</u> $(\omega\mathcal{L} \gg \mathcal{R},\ \omega\mathcal{C} \gg \mathcal{G})$

$$\bar{\gamma} = \sqrt{j\omega\mathcal{L}\left(1 + \frac{\mathcal{R}}{j\omega\mathcal{L}}\right) j\omega\mathcal{C}\left(1 + \frac{\mathcal{G}}{j\omega\mathcal{C}}\right)}$$

$$\simeq j\omega\sqrt{\mathcal{L}\mathcal{C}}\sqrt{1 + \frac{\mathcal{R}}{j\omega\mathcal{L}} + \frac{\mathcal{G}}{j\omega\mathcal{C}}}$$

$$\simeq j\omega\sqrt{\mathcal{L}\mathcal{C}}\left[1 + \frac{1}{2}\left(\frac{\mathcal{R}}{j\omega\mathcal{L}} + \frac{\mathcal{G}}{j\omega\mathcal{C}}\right)\right]$$

$$= \frac{1}{2}\left(\mathcal{R}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}} + \mathcal{G}\sqrt{\frac{\mathcal{L}}{\mathcal{C}}}\right) + j\omega\sqrt{\mathcal{L}\mathcal{C}}$$

$\sqrt{1+x} = 1 + \frac{1}{2}x$ for small $x \ll 1$

$$\therefore \alpha \approx \frac{1}{2}\left(\mathcal{R}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}} + \mathcal{G}\sqrt{\frac{\mathcal{L}}{\mathcal{C}}}\right)$$

$\frac{\omega}{v_p}$

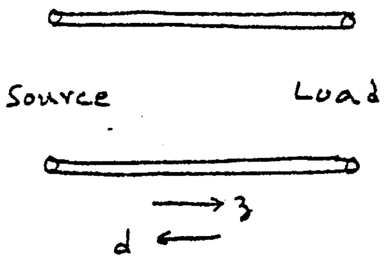
$$\beta \approx \omega\sqrt{\mathcal{L}\mathcal{C}} \longrightarrow v_p \approx \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$$

only for narrowband signals

$$\text{Also, } \bar{Z}_0 = \sqrt{\frac{j\omega\mathcal{L}(1 + \mathcal{R}/j\omega\mathcal{L})}{j\omega\mathcal{C}(1 + \mathcal{G}/j\omega\mathcal{C})}}$$

$$\approx \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}.$$

Standing Waves



$$\bar{V}(d) = \bar{V}^{+} e^{\bar{\gamma} d} + \bar{V}^{-} e^{-\bar{\gamma} d}$$

$$\bar{I}(d) = \frac{1}{\bar{Z}_0}\left(\bar{V}^{+} e^{\bar{\gamma} d} - \bar{V}^{-} e^{-\bar{\gamma} d}\right)$$

Define $\bar{\Gamma}(d) = \dfrac{\bar{V}^{-}(d)}{\bar{V}^{+}(d)} = \dfrac{\bar{V}^{-}}{\bar{V}^{+}} e^{-2\bar{\gamma} d}$

$$= \bar{\Gamma}_R e^{-2\bar{\gamma} d} = \bar{\Gamma}_R e^{-2\alpha d} e^{-j2\beta d}$$

Note $\boxed{|\bar{\Gamma}(d)| = |\bar{\Gamma}_R| e^{-2\alpha d}}$

$$\bar{V}(d) = \bar{V}^{+} e^{\bar{\gamma} d}\left(1 + \bar{\Gamma}_R e^{-2\bar{\gamma} d}\right)$$

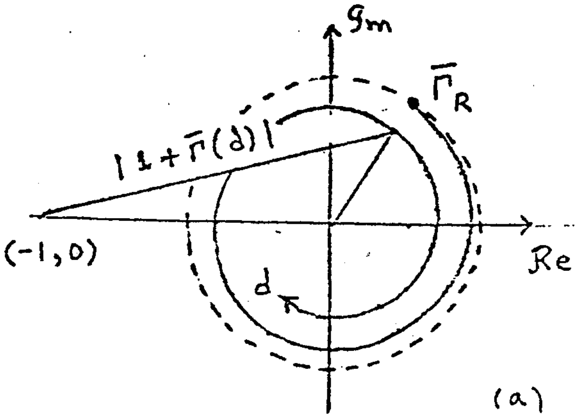
$$= \bar{V}^{+} e^{\alpha d} e^{j\beta d}\left[1 + \bar{\Gamma}(d)\right]$$

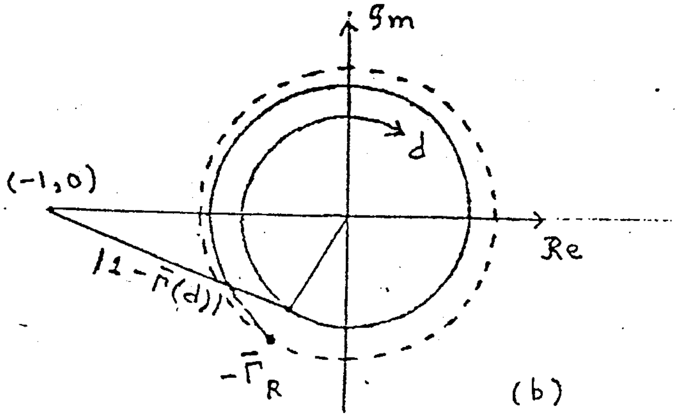
$$\bar{I}(d) = \frac{\bar{V}^{+}}{\bar{Z}_0} e^{\bar{\gamma} d}\left(1 - \bar{\Gamma}_R e^{-2\bar{\gamma} d}\right)$$

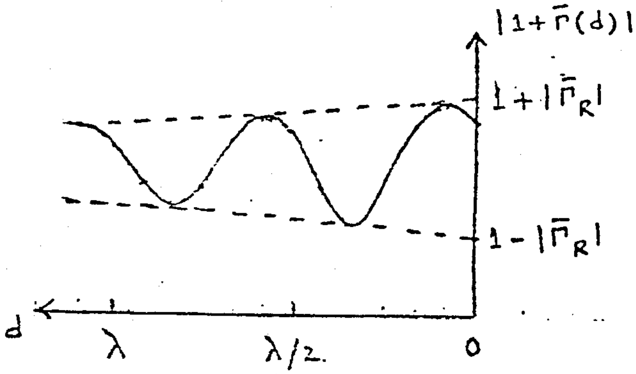
$$= \frac{\bar{V}^{+}}{\bar{Z}_0} e^{\alpha d} e^{j\beta d}\left[1 + \bar{\Gamma}(d)\right]$$

$$\boxed{\begin{aligned} |\bar{V}(d)| &= |\bar{V}^{+}| e^{\alpha d} |1 + \bar{\Gamma}(d)| \\ |\bar{I}(d)| &= \frac{|\bar{V}^{+}|}{|\bar{Z}_0|} e^{\alpha d} |1 - \bar{\Gamma}(d)| \end{aligned}}$$ S.W. Patterns

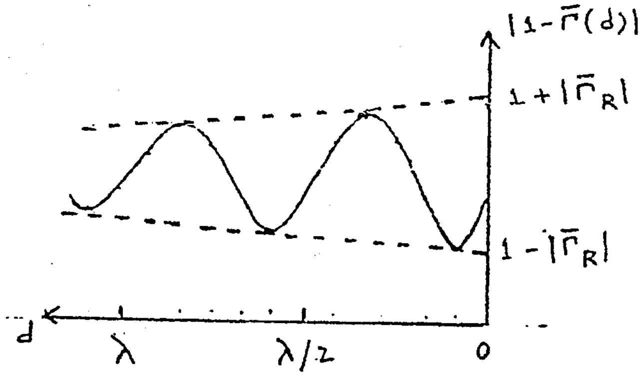
$$\text{SWR at a point} = \frac{|\bar{V}|_{\text{upper envelope}}}{|\bar{V}|_{\text{lower envelope}}} = \frac{1 + |\bar{\Gamma}_R| e^{-2\alpha d}}{1 - |\bar{\Gamma}_R| e^{-2\alpha d}}$$


Im
$\bar{\Gamma}_R$
$|1+\bar{\Gamma}(d)|$
(-1,0)
Re
d
(a)

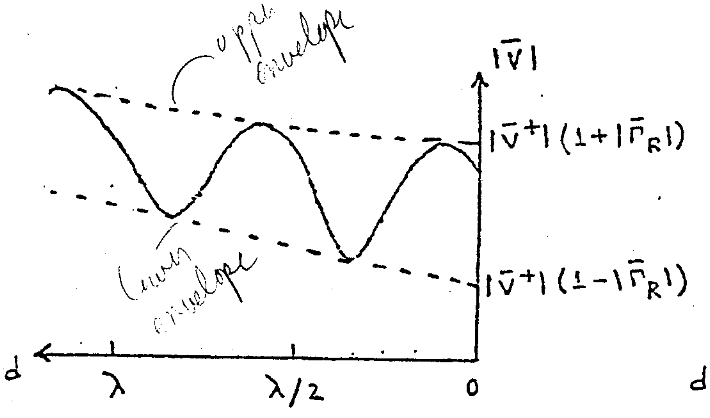

Im
d
(-1,0)
Re
$|1-\bar{\Gamma}(d)|$
$-\bar{\Gamma}_R$
(b)


$|1+\bar{\Gamma}(d)|$
$1+|\bar{\Gamma}_R|$
$1-|\bar{\Gamma}_R|$
d
λ
λ/2
0

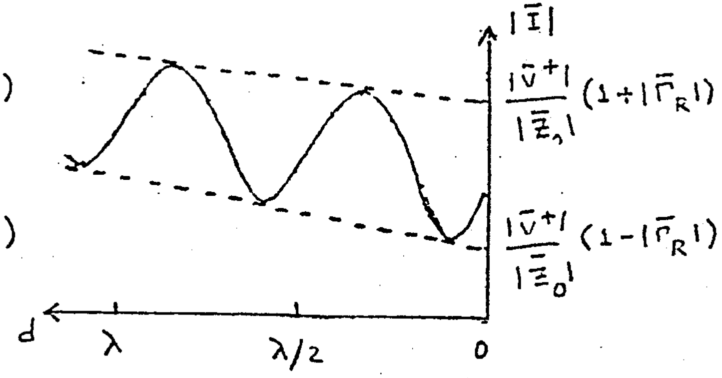
(a)


$|1-\bar{\Gamma}(d)|$
$1+|\bar{\Gamma}_R|$
$1-|\bar{\Gamma}_R|$
d
λ
λ/2
0

(b)


upper envelope
lower envelope
$|\bar{V}|$
$|\bar{V}^+|(1+|\bar{\Gamma}_R|)$
$|\bar{V}^+|(1-|\bar{\Gamma}_R|)$
d
λ
λ/2
0

(a)


$|\bar{I}|$
$\frac{|\bar{V}^+|}{|\bar{Z}_0|}(1+|\bar{\Gamma}_R|)$
$\frac{|\bar{V}^+|}{|\bar{Z}_0|}(1-|\bar{\Gamma}_R|)$
d
λ
λ/2
0

(b)

-1 . Impedance and Power Flow

$$\bar{Z}(d) = \frac{\bar{V}(d)}{\bar{I}(d)} = \bar{Z}_0 \frac{1+\bar{\Gamma}(d)}{1-\bar{\Gamma}(d)}$$

$$\boxed{\bar{Z}(d) = \bar{Z}_0 \frac{1+\bar{\Gamma}_R e^{-2\bar{\gamma}d}}{1-\bar{\Gamma}_R e^{-2\bar{\gamma}d}}}$$

$$\langle P \rangle = \frac{1}{2} \mathcal{R}e \left[\bar{V}(d)\, \bar{I}^*(d) \right]$$

$$= \frac{1}{2} \mathcal{R}e \left\{ \bar{V}^+ e^{\bar{\gamma}d} [1+\bar{\Gamma}(d)] \frac{(\bar{V}^+)^*}{\bar{Z}_0^*} e^{\bar{\gamma}^* d} [1-\bar{\Gamma}^*(d)] \right\}$$

$$= \frac{1}{2} \mathcal{R}e \left\{ \frac{|\bar{V}^+|^2}{\bar{Z}_0^*} e^{2\alpha d} [1-|\bar{\Gamma}(d)|^2 + \bar{\Gamma}(d) - \bar{\Gamma}^*(d)] \right\}$$

$$= \frac{1}{2} \mathcal{R}e \left\{ |\bar{V}^+|^2 \bar{Y}_0^* e^{2\alpha d} [1-|\bar{\Gamma}(d)|^2 + j\,2\, \mathcal{I}m\, \bar{\Gamma}(d)] \right\}$$

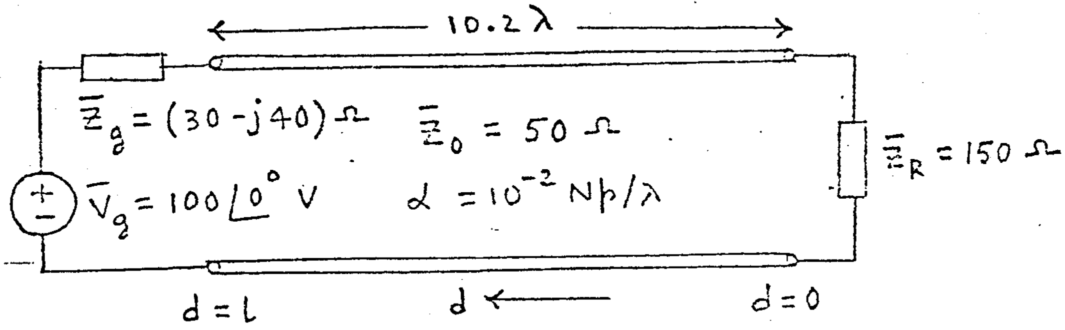
$$= \frac{1}{2} |\bar{V}^+|^2 e^{2\alpha d} \left\{ G_0 [1-|\bar{\Gamma}(d)|^2] + 2 B_0\, \mathcal{I}m\, \bar{\Gamma}(d) \right\}$$

where

$$\bar{Y}_0 = \frac{1}{\bar{Z}_0} = G_0 + j B_0$$

Note that $|\bar{\Gamma}(d)|$ can exceed 1 without causing $\langle P \rangle$ to be negative.

-xample:



$$\bar{\Gamma}_R = \frac{\bar{Z}_R - Z_0}{\bar{Z}_R + Z_0} = \frac{150 - 50}{150 + 50} = 0.5$$

$$\text{SWR at the load end} = \frac{1 + 0.5}{1 - 0.5} = 3.0$$

$2\beta l = 40.8\pi$

$\beta = \frac{40.8\pi}{2l}$

$$\bar{\Gamma}(l) = \bar{\Gamma}_R\, e^{-2\bar{\gamma} l} = \bar{\Gamma}_R\, e^{-2\alpha l}\, e^{-j2\beta l}$$

$$= 0.5\, e^{-0.204}\, e^{-j40.8\pi} = 0.4077 \angle -144^\circ$$

$$\text{SWR at the input end} = \frac{1 + 0.4077}{1 - 0.4077} = 2.38$$

$$\bar{Z}_{in} = \bar{Z}(l) = \bar{Z}_0\, \frac{1 + \bar{\Gamma}(l)}{1 - \bar{\Gamma}(l)}$$

$$= 50\, \frac{1 + 0.4077 \angle -144^\circ}{1 - 0.4077 \angle -144^\circ} = 50\, \frac{1 + (-0.33 - j0.24)}{1 - (-0.33 - j0.24)}$$

$$= 50\, \frac{0.7117 \angle -19.708^\circ}{1.3515 \angle 10.229^\circ} = 26.63 \angle -29.927^\circ$$

$$= (22.817 - j13.140)\,\Omega$$

$$\bar{I}(l) = \frac{\bar{V}_g}{\bar{Z}_g + \bar{Z}_{in}} = \frac{100\angle 0^\circ}{(30 - j40) + (22.817 - j13.140)}$$

$$= \frac{100\angle 0^\circ}{74.923\angle -45.175^\circ} = 1.3347\angle 45.175^\circ \text{ A}$$

$$\bar{V}(l) = \bar{Z}_{in}\,\bar{I}(l) = 26.33\angle -29.927^\circ \times 1.3347\angle 45.175^\circ$$

$$= 35.143\angle 15.238^\circ \text{ V}$$

$$< P(l) > = \frac{1}{2}\,\mathrm{Re}\left[\bar{V}(l)\,\bar{I}^*(l)\right]$$

$$= \frac{1}{2}\,\mathrm{Re}\left[35.143\angle 15.238^\circ \times 1.3347\angle -45.175^\circ\right]$$

$$= 20.32 \text{ W}$$

$$|\bar{V}^+| = \sqrt{\frac{2 < P(l) > e^{-2\alpha l}}{G_0\left[1 - |\bar{\Gamma}(l)|^2\right]}} \qquad \text{since } B_0 = 0$$

$$= \sqrt{\frac{2 \times 20.32 \times e^{-0.204}}{0.02\,(1 - 0.4077^2)}} = 44.58 \text{ V}$$

$$< P(0) > = \frac{1}{2}\,|\bar{V}^+|^2\,G_0\,(1 - |\bar{\Gamma}_R|^2)$$

$$= \frac{1}{2} \times 44.58^2 \times 0.02\,(1 - 0.25)$$

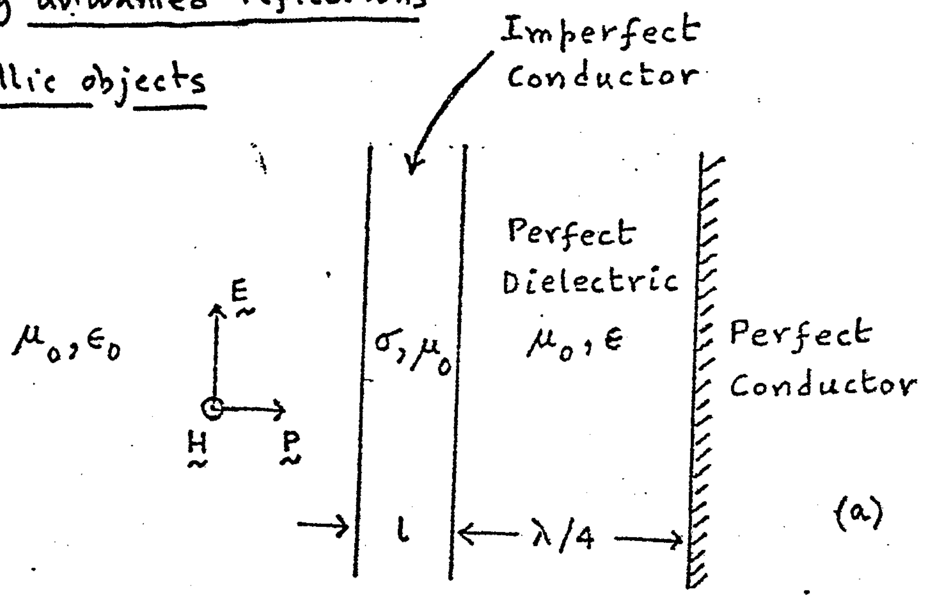
$$= 14.91 \text{ W}$$

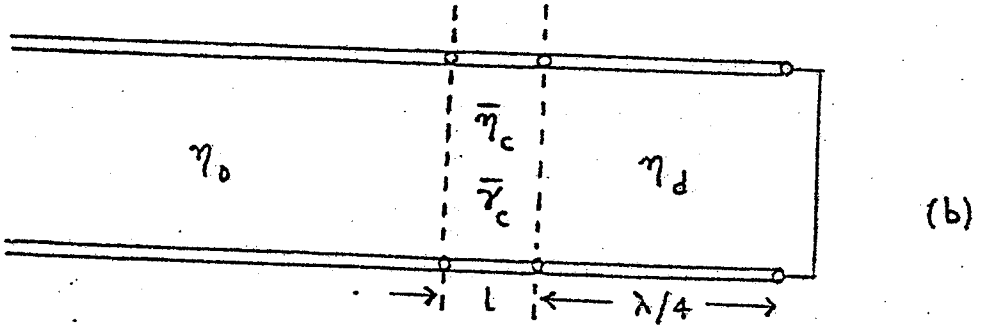
$$< P_d > = < P(l) > - < P(0) >$$

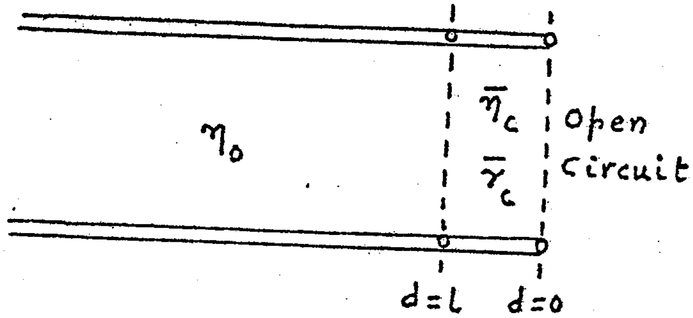
$$= 20.32 - 14.91$$

$$= 5.41 \text{ W}$$

Eliminating unwanted reflections from metallic objects







$$\bar{Z}(l) = \bar{\eta}_c \frac{1 + \bar{\Gamma}(l)}{1 - \bar{\Gamma}(l)} = \eta_c \frac{1 + \bar{\Gamma}_R e^{-2\bar{\gamma}_c l}}{1 - \bar{\Gamma}_R e^{-2\bar{\gamma}_c l}}$$

$$= \bar{\eta}_c \frac{1 + e^{-2\bar{\gamma}_c l}}{1 - e^{-2\bar{\gamma}_c l}}$$

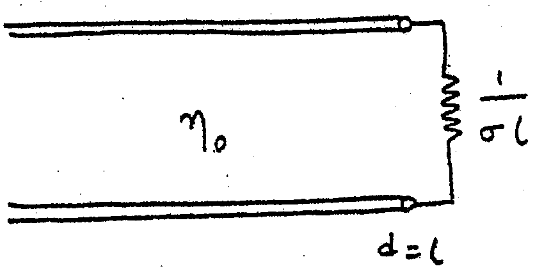
$\bar{\Gamma}_R = 1$ for open circuit

For $|\bar{\gamma}_c l| \ll 1$,

$$e^{-2\bar{\gamma}_c l} \approx 1 - 2\bar{\gamma}_c l$$

$$\bar{Z}(l) \approx \bar{\eta}_c \frac{1 + 1 - 2\bar{\gamma}_c l}{1 - 1 + 2\bar{\gamma}_c l} = \frac{\bar{\eta}_c}{\bar{\gamma}_c l}$$

$$= \frac{(1+j)\sqrt{\pi f \mu / \sigma}}{(1+j)\sqrt{\pi f \mu \sigma}\; l} = \frac{1}{\sigma l}$$

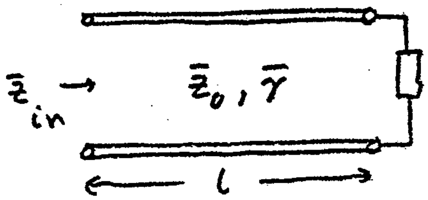


If $\sigma l = \frac{1}{\eta_0}$, no reflection occurs.

All incident power is dissipated in The imperfect conductor.

Determination of $\bar{Z}_0$ and $\bar{\gamma}$

$$\bar{Z}_{in} = \bar{Z}(l) = \bar{Z}_0 \frac{1 + \bar{\Gamma}_R \, e^{-2\bar{\gamma} l}}{1 - \bar{\Gamma}_R \, e^{-2\bar{\gamma} l}}$$



with output short circuited, $\bar{\Gamma}_R = -1$

$$\bar{Z}_{in}^{s} = \bar{Z}_0 \frac{1 - e^{-2\bar{\gamma} l}}{1 + e^{-2\bar{\gamma} l}} = \bar{Z}_0 \frac{e^{\bar{\gamma} l} - e^{-\bar{\gamma} l}}{e^{\bar{\gamma} l} + e^{-\bar{\gamma} l}} = \bar{Z}_0 \tanh \bar{\gamma} l$$

with output open circuited, $\bar{\Gamma}_R = +1$

$$\bar{Z}_{in}^{o} = \bar{Z}_0 \frac{1 + e^{-2\bar{\gamma} l}}{1 - e^{-2\bar{\gamma} l}} = \bar{Z}_0 \frac{e^{\bar{\gamma} l} + e^{-\bar{\gamma} l}}{e^{\bar{\gamma} l} - e^{-\bar{\gamma} l}} = \bar{Z}_0 \coth \bar{\gamma} l$$

$$\therefore \quad \bar{Z}_{in}^{s} \cdot \bar{Z}_{in}^{o} = \bar{Z}_0^{2}$$

$$\frac{\bar{Z}_{in}^{s}}{\bar{Z}_{in}^{o}} = \tanh^2 \bar{\gamma} l$$

$$\boxed{\begin{aligned} \bar{Z}_0 &= \sqrt{\bar{Z}_{in}^{s} \cdot \bar{Z}_{in}^{o}} \\ \bar{\gamma} l &= \tanh^{-1} \sqrt{\frac{\bar{Z}_{in}^{s}}{\bar{Z}_{in}^{o}}} \end{aligned}}$$

EXAMPLE

$$\bar{Z}_{in}^{s} = (30 - j40)\ \Omega$$

$$\bar{Z}_{in}^{o} = (30 + j40)\ \Omega$$

$$\bar{Z}_0 = \sqrt{(30-j40)(30+j40)} = 50\ \Omega$$

$$\bar{\gamma} l = \tanh^{-1}\sqrt{\frac{30-j40}{30+j40}} = \tanh^{-1}\sqrt{\frac{50\angle -53.13^\circ}{50\angle 53.13^\circ}}$$

$$= \tanh^{-1}\sqrt{1\angle -106.26^\circ} = \tanh^{-1}(1\angle -53.13^\circ)$$

$$= \tanh^{-1}(0.6 - j0.8)$$

Using $\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$, we have

$$\bar{\gamma} l = \frac{1}{2}\ln\frac{1.6-j0.8}{0.4+j0.8} = \frac{1}{2}\ln\frac{1.789\angle -26.565^\circ}{0.894\angle 63.435^\circ}$$

$$= \frac{1}{2}\ln(2\angle -90^\circ) = \frac{1}{2}\ln 2e^{j(2n\pi - \pi/2)}$$

$$= \frac{1}{2}[\ln 2 + j(2n\pi - \pi/2)]$$

$$= 0.3466 + j(n\pi - \pi/4) \qquad n = 0, 1, 2, \ldots$$

$\therefore\ \alpha l = 0.3466$ or $\alpha = 0.3466/l$

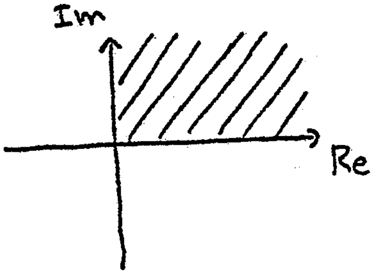
$\beta l = n\pi - \pi/4, \quad n = 1, 2, \ldots$

If the approximate value of β is known, then the correct value of n and hence of β can be determined.

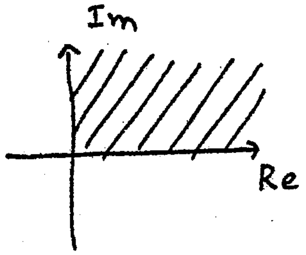
Extension of Smith chart for Lossy Lines

$$\bar{z}(d) = \frac{\bar{Z}(d)}{\bar{Z}_0}$$

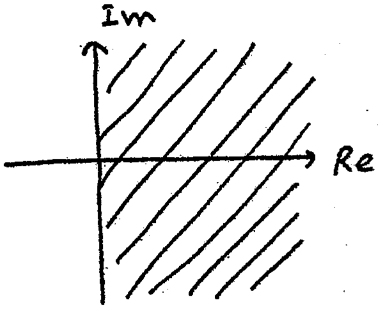
$$\bar{Z}_0 = \sqrt{\frac{\mathcal{R}+j\omega\mathcal{L}}{\mathcal{G}+j\omega\mathcal{C}}}$$



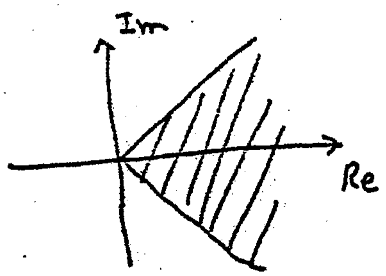
$0^\circ < \angle \mathcal{R}+j\omega\mathcal{L} < 90^\circ$



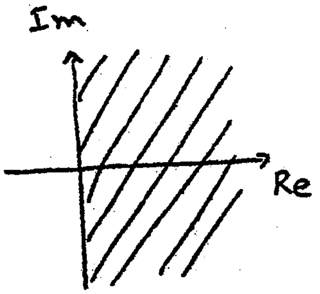
$0^\circ < \angle \mathcal{G}+j\omega\mathcal{C} < 90^\circ$



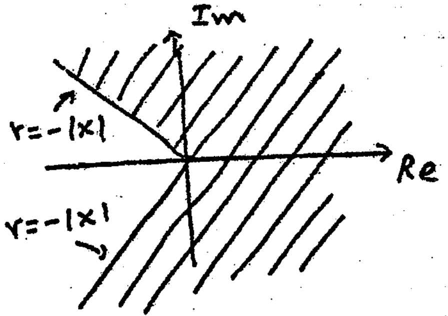
$-90^\circ < \angle \frac{\mathcal{R}+j\omega\mathcal{L}}{\mathcal{G}+j\omega\mathcal{C}} < 90^\circ$



$-45^\circ < \angle \bar{Z}_0 < 45^\circ$



$-90^\circ < \angle \bar{Z}(d) < 90^\circ$



$-135^\circ < \angle \bar{z}(d) < 135^\circ$

Thus the boundary of the permissible region of $\bar{z}(d)$ in the complex $\bar{z}$-plane is given by

$$r = -|x|$$

The corresponding loci in the $\bar{\Gamma}$-plane are given by

$$\bar{\Gamma} = \frac{\bar{z}-1}{\bar{z}+1} = \frac{r+jx-1}{r+jx+1}$$

$$= \frac{(-|x|-1)+jx}{(-|x|+1)+jx}$$

$$= \frac{[((-|x|-1)+jx][(-|x|+1)-jx]}{(-|x|-1)^2+x^2}$$

$$= \frac{2x^2-1+j2x}{2x^2-2|x|+1}$$

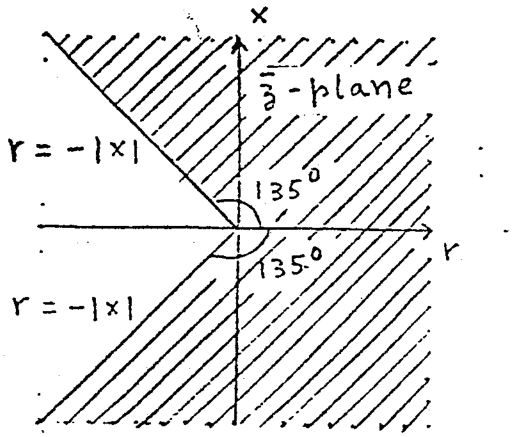
$$\mathrm{Re}\,\bar{\Gamma} = \frac{2x^2-1}{2x^2-2|x|+1} = \frac{2x^2-1}{2x^2\mp 2x+1} \quad \text{for } x \gtrless 0$$

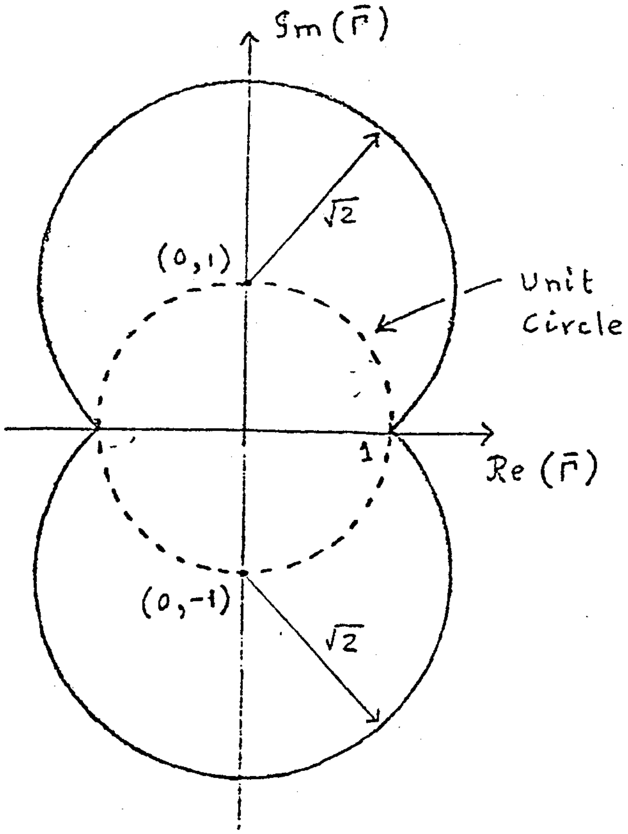
$$\mathrm{Im}\,\bar{\Gamma} = \frac{2x}{2x^2-2|x|+1} = \frac{2x}{2x^2\mp 2x+1} \quad \text{for } x \gtrless 0$$

$$\left(\frac{2x^2-1}{2x^2\mp 2x+1}\right)^2 + \left(\frac{2x}{2x^2\mp 2x+1} \mp 1\right)^2 = 2 \quad \text{for } x \gtrless 0$$

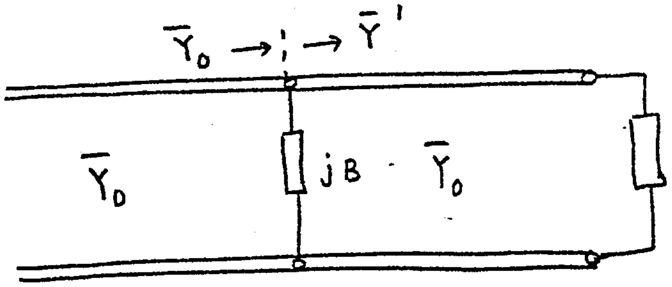
$$[\mathrm{Re}\,\bar{\Gamma}]^2 + [\mathrm{Im}\,\bar{\Gamma} \mp 1]^2 = (\sqrt{2})^2 \quad \text{for } x \gtrless 0$$

These are circles with centers at $(0, \pm 1)$ and radii equal to $\sqrt{2}$.


x
$\bar{z}$-plane
$r = -|x|$
135^0
135^0
$r = -|x|$
r


$\mathcal{I}m(\bar{\Gamma})$
$\sqrt{2}$
(0,1)
Unit
Circle
1
$\mathcal{R}e(\bar{\Gamma})$
(0,-1)
$\sqrt{2}$

Stub Matching



$$\bar{Y}_0 = \bar{Y}' + jB$$

$$\bar{Y}' = \bar{Y}_0 - jB$$

$$\bar{y}' = \frac{\bar{Y}'}{\bar{Y}_0} = 1 - j\frac{B}{\bar{Y}_0}$$

$$= 1 - j\frac{B}{G_0 + jB_0}$$

$\bar{Y}_0 = G_0 + jB_0$

$$= 1 - j\frac{B(G_0 - jB_0)}{G_0^2 + B_0^2}$$

$$= \left(1 - \frac{BB_0}{G_0^2 + B_0^2}\right) - j\frac{BG_0}{G_0^2 + B_0^2}$$

$$\text{Re}\,\bar{y}' = 1 - \frac{BB_0}{G_0^2 + B_0^2}$$

$$\text{Im}\,\bar{y}' = -\frac{BG_0}{G_0^2 + B_0^2}$$

$$\boxed{\text{Re}\,\bar{y}' = 1 + \frac{B_0}{G_0}\,\text{Im}\,\bar{y}'}$$

Locus of $\bar{y}'$ for possible match

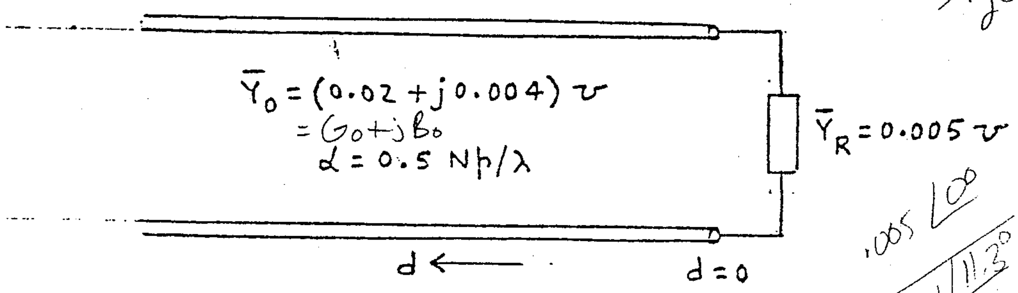
Example

,0204 ∠11.3

49∠-11.3

48-j9.6

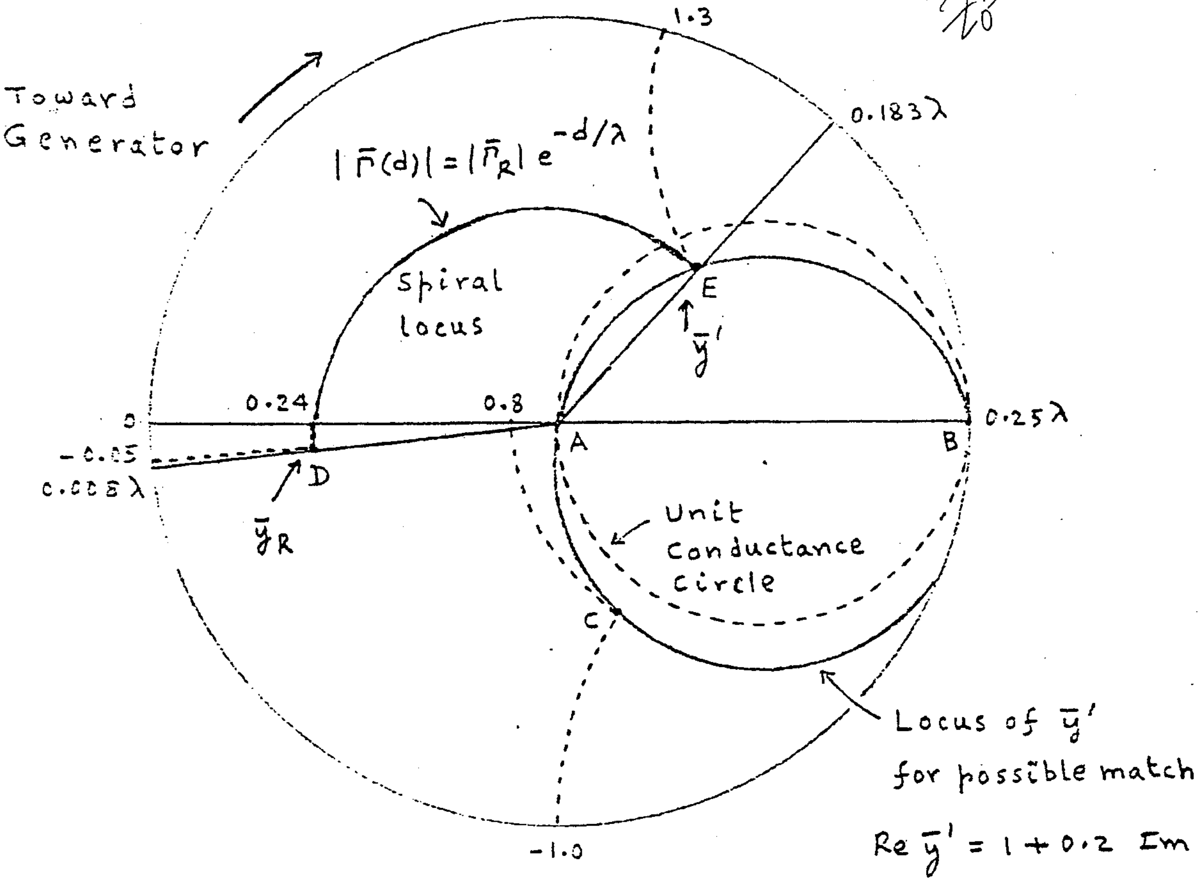
$\frac{49}{200}$



$\frac{,005 \angle 0^\circ}{,0204 \angle 11.3^\circ}$

,0245 ∠-11.3°

$\bar{z}_R = 200 / \bar{z}_0$



$\text{Re}\,\bar{y}' = 1 + 0.2\ \text{Im}\,\bar{y}'$

$\frac{B_0}{G_0} = .20$

stub Location = (0.183 + 0.008) λ = 0.191 λ

$$B = -\frac{G_0^2 + B_0^2}{G_0}\ \text{Im}\,\bar{y}' = -\frac{0.02^2 + 0.004^2}{0.02} \times 1.3$$

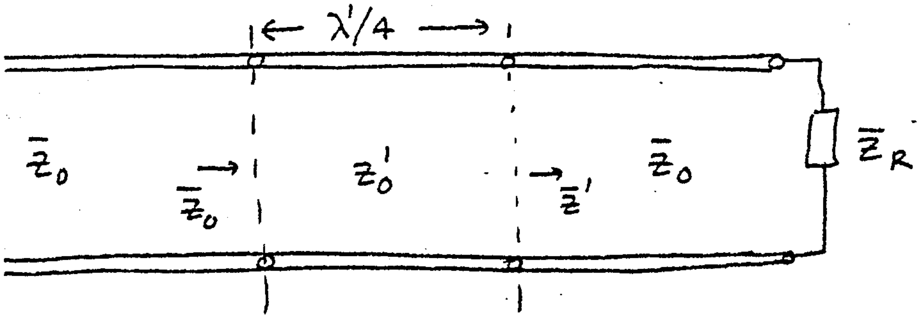
$$= -0.027\ \mho.$$

$Y_0 = ,204 \angle 11.30$

,94

d = [illegible]

Quarter-Wave Transformer Matching



$$\bar{Z}_0 \, \bar{Z}' = (Z_0')^2$$

$$\bar{Z}' = \frac{(Z_0')^2}{\bar{Z}_0}$$

$$\bar{z}' = \frac{\bar{Z}'}{\bar{Z}_0} = \frac{(Z_0')^2}{\bar{Z}_0^2}$$

$$\angle \bar{z}' = \angle (Z_0')^2 - \angle \bar{Z}_0^2$$

$$= 0 - 2 \angle \bar{Z}_0$$

$$\angle \bar{z}' = -2 \angle \bar{Z}_0$$

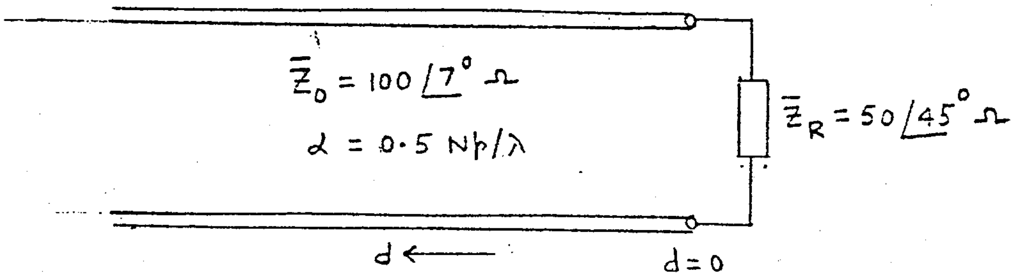
$$\tan^{-1} \frac{\mathrm{Im}\, \bar{z}'}{\mathrm{Re}\, \bar{z}'} = -2 \angle \bar{Z}_0$$

$$\frac{\mathrm{Im}\, \bar{z}'}{\mathrm{Re}\, \bar{z}'} = \tan(-2 \angle \bar{Z}_0)$$

$$\boxed{\mathrm{Im}\, \bar{z}' = \tan(-2 \angle \bar{Z}_0) \cdot \mathrm{Re}\, \bar{z}'}$$

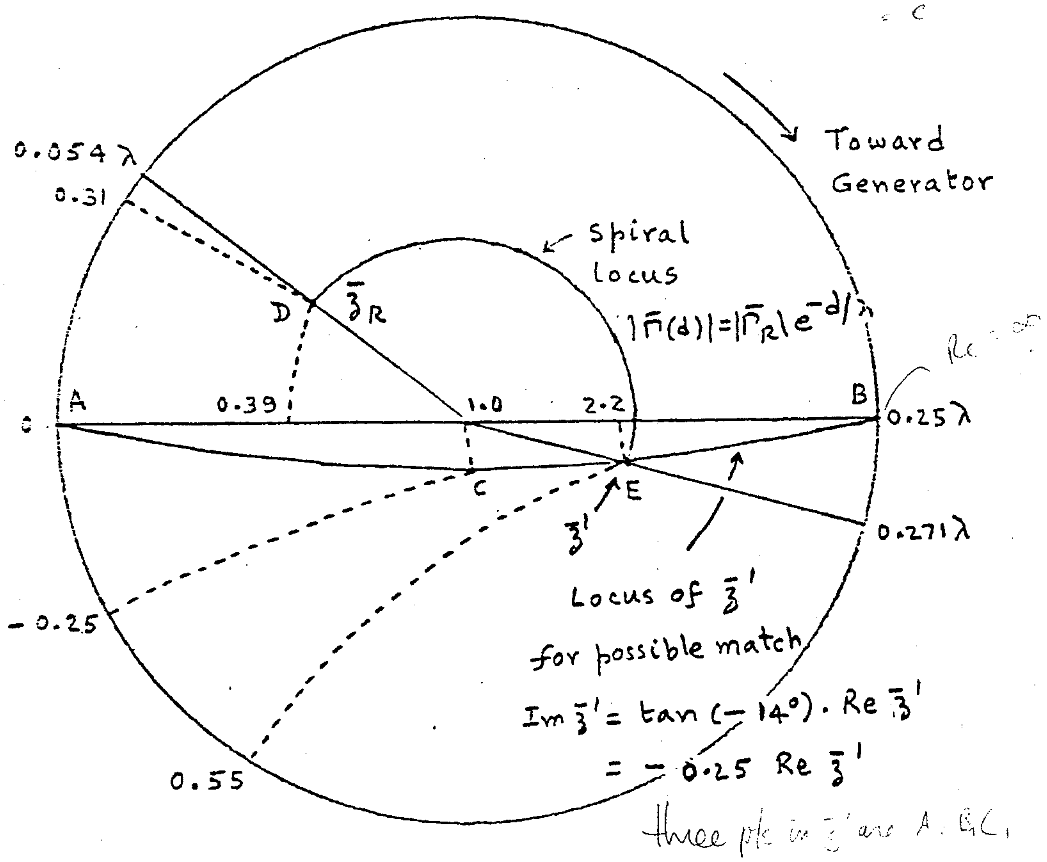
Locus of $\bar{z}'$
for possible match

Example



$$\bar{z}_R = \frac{50\angle 45^\circ}{100\angle 7^\circ} = .39 + j0.31$$

$$e^{-2\alpha d} = e^{-2\times .5\, d/\lambda} = e^{-d/\lambda}$$



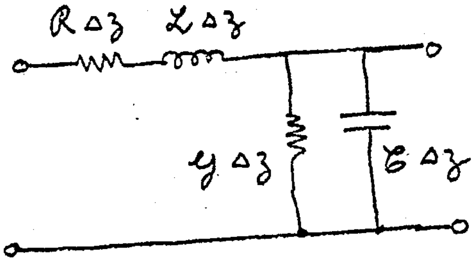
$$\text{Location} = (0.271 - 0.054)\lambda = 0.217\lambda$$

$$Z_0' = \sqrt{\bar{z}'\,\bar{Z}_0^2} = \sqrt{(2.2 - j0.55)\times 10^4 \angle 14^\circ}$$

$$= 148.35\ \Omega$$

Distortionless Line

$$\boxed{\frac{\mathcal{R}}{\mathcal{L}} = \frac{\mathcal{G}}{\mathcal{C}}}$$



$$\begin{aligned}
\bar{\gamma} &= \sqrt{(\mathcal{R}+j\omega\mathcal{L})(\mathcal{G}+j\omega\mathcal{C})} \\
&= \sqrt{\mathcal{R}\left(1+j\frac{\omega\mathcal{L}}{\mathcal{R}}\right)\mathcal{G}\left(1+j\frac{\omega\mathcal{C}}{\mathcal{G}}\right)} \\
&= \sqrt{\mathcal{R}\mathcal{G}}\left(1+j\omega\frac{\mathcal{L}}{\mathcal{R}}\right) \\
&= \sqrt{\mathcal{R}\mathcal{G}} + j\omega\mathcal{L}\sqrt{\frac{\mathcal{G}}{\mathcal{R}}} \\
&= \sqrt{\mathcal{R}\mathcal{G}} + j\omega\mathcal{L}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}} \\
&= \sqrt{\mathcal{R}\mathcal{G}} + j\omega\sqrt{\mathcal{L}\mathcal{C}}
\end{aligned}$$

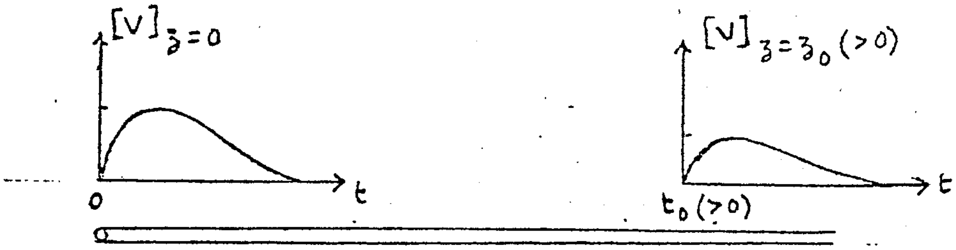
$\therefore\ \alpha = \sqrt{\mathcal{R}\mathcal{G}}$ is independent of frequency

$\beta = \omega\sqrt{\mathcal{L}\mathcal{C}} \longrightarrow v_p = \dfrac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$ is independent of frequency

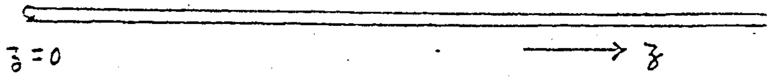
$$\begin{aligned}
\bar{Z}_0 &= \sqrt{\frac{\mathcal{R}+j\omega\mathcal{L}}{\mathcal{G}+j\omega\mathcal{C}}} = \sqrt{\frac{\mathcal{R}(1+j\omega\mathcal{L}/\mathcal{R})}{\mathcal{G}(1+j\omega\mathcal{C}/\mathcal{G})}} \\
&= \sqrt{\frac{\mathcal{R}}{\mathcal{G}}} = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}
\end{aligned}$$

is purely real and independent of frequency.

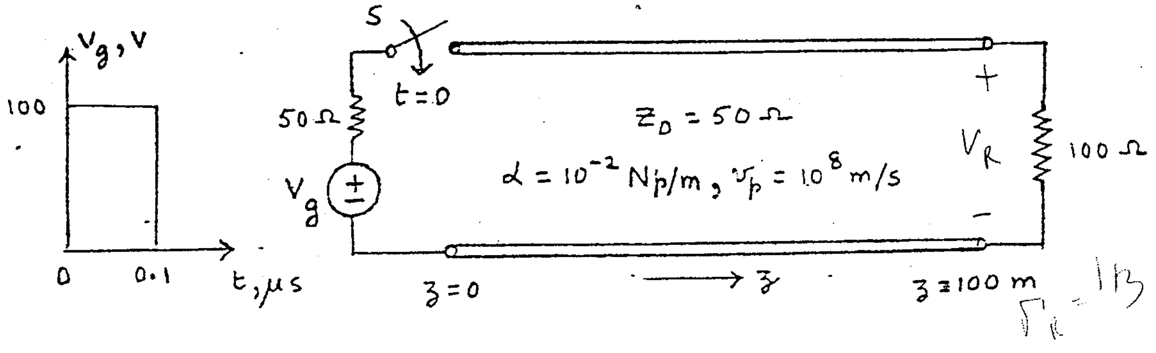
everything similar t lossless case
but have attenuation $\alpha \neq 0$



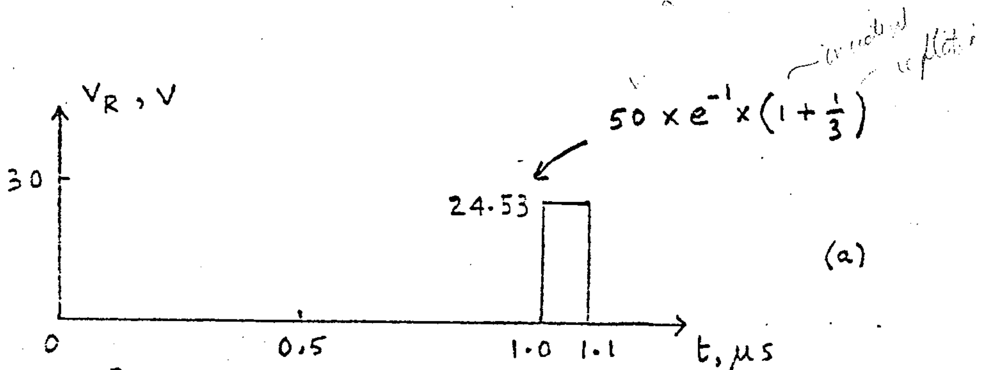
Distortionless Line



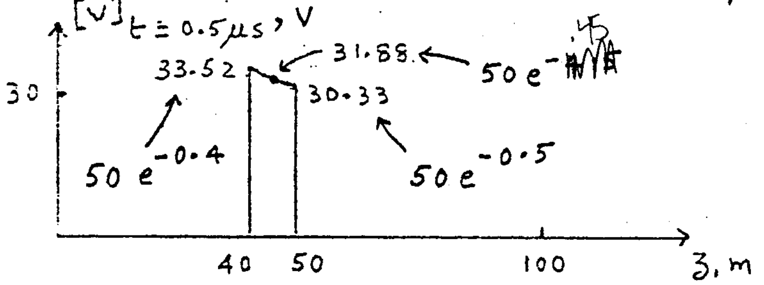
Example



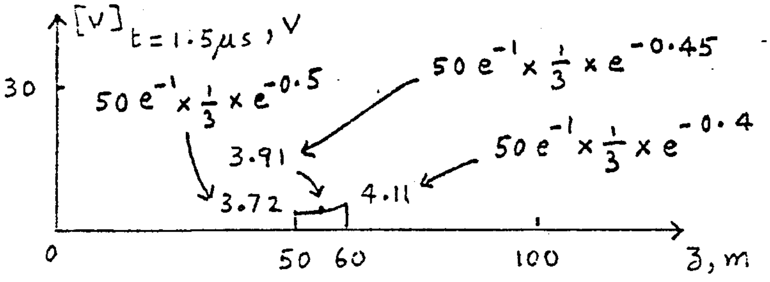
$\frac{100}{2} e^{-10^{-2} \cdot 100}$



(a)



(b)

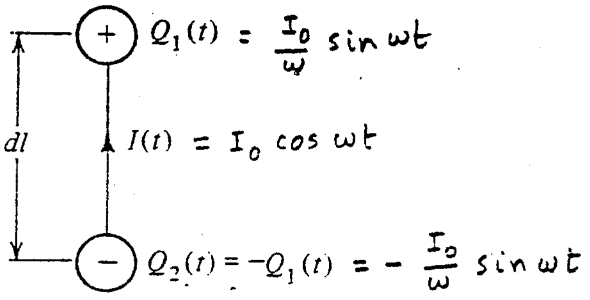


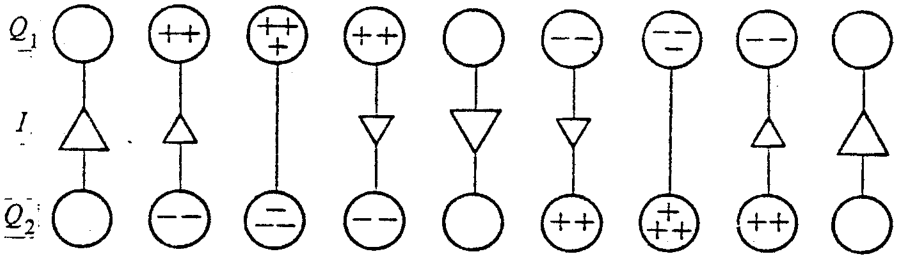
(c)

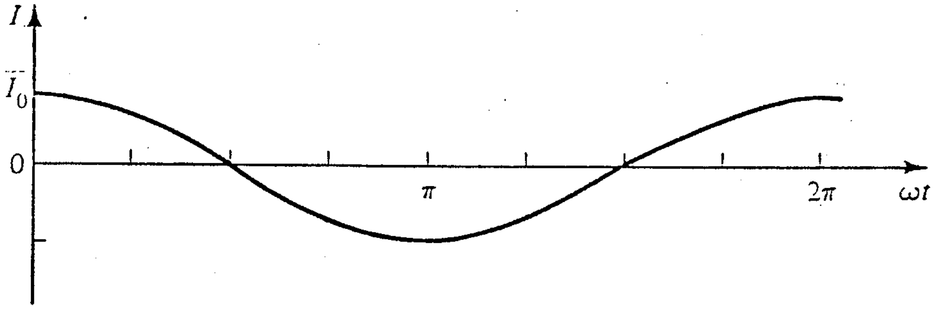
ANTENNAS

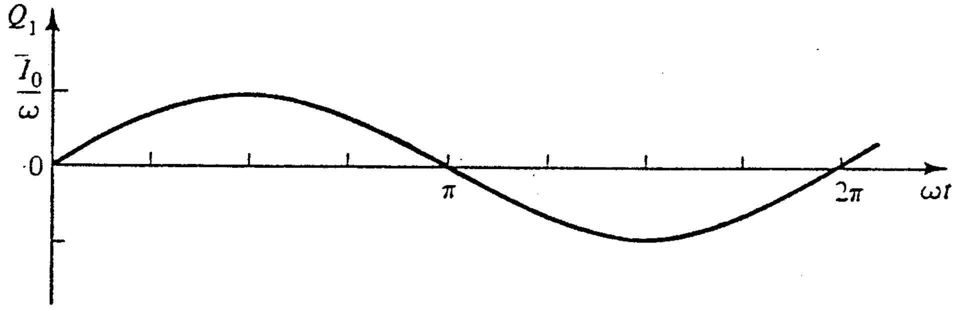
(32-36)

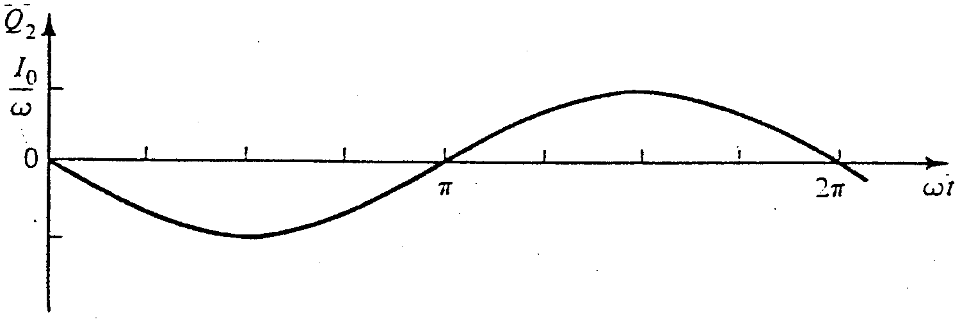
HERTZIAN DIPOLE







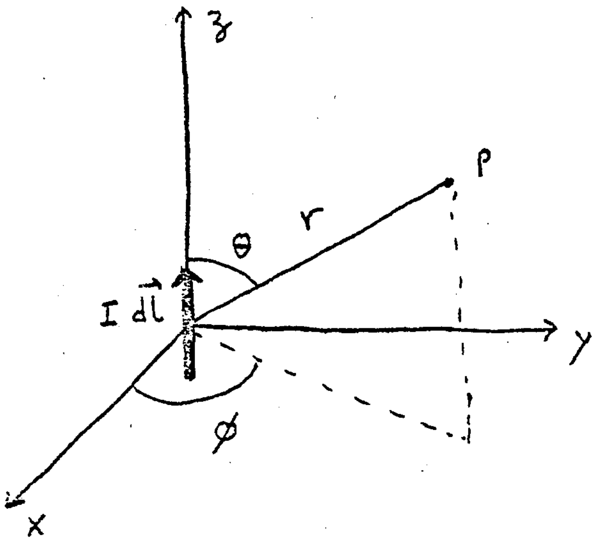




Magnetic Vector Potential Due to a Current Element

$$\vec{\nabla} \circ \vec{B} = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$



$$\vec{B} = \frac{\mu}{4\pi} \; \frac{I\,d\vec{l} \times \vec{i}_r}{r^2}$$

$$= \frac{\mu}{4\pi} \; I\,d\vec{l} \times \left(-\vec{\nabla}\frac{1}{r}\right) \qquad \vec{A} \times \vec{\nabla} V = V\,\vec{\nabla} \times \vec{A} - \vec{\nabla} \times (V\vec{A})$$

$$= -\frac{\mu I}{4\pi}\,\vec{\nabla} \times d\vec{l} + \vec{\nabla} \times \left(\frac{\mu I\,d\vec{l}}{4\pi r}\right)$$

$$= 0 + \vec{\nabla} \times \frac{\mu I\,d\vec{l}}{4\pi r}$$

$$\therefore \boxed{\vec{A} = \frac{\mu I\,d\vec{l}}{4\pi r}}$$

Hertzian Dipole

$$I = I_0 \cos \omega t$$

$$\vec{A} = \frac{\mu I_0 d\vec{l}}{4\pi r} \cos \omega\left(t - \frac{r}{v_p}\right)$$

$$= \frac{\mu I_0 d\vec{l}}{4\pi r} \cos(\omega t - \beta r) \qquad \text{Retarded Potential.}$$

For $d\vec{l} = dz\, \vec{i}_z$

$$\vec{A} = \frac{\mu I_0 dz\, \vec{i}_z}{4\pi r} \cos(\omega t - \beta r)$$

$$= \frac{\mu I_0 dz(\cos\theta\, \vec{i}_r - \sin\theta\, \vec{i}_\theta)}{4\pi r} \cos(\omega t - \beta r)$$

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{1}{\mu} \vec{\nabla} \times \vec{A} = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right] \vec{i}_\phi$$

$$= \frac{I_0 dl \sin\theta}{4\pi}\left[\frac{\cos(\omega t - \beta r)}{r^2} - \frac{\beta \sin(\omega t - \beta r)}{r}\right] \vec{i}_\phi$$

Then from $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t}$,

$$\vec{E} = \frac{2 I_0 dl \cos\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta \cos(\omega t - \beta r)}{r^2}\right] \vec{i}_r$$

$$+ \frac{I_0 dl \sin\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2} - \frac{\beta^2 \sin(\omega t - \beta r)}{r}\right] \vec{i}_\theta$$

$1 - \frac{2\pi}{\lambda}(.2\lambda) = \left(\frac{2\pi}{\cancel{\lambda}}(.2\cancel{\lambda})\right)^2$

$1 - \frac{2\pi}{\lambda}$

$1 - .4\pi = 4\pi^2 .04$

$= 1.6\pi^2$

$= .16\pi^2$

$=$

Radiation fields

For large r, terms in $\frac{1}{r} \gg$ terms in $\frac{1}{r^2}$ and $\frac{1}{r^3}$.

How large? Consider the $\vec{H}$ field. Then

$$\frac{\beta}{r} \gg \frac{1}{r^2}$$

$$\beta r \gg 1$$

$$r \gg \frac{1}{\beta}$$

$$r \gg \frac{\lambda}{2\pi}$$

$\therefore$ Even at a distance of a few wavelengths from the dipole, the terms in $\frac{1}{r^2}$ and $\frac{1}{r^3}$ are negligible compared to the $\frac{1}{r}$ terms. Then

$$\underset{\sim}{E} \approx -\frac{\beta^2 I_0 \, dl \sin\theta}{4\pi \epsilon \omega r} \sin(\omega t - \beta r) \, \underset{\sim}{i}_\theta$$

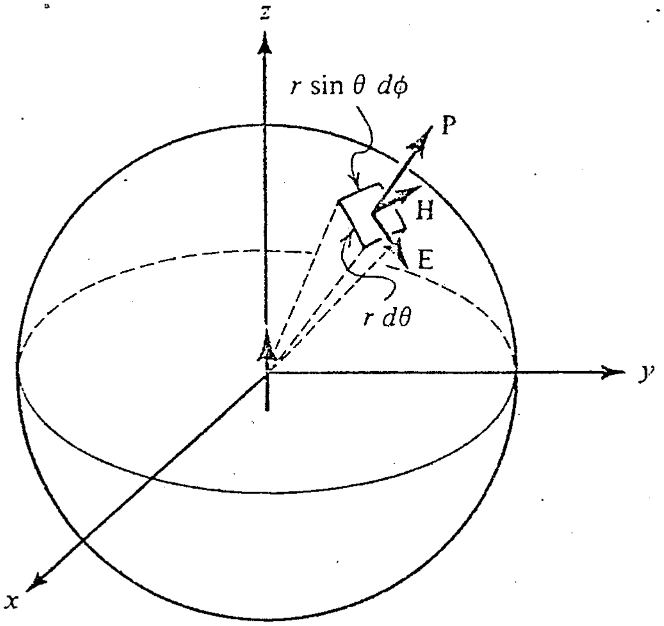
$$= -\frac{\eta \beta I_0 \, dl \sin\theta}{4\pi r} \sin(\omega t - \beta r) \, \underset{\sim}{i}_\theta$$

$$\underset{\sim}{H} \approx -\frac{\beta I_0 \, dl \sin\theta}{4\pi r} \sin(\omega t - \beta r) \, \underset{\sim}{i}_\phi$$

These are called the "radiation fields."

Note $\underset{\sim}{E} \perp \underset{\sim}{H} \perp$ r direction, and $\frac{E_\theta}{H_\phi} = \eta$.

Radiation Resistance of Hertzian Dipole



$$\underset{\sim}{E} = -\frac{\beta^2 I_0\, dl \sin\theta}{4\pi\epsilon\omega r}\sin(\omega t-\beta r)\,\underset{\sim}{i}_\theta$$

$$= -\frac{\eta\beta I_0\, dl \sin\theta}{4\pi r}\sin(\omega t-\beta r)\,\underset{\sim}{i}_\theta$$

$$\underset{\sim}{H} = -\frac{\beta I_0\, dl \sin\theta}{4\pi r}\sin(\omega t-\beta r)\,\underset{\sim}{i}_\phi$$

$$\underset{\sim}{P} = \underset{\sim}{E}\times\underset{\sim}{H}$$

$$= E_\theta H_\phi\,\underset{\sim}{i}_r$$

$$= \frac{\eta\beta^2 I_0^2 (dl)^2 \sin^2\theta}{16\pi^2 r^2}\sin^2(\omega t-\beta r)\,\underset{\sim}{i}_r$$

$\oint \vec{P}\cdot d\vec{S}$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\underset{\sim}{P}\cdot r^2\sin\theta\, d\theta\, d\phi\,\underset{\sim}{i}_r$$

$$= \frac{2\pi\eta}{3} I_0^2\left(\frac{dl}{\lambda}\right)^2\sin^2(\omega t-\beta r)$$

$<\sin^2\theta> = 1/2$

$$<P_{rad}> = \frac{2\pi\eta}{3} I_0^2\left(\frac{dl}{\lambda}\right)^2\left(<\sin^2(\omega t-\beta r)>\right) = 1/2$$

since un lossy

η Resist is indep of R

$$= \frac{1}{2} I_0^2\left[\frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2\right]$$

$$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2$$

$I = I_0 \cos\omega t$

R_{rad}

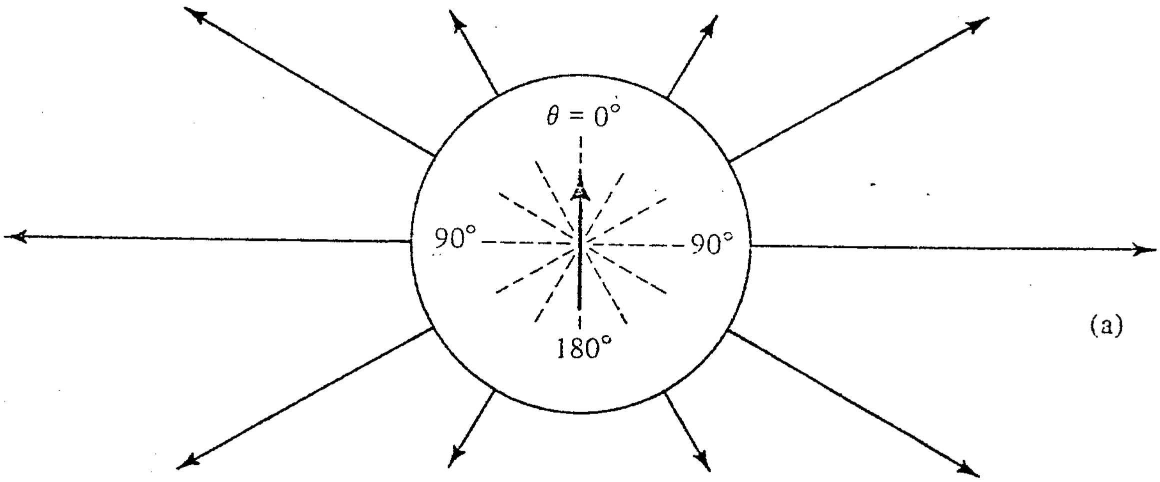
$<P_d> = 1/2\, I_0^2 R$

$$\boxed{R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2}$$ for free space

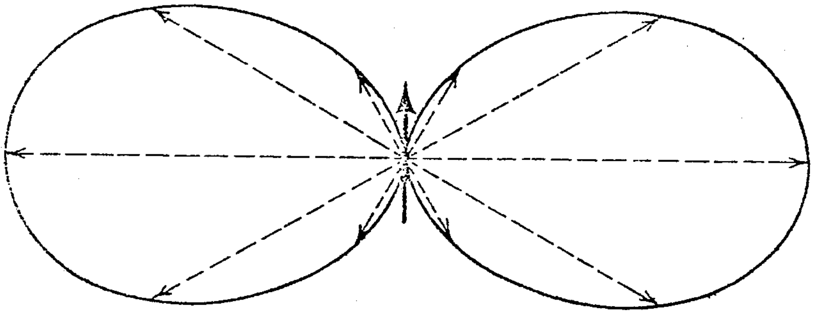
$<P_d> = <P_{rad}>$

Radiation Resistance of an antenna is that value of a resistor which would dissipate the same amount of average power as that radiated by the antenna when the same current is passed through

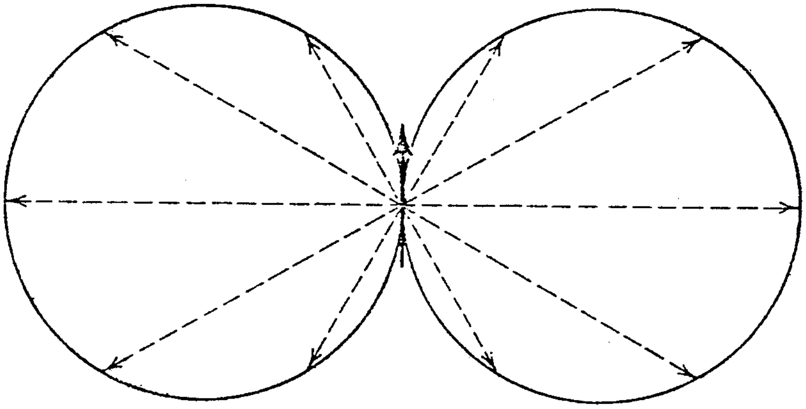
Radiation Patterns



(a)



Power Pattern
varies of
$\sin^2\theta$
where θ is defined as
(b)
in (a)



Field Pattern
$\sin\theta$

(c)

Directivity of Hertzian Dipole

$$\underset{\sim}{P} = \frac{\eta \beta^2 I_0^2 (dl)^2 \sin^2\theta}{16 \pi^2 r^2} \sin^2(\omega t - \beta r) \, \underset{\sim}{i}_r$$

$$P_r = \frac{P_0}{r^2} \sin^2(\omega t - \beta r) \sin^2\theta$$

$$[P_r]_{max} = \frac{P_0}{r^2} \sin^2(\omega t - \beta r) \cdot [\sin^2\theta]_{max}$$

$$[P_r]_{av} = \frac{P_{rad}}{4\pi r^2}$$

$$= \frac{1}{4\pi r^2} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{P_0}{r^2} \sin^2(\omega t - \beta r) \cdot \sin^2\theta \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{P_0 \sin^2(\omega t - \beta r)}{4\pi r^2} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin^2\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$D = \frac{[P_r]_{max}}{[P_r]_{av}} = 4\pi \frac{[\sin^2\theta]_{max} \; \updownarrow {\scriptstyle \text{Same fn}}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin^2\theta \cdot \sin\theta \, d\theta \, d\phi}$$

$$= 4\pi \frac{1}{2\pi \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta} = 4\pi \frac{1}{4\pi \int_{\theta=0}^{\pi/2} \sin^3\theta \, d\theta}$$

$$= \frac{1}{2/3} = \frac{3}{2} = 1.5$$

In general,

$$P_r = \frac{P_0 \sin^2(\omega t - \beta r)}{r^2} \quad f(\theta, \phi)$$

$$\boxed{D = 4\pi \frac{[f(\theta,\phi)]_{max}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} f(\theta,\phi) \sin\theta \, d\theta \, d\phi}}$$

Example:

$$f(\theta, \phi) = \sin^4\theta$$

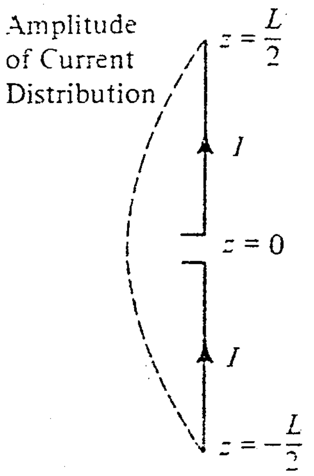
$$D = 4\pi \frac{[\sin^4\theta]_{max}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin^5\theta \, d\theta \, d\phi}$$

$$= \frac{1}{\int_{\theta=0}^{\pi/2} \sin^5\theta \, d\theta}$$

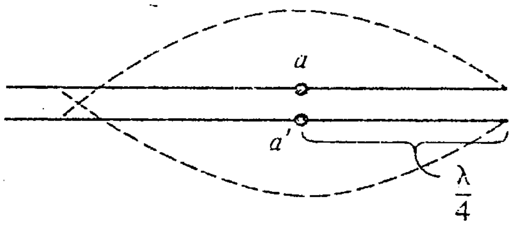
.250

$$= \frac{1}{\frac{2 \times 4}{1 \times 3 \times 5}} = \frac{15}{8} = 1\frac{7}{8} \, .$$

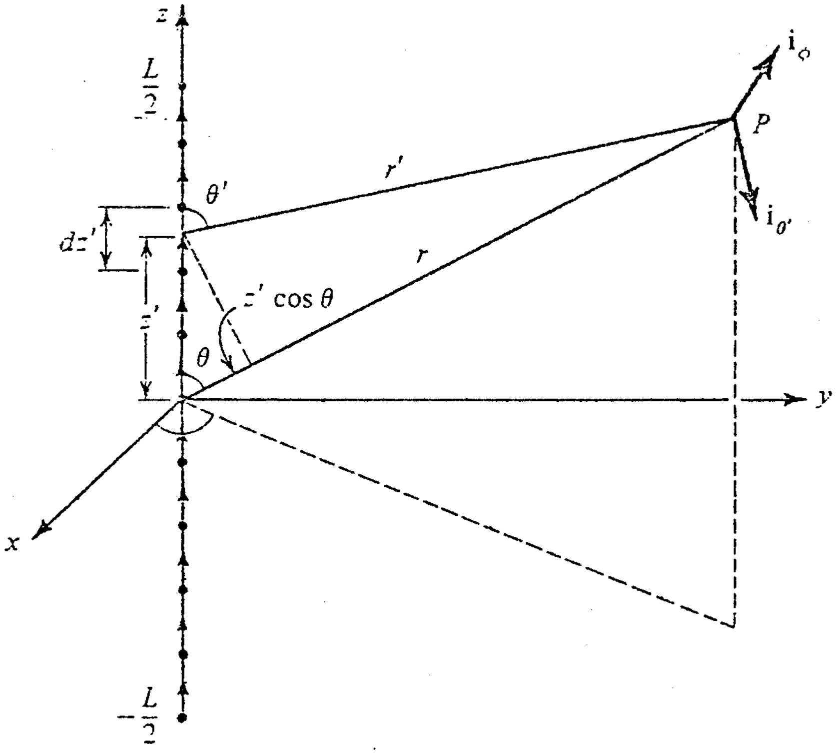
HALF-WAVE DIPOLE



(a)



(b)



Radiation Fields for Half-Wave Dipole

$$I(z) = I_0 \cos\frac{\pi z}{L}\cos\omega t$$

Consider an element dz' at $z = z'$. Then

$$I(z') = \left(I_0 \cos\frac{\pi z'}{L}\right)\cos\omega t$$

$$d\underset{\sim}{E} = \frac{-\eta\beta I_0 \cos\frac{\pi z'}{L}\, dz' \sin\theta'}{4\pi r'} \sin(\omega t - \beta r')\, \underset{\sim}{i}_{\theta'}$$

$$d\underset{\sim}{H} = -\frac{\beta I_0 \cos\frac{\pi z'}{L}\, dz' \sin\theta'}{4\pi r'} \sin(\omega t - \beta r')\, \underset{\sim}{i}_{\phi}$$

$$\underset{\sim}{E} = \int_{z'=-\frac{L}{2}}^{\frac{L}{2}} d\underset{\sim}{E} = -\int_{z'=-\frac{L}{2}}^{\frac{L}{2}} \frac{\eta\beta I_0 \cos\frac{\pi z'}{L}\sin\theta'\, dz'}{4\pi r'} \sin(\omega t - \beta r')\, \underset{\sim}{i}_{\theta'}$$

$$\underset{\sim}{H} = \int_{z'=-\frac{L}{2}}^{\frac{L}{2}} d\underset{\sim}{H} = -\int_{z'=-\frac{L}{2}}^{\frac{L}{2}} \underbrace{\frac{\beta I_0 \cos\frac{\pi z'}{L}\sin\theta'\, dz'}{4\pi r'}}_{\text{amplitude}} \underbrace{\sin(\omega t - \beta r')}_{\text{phase}}\, \underset{\sim}{i}_{\phi}$$

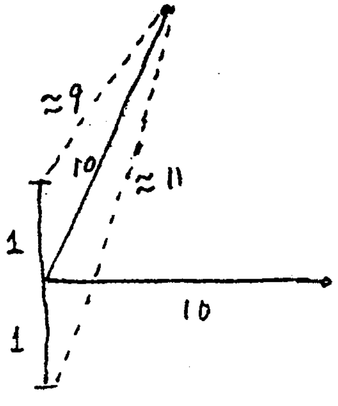
For radiation fields,

$r' \gg L$

$\underset{\sim}{i}_{\theta'} \approx \underset{\sim}{i}_{\theta}$, $\theta' \approx \theta$

$r' \approx r$ in the amplitude factors

$r' \approx r - z'\cos\theta$ in the phase factors



$L = 2$ m

$9 < r' < 11$

$810^\circ < \beta r' < 990^\circ$

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$E_\theta = -\int_{z'=-\frac{L}{2}}^{\frac{L}{2}} \frac{\eta \beta I_0 \cos \frac{\pi z'}{L} \sin\theta}{4\pi r} \sin(\omega t - \beta r + \beta z' \cos\theta)\, dz'$$

$$= -\frac{\eta \frac{\pi}{L} I_0 \sin\theta}{4\pi r} \int_{z'=-\frac{L}{2}}^{\frac{L}{2}} \cos\frac{\pi z'}{L} \sin\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{L} z' \cos\theta\right) dz'$$

$$= -\frac{\eta \frac{\pi}{L} I_0 \sin\theta}{8\pi r} \int_{z'=-\frac{L}{2}}^{\frac{L}{2}} \left[\sin\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{L} z' \cos\theta + \frac{\pi}{L} z'\right) + \sin\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{L} z' \cos\theta - \frac{\pi}{L} z'\right)\right] dz'$$

$$= -\frac{\eta \frac{\pi}{L} I_0 \sin\theta}{8\pi r} \left[\frac{-\cos\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{L} z' \cos\theta + \frac{\pi}{L} z'\right)}{\frac{\pi}{L}(\cos\theta + 1)} - \frac{\cos\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{L} z' \cos\theta - \frac{\pi}{L} z'\right)}{\frac{\pi}{L}(\cos\theta - 1)} \right]_{z'=-\frac{L}{2}}^{\frac{L}{2}}$$

$$= -\frac{\eta I_0 \sin\theta}{8\pi r} \left[\frac{\sin\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{2}\cos\theta\right) + \sin\left(\omega t - \frac{\pi}{L} r - \frac{\pi}{2}\cos\theta\right)}{\cos\theta + 1} - \frac{\sin\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{2}\cos\theta\right) + \sin\left(\omega t - \frac{\pi}{L} r - \frac{\pi}{2}\cos\theta\right)}{\cos\theta - 1} \right]$$

$$= -\frac{\eta I_0 \sin\theta}{8\pi r} \Bigg[\frac{2\sin\left(\omega t - \frac{\pi}{L} r\right) \cos\left(\frac{\pi}{2}\cos\theta\right)}{\cos\theta + 1} - \frac{2\sin\left(\omega t - \frac{\pi}{L} r\right) \cos\left(\frac{\pi}{2}\cos\theta\right)}{\cos\theta - 1}$$

$$= -\frac{\eta I_0}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \sin\left(\omega t - \frac{\pi}{L} r\right)$$

Similarly,

$$H_\phi = -\frac{I_0}{2\pi r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\sin\left(\omega t - \frac{\pi}{L}r\right)$$

Then

$$\underset{\sim}{P} = \underset{\sim}{E}\times\underset{\sim}{H} = E_\theta H_\phi \underset{\sim}{i}_r$$

$$= \frac{\eta I_0^2}{4\pi^2 r^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\cdot\sin^2\left(\omega t - \frac{\pi}{L}r\right)\underset{\sim}{i}_r$$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\underset{\sim}{P}\cdot r^2\sin\theta\, d\theta\, d\phi\, \underset{\sim}{i}_r$$

$$= \frac{0.609\,\eta I_0^2}{\pi}\sin^2\left(\omega t - \frac{\pi}{L}r\right)$$

$$\langle P_{rad}\rangle = \frac{0.609\,\eta I_0^2}{2\pi} = \frac{1}{2}I_0^2\left(\frac{0.609\,\eta}{\pi}\right)$$

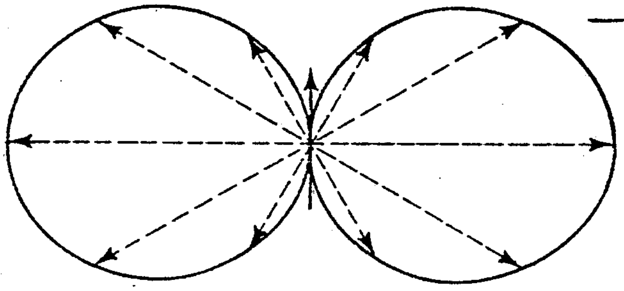
$$\boxed{R_{rad} = \frac{0.609\,\eta}{\pi}} = 73\ \Omega \text{ for free space}$$

$$D = 4\pi\frac{\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]_{max}}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\sin\theta\, d\theta\, d\phi}$$

$$= \frac{4\pi}{2\pi\int_{\theta=0}^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}d\theta} = \frac{4\pi}{2\pi\times 2\times 0.60943}$$

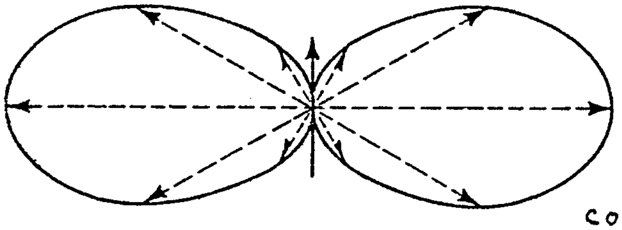
$$\boxed{D = 1.642}$$

Radiation Patterns for Half-Wave Dipole



$$\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

(a)



(b)

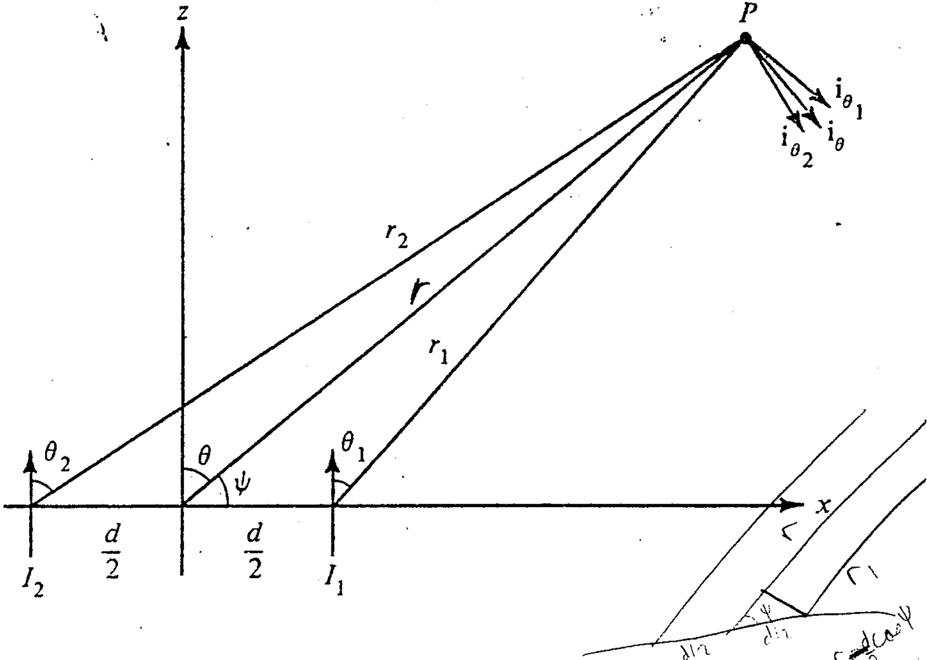
$$\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

530
52
2650
2625
27300
360

Antenna Arrays

$$I_1 = I_0 \cos\left(\omega t + \frac{\alpha}{2}\right)$$

$$I_2 = I_0 \cos\left(\omega t - \frac{\alpha}{2}\right)$$



$r_1 \approx r - \frac{d}{2}\cos\psi$

for $r \gg \lambda$

$$\underset{\sim}{E}_1 = -\frac{\eta\beta I_0\, dl \sin\theta_1}{4\pi r_1} \sin\left(\omega t - \beta r_1 + \frac{\alpha}{2}\right) \underset{\sim}{i}_{\theta_1}$$

$$\approx -\frac{\eta\beta I_0\, dl \sin\theta}{4\pi r} \sin\left[\omega t - \beta\left(r - \frac{d}{2}\cos\psi\right) + \frac{\alpha}{2}\right] \underset{\sim}{i}_{\theta}$$

$$\underset{\sim}{E}_2 = -\frac{\eta\beta I_0\, dl \sin\theta_2}{4\pi r_2} \sin\left(\omega t - \beta r_2 - \frac{\alpha}{2}\right) \underset{\sim}{i}_{\theta_2}$$

$$\approx -\frac{\eta\beta I_0\, dl \sin\theta}{4\pi r} \sin\left[\omega t - \beta\left(r + \frac{d}{2}\cos\psi\right) - \frac{\alpha}{2}\right] \underset{\sim}{i}_{\theta}$$

$$\underset{\sim}{E} = \underset{\sim}{E}_1 + \underset{\sim}{E}_2$$

$$= -\frac{\eta\beta I_0\, dl \sin\theta}{4\pi r}\left[\sin\left(\omega t - \beta r + \frac{\beta d}{2}\cos\psi + \frac{\alpha}{2}\right) + \sin\left(\omega t - \beta r - \frac{\beta d}{2}\cos\psi - \frac{\alpha}{2}\right)\right] \underset{\sim}{i}_{\theta}$$

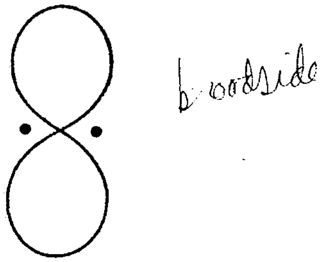
$$= -\frac{2\eta\beta I_0\, dl \sin\theta}{4\pi r} \cos\left(\frac{\beta d\cos\psi + \alpha}{2}\right) \sin(\omega t - \beta r)\, \underset{\sim}{i}_{\theta}$$

$$\underset{\sim}{E} = -\frac{2\eta\beta I_0\, dl}{4\pi r}\;\underbrace{\underbrace{\sin\theta}_{\text{Unit pattern}}\;\underbrace{\cos\left(\frac{\beta d\cos\psi+\alpha}{2}\right)}_{\text{Group pattern}}}_{\text{Resultant Pattern}}\;\sin(\omega t-\beta r)\;\underset{\sim}{i}_\theta$$

Group Patterns for Several Cases

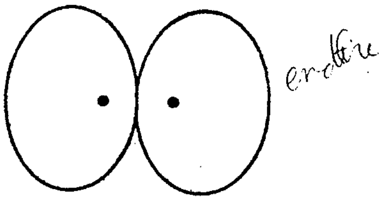
a) $d = \frac{\lambda}{2}$, $\alpha = 0$

$$\left|\cos\left(\frac{\beta\lambda}{4}\cos\psi\right)\right| = \cos\left(\frac{\pi}{2}\cos\psi\right)$$



b) $d = \frac{\lambda}{2}$, $\alpha = \pi$

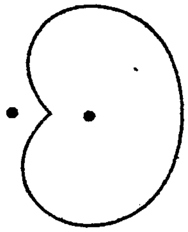
$$\left|\cos\left(\frac{\beta\lambda}{4}\cos\psi+\frac{\pi}{2}\right)\right| = \left|\sin\left(\frac{\pi}{2}\cos\psi\right)\right|$$



c) $d = \frac{\lambda}{4}$, $\alpha = -\frac{\pi}{2}$

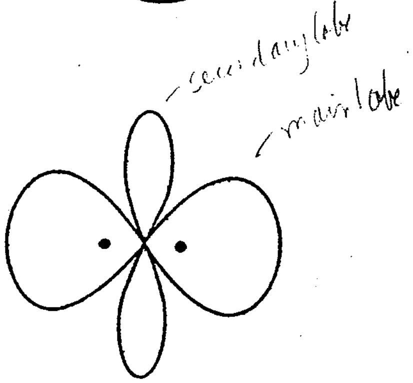
$$\left|\cos\left(\frac{\beta\lambda}{8}\cos\psi-\frac{\pi}{4}\right)\right|$$

$$= \cos\left(\frac{\pi}{4}\cos\psi-\frac{\pi}{4}\right)$$

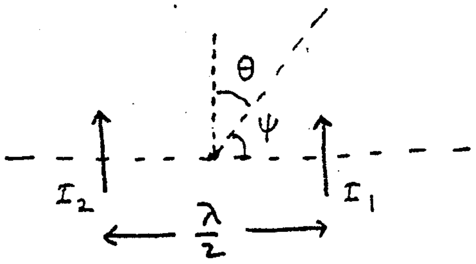


d) $d = \lambda$, $\alpha = 0$

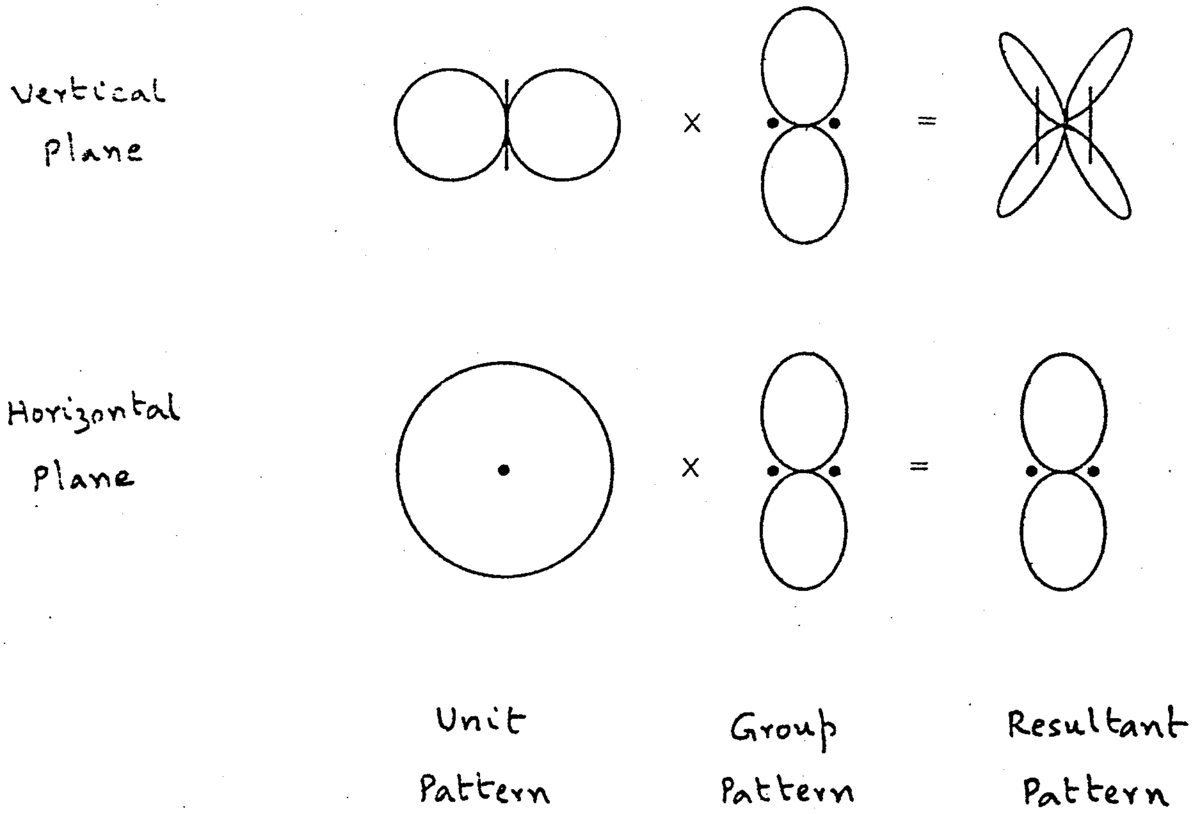
$$\left|\cos\left(\frac{\beta\lambda}{2}\cos\psi\right)\right| = \left|\cos(\pi\cos\psi)\right|$$



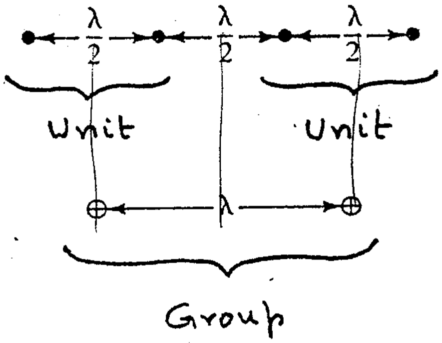
Resultant Patterns for an Array of Two Hertzian Dipoles

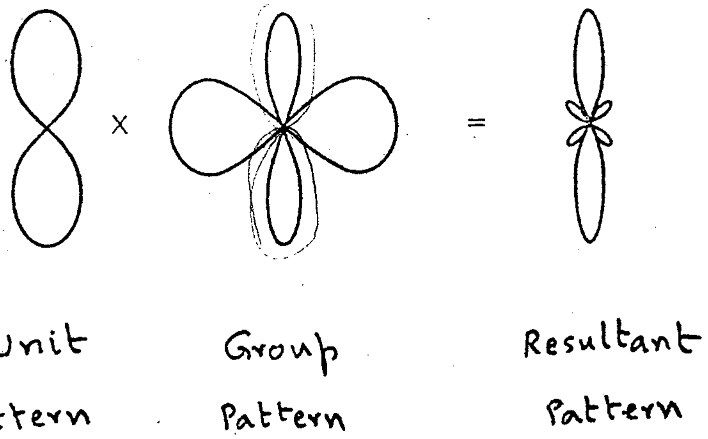


$$I_1 = I_2 = I_0 \cos \omega t \quad ; (\alpha = 0)$$



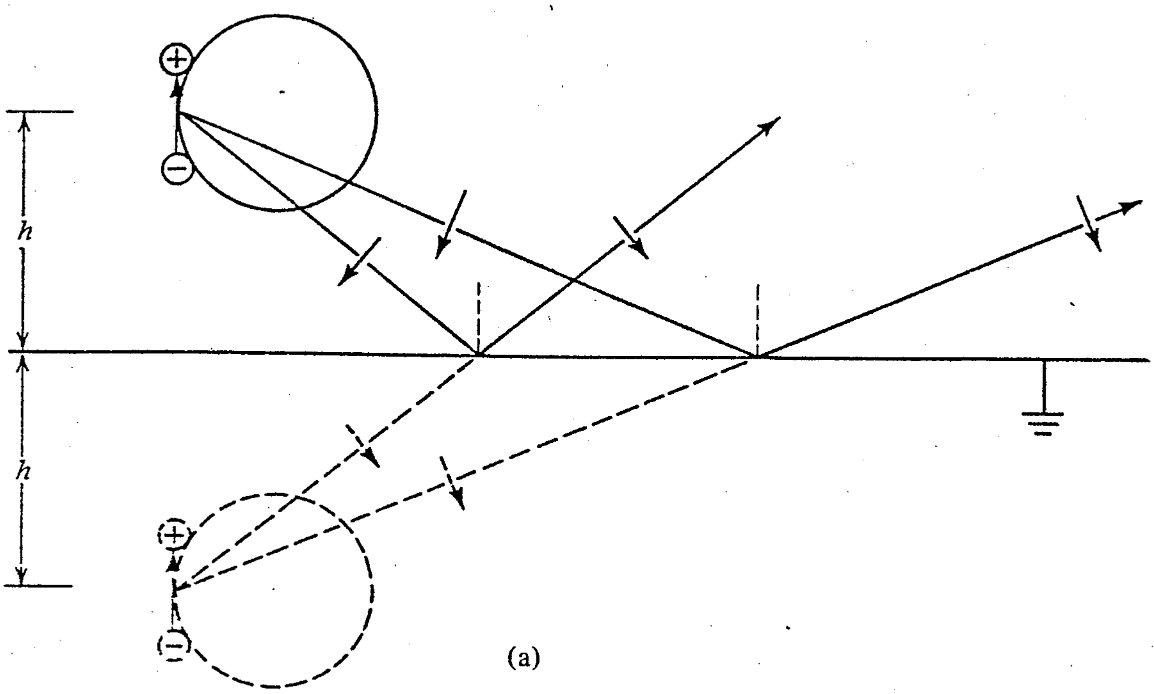
Another Example of Pattern Multiplication



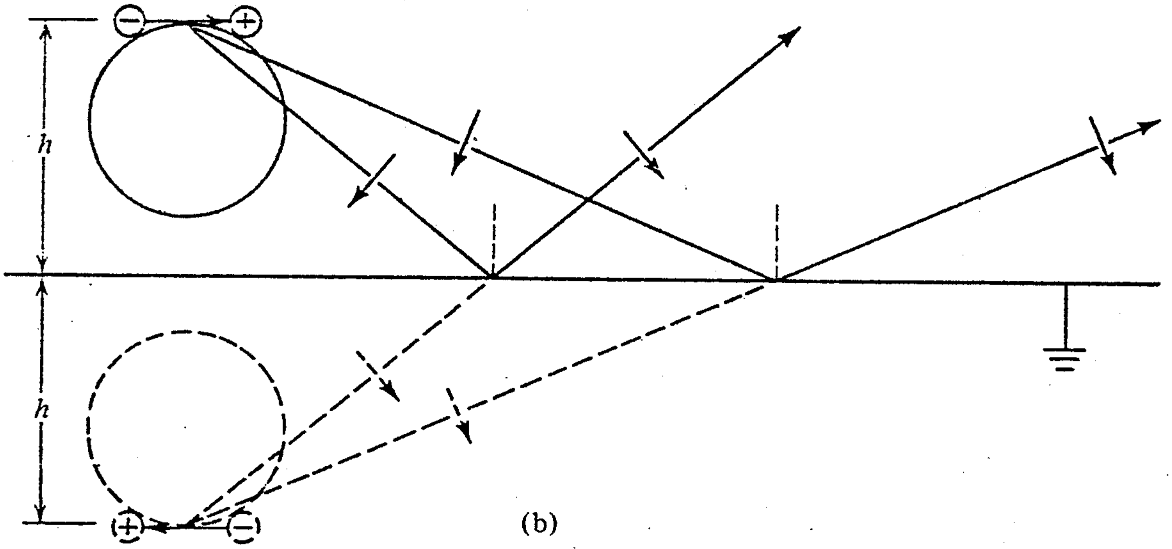


see case (d)
pg 35-2

Image Antennas



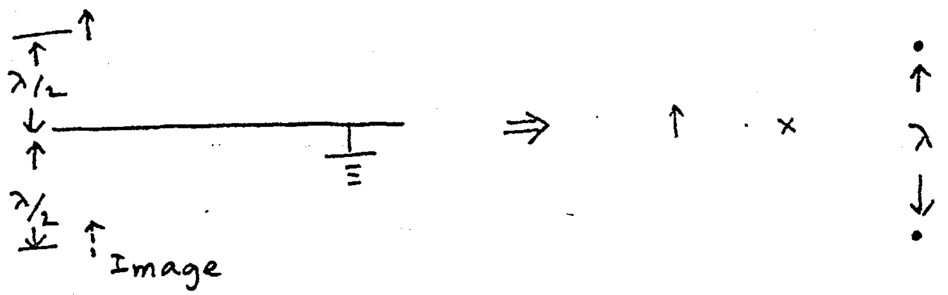
(a)

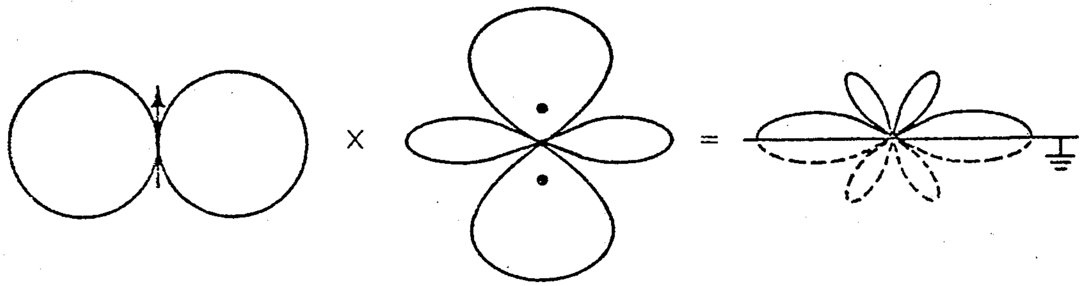


(b)

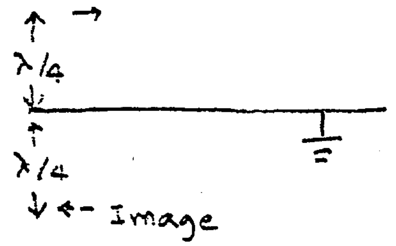
Examples of Radiation Patterns

1) Vertical Hertzian Dipole $\frac{\lambda}{2}$ above ground

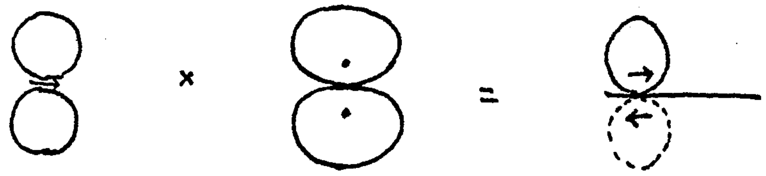




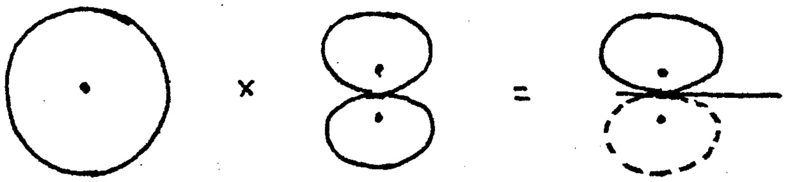
2) Horizontal Hertzian Dipole $\frac{\lambda}{4}$ above ground



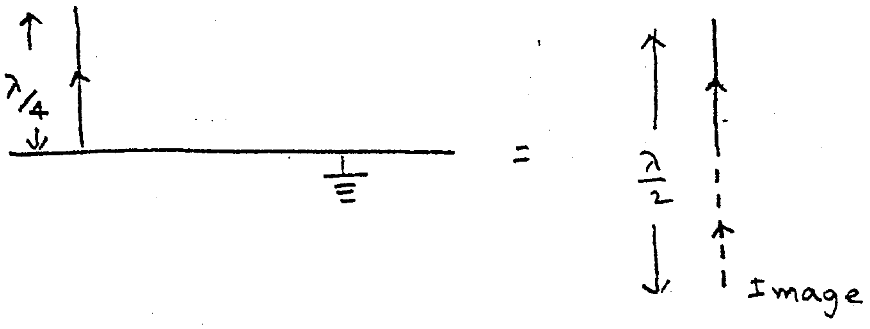
In The vertical plane containing The axis of The antenna

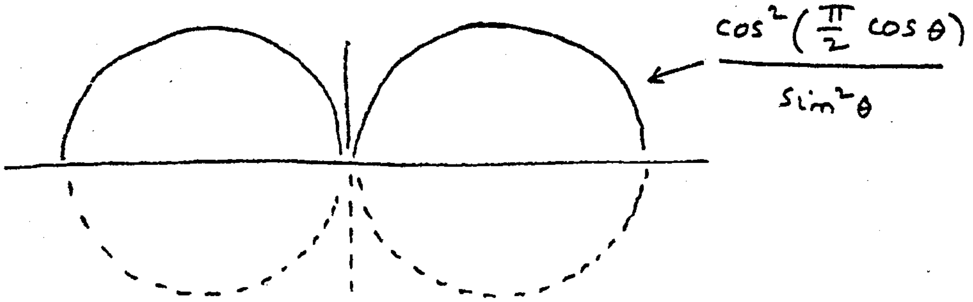


In The vertical plane perpendicular to the axis of The antenna



3) Vertical antenna of length $\frac{\lambda}{4}$ above ground



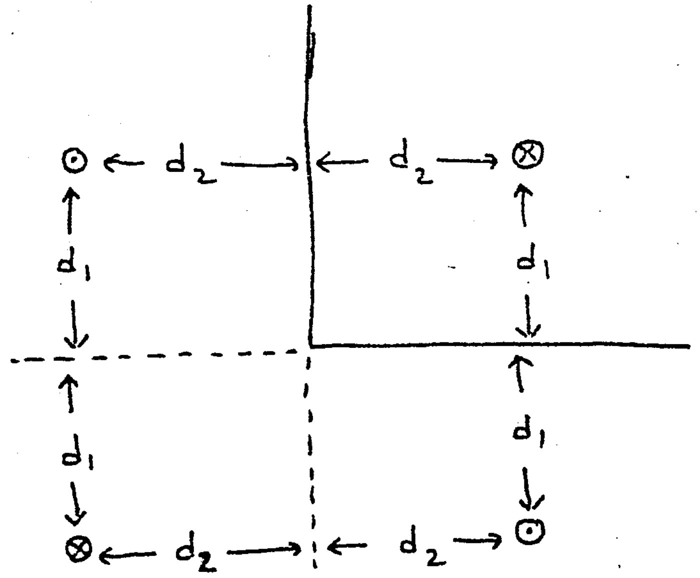


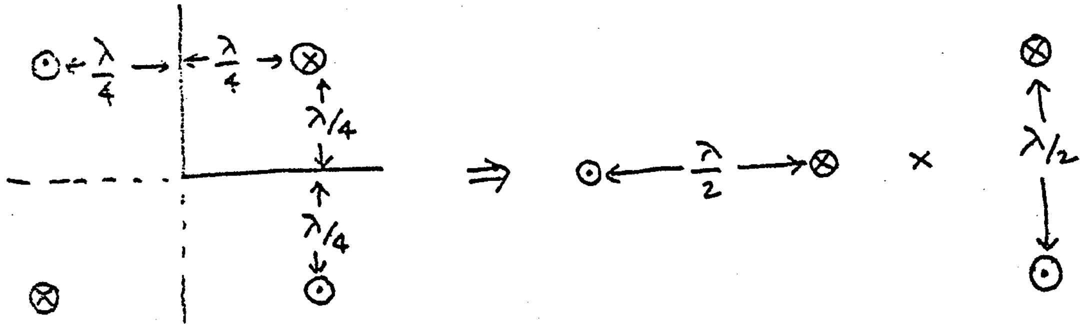
$$D = 4\pi \frac{\left[f(\theta,\phi)\right]_{max}}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} f(\theta,\phi)\,\sin\theta\;d\theta\,d\phi} = \frac{\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]_{max}}{\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,d\theta\,d\phi}$$

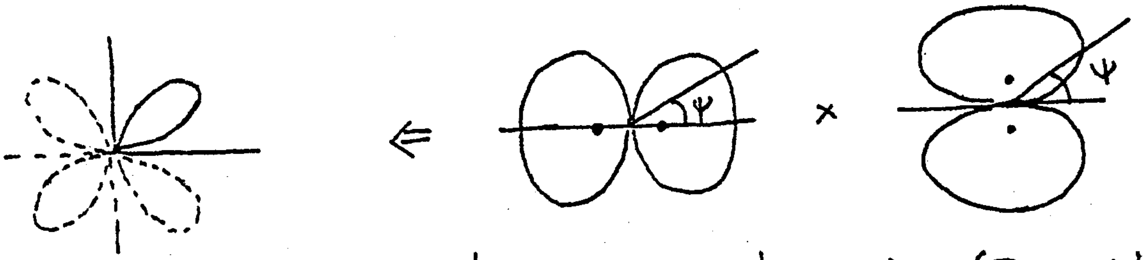
$$= 4\pi\frac{1}{2\pi\int_{\theta=0}^{\pi/2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,d\theta}$$

$$= \frac{2}{0.60943} = 3.28$$ (twice that of of $\lambda/2$ dipole in the absence of ground)

Corner Reflector







$$\left| \sin\left(\frac{\pi}{2} \cos\psi \right) \right| \times \left| \sin\left(\frac{\pi}{2} \sin\psi \right) \right|$$

PROBLEMS

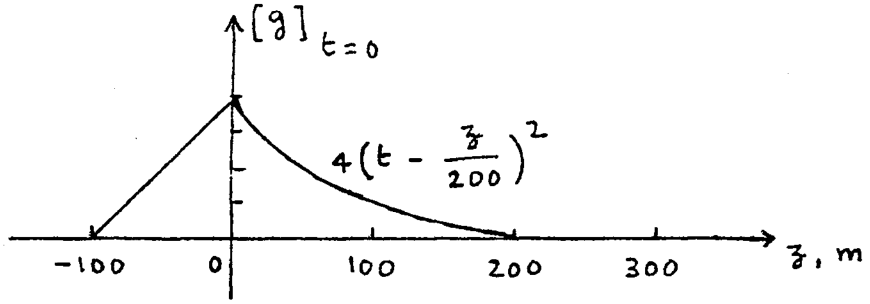
EE350
(NNR)

PROBLEMS

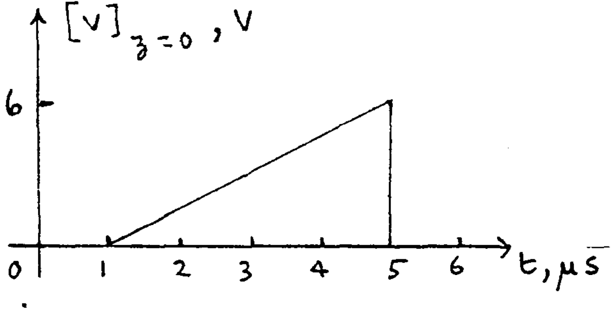
TIME DOMAIN ANALYSIS

1. Write expressions for traveling wave functions for each of the following cases: (a) Time variation at y = 0 in the manner $10e^{-4t^2}$ and propagating in the +y direction with velocity 0.4 m/s. (b) Distance variation at t = 0 in the manner 5 cos 2x and propagating in the −x direction with velocity 10 m/s. (c) Distance variation at t = 0 in the manner $z^2\ e^{8z^3}$ and propagating in the +z direction with velocity 0.5 m/s.

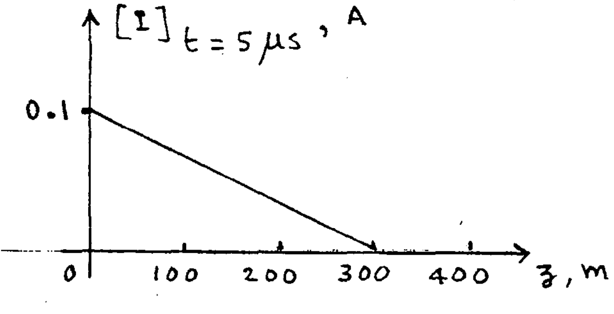
2. The variation with z for t = 0 of a function g(z,t) representing a traveling wave propagating in the −z direction with velocity 100 m/s is shown in the accompanying figure. Find and sketch: (a) g versus z for t = 1 s, (b) g versus t for z = 0, and (c) g versus t for z = 50 m.



3. The line voltage versus t at z = 0 and the line current versus z for t = 5 μs for a wave propagating in the +z direction along an infinitely long lossless line are as shown in Figs. (a) and (b), respectively. Find L and C of the line.

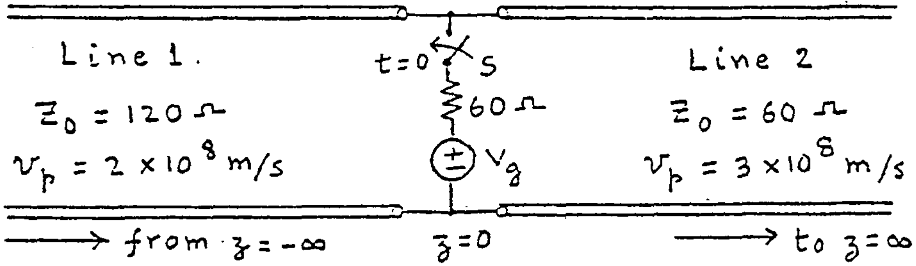


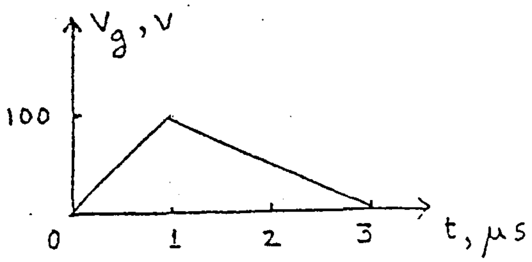
(a)



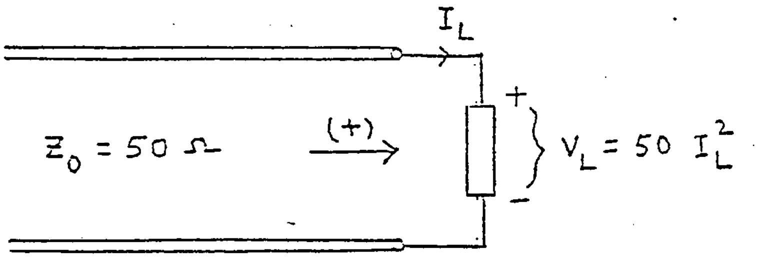
(b)

4. In the system shown, two semi-infinitely long lines extend away from the voltage source in either direction. The switch S is closed at t = 0. Find and sketch the following quantities: (a) line voltage versus t at z = 300m; (b) line current versus t at z = -400 m; (c) line voltage versus z for t = 2.0 μs; (d) line current versus z for t = 4.0 μs.

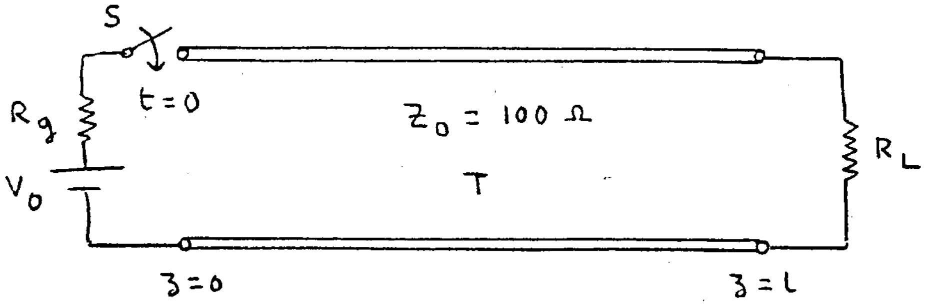




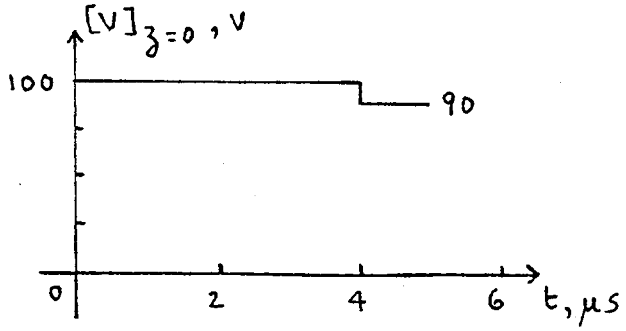
5. In the figure below, a transmission line of characteristic impedance 50 Ω is terminated by a passive nonlinear element having the volt-ampere characteristic indicated in the figure. If a (+) wave of constant voltage 10 V is incident on the termination, find the resulting (−) wave voltage.



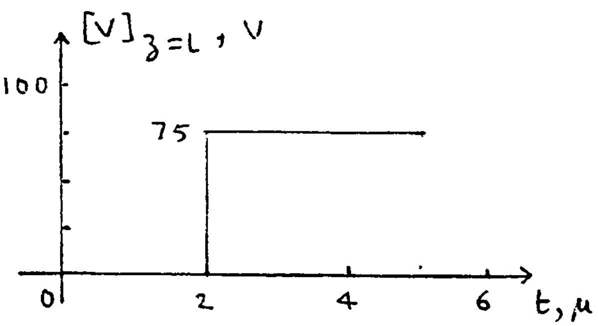
6. In the system shown in Fig. (a), the switch S is closed at t = 0. The line voltage variations with time at z = 0 and z = l for the first 5 μs are observed to be as shown in Figs. (b) and (c), respectively. Find the values of V_0, R_g, R_L, and T.



(a)



(b)



(c)

12 · +1/3
+36/15

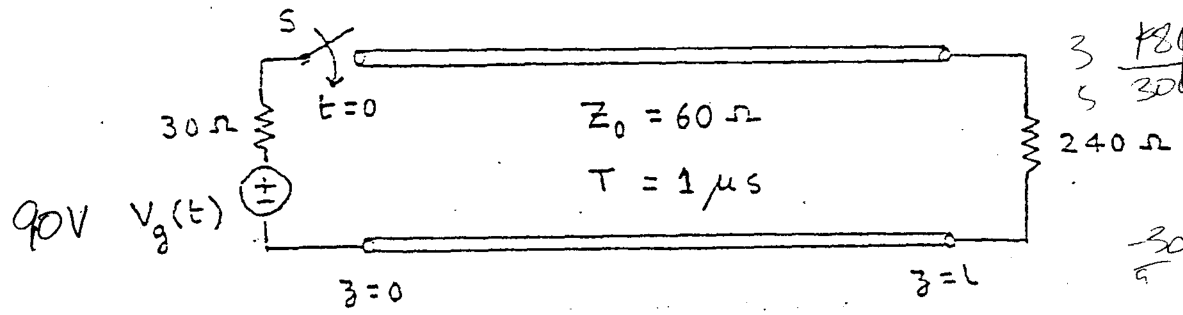
−12 · 3/5   −36/5

36 · −1/3
36/25 · −1/3
−12/4   −12/25 · 3/5

60 · 3/1
12

180/5
· 3/5
12/5

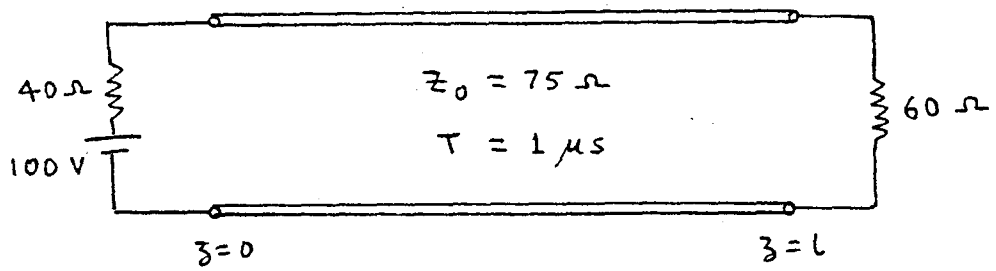
7. In the system shown, the switch S is closed at t = 0. Assume $V_g(t)$ to be a direct voltage of 90 V and draw the voltage and current bounce diagrams. From these bounce diagrams, sketch the following: (a) line voltage and line current versus t (up to t = 7.25 μs) at z = 0, z = l, and z = $l/2$; (b) line voltage and line current versus z for t = 1.2 μs and t = 3.5 μs.



3/5   180/300 · 30   18/30   9/15

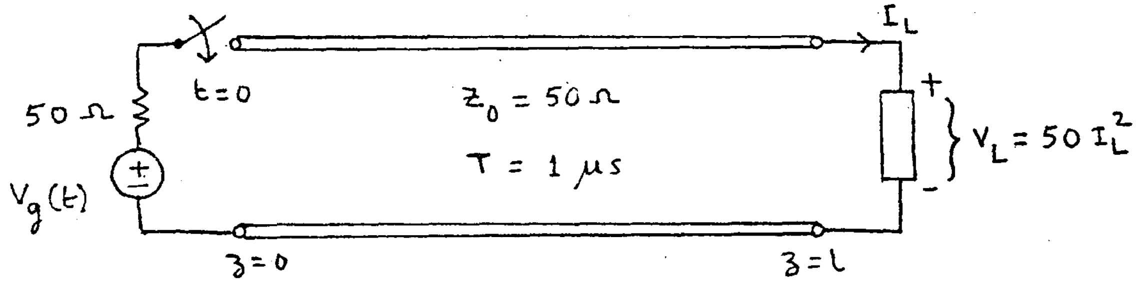
−30/5

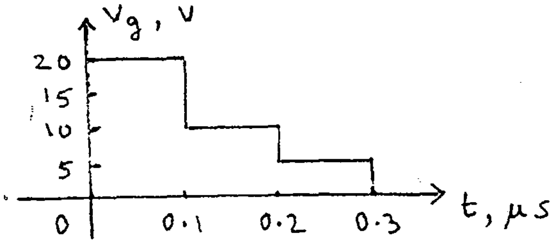
8. The system shown in the figure below is in the steady state. Find (a) the line voltage and current, (b) the (+) wave voltage and current, and (c) the (−) wave voltage and current.



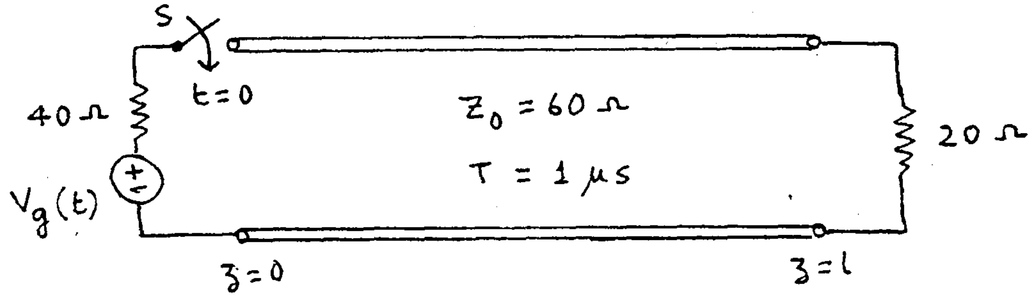
9. For the system of Prob. 7, assume that the voltage source is of 0.3 μs duration, instead of being of infinite duration. Find and sketch the line voltage and line current versus z for t = 1.2 μs and t = 3.5 μs.

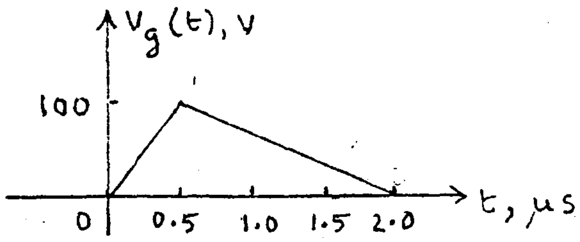
10 In the system shown, the switch S is closed at t = 0. The load is a passive nonlinear element having the indicated volt-ampere characteristic. Find and sketch the line voltage and line current versus time at z = 0 and z = l for $0 < t < \infty$.





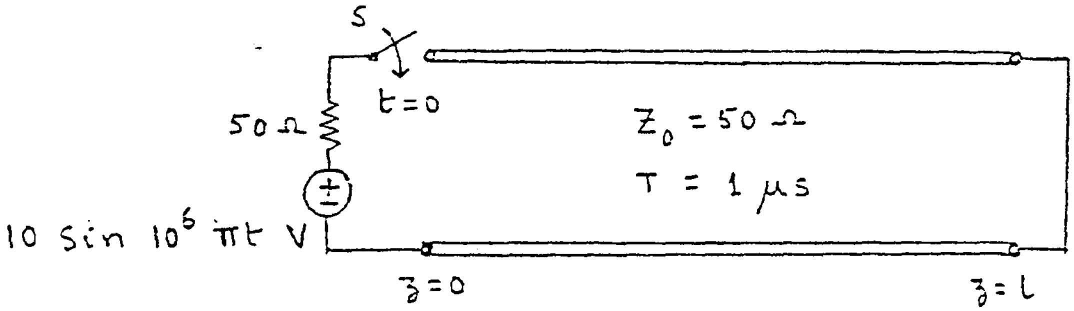
11. In the system shown, the switch S is closed at t = 0. Find and sketch (a) the line voltage versus z for $t = 2\frac{1}{2}$ μs and (b) the line current versus z for $t = 2\frac{1}{2}$ μs.



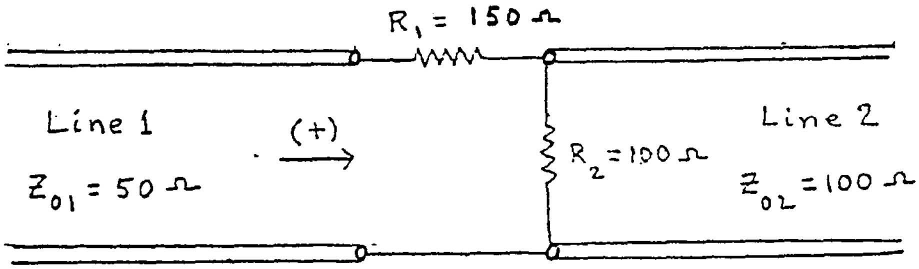


12. In the system shown, the switch S is closed at t = 0. Draw the voltage and current bounce diagrams and sketch the following: (a) line voltage and line current versus t for z = 0 and z = l; (b) line voltage and line current versus z for t = 2, 9/4, 5/2, 11/4, and 3 μs.

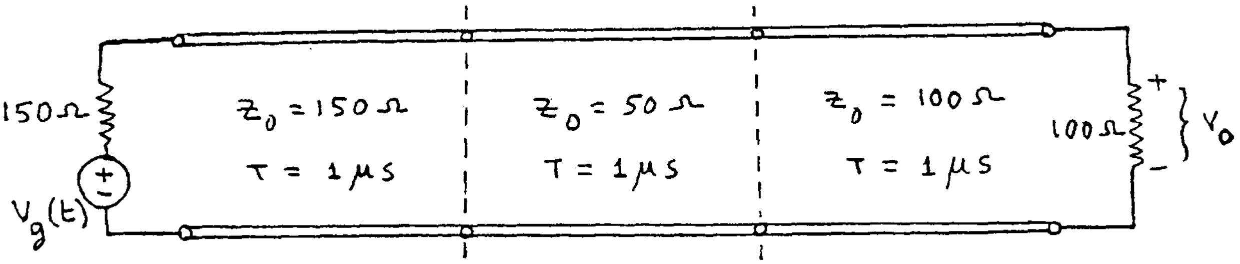
NOTE: The period of the source voltage is 2 μs, which is equal to the two-way travel time on the line.



13. In the system shown, an incident wave of voltage V^+ strikes the discontinuity from the left, that is, from line 1. Find the reflected wave voltage and current into line 1 and the transmitted wave voltage and current into line 2.

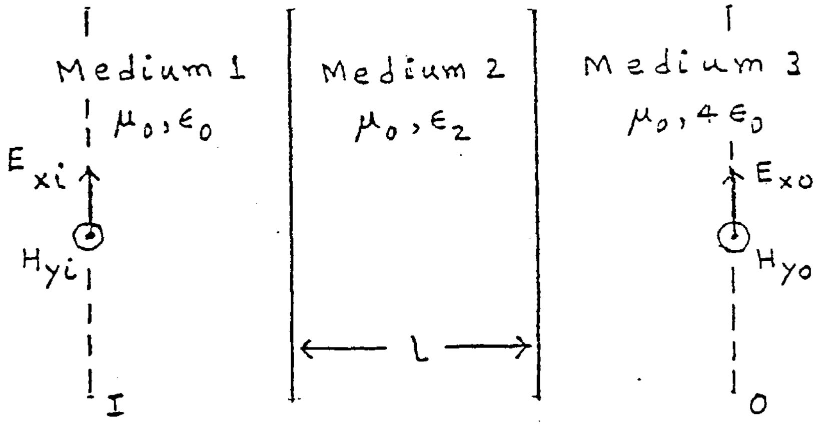


14. In the system shown, find and sketch the output voltage V_o across the 100 Ω resistor for $V_g(t) = \delta(t)$.

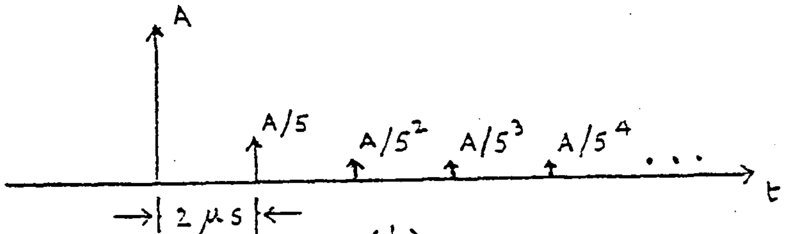


15. In Fig. (a) below, the plane I is the input plane from which a uniform plane wave is incident normally on the interface between medium 1 and medium 2, and the plane 0 is the output plane in which the response of the system is observed. For an incident wave of $E_{xi}(t) = \delta(t)$, find the permittivity $\varepsilon_2 (> \varepsilon_0)$ and the thickness l of medium 2 required to obtain the electric field $E_{xo}(t)$ in the output plane, as shown in Fig. (b), in which the interval between successive impulses is 2 μs. Then find the value of A.

<u>HINT:</u> Use transmission line analogy. First find ε_2 and then l.

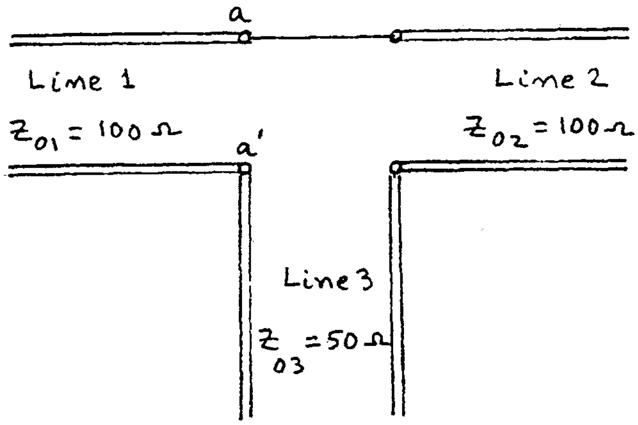


(a)

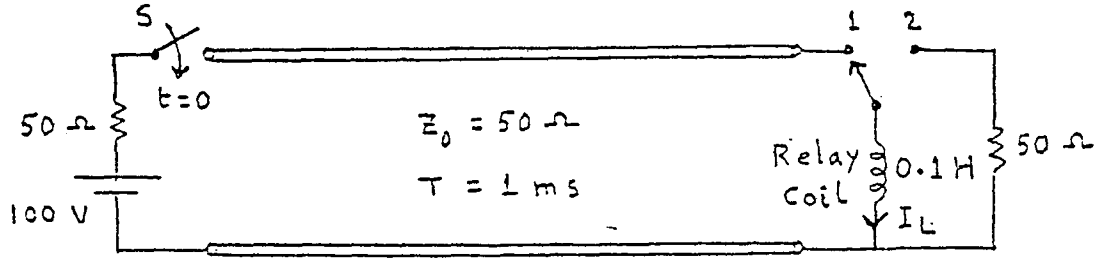


(b)

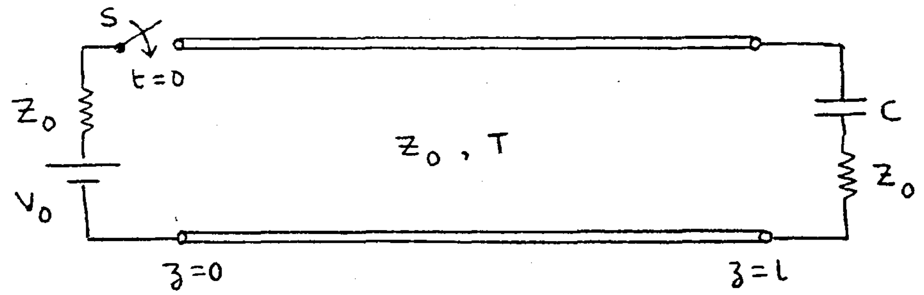
16. In the figure shown, a (+) wave carrying power P watts is incident on the junction a - a' from line 1. Find (a) the power reflected into line 1, (b) the power transmitted into line 2 and (c) the power transmitted into line 3.



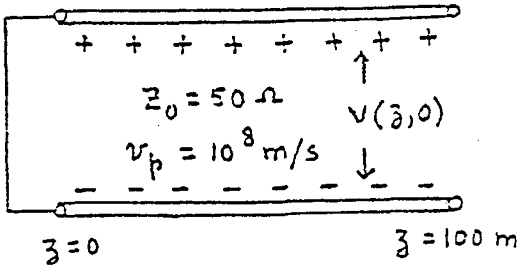
17. In the system shown, the switch S is closed at t = 0, with no current in the relay coil and with the relay in position 1. When the relay coil current I_L reaches 1.73 A, the relay switches to position 2; when the current drops to 0.636 A, the relay switches back to position 1. (a) Find the time t_1 at which the relay switches to position 2. (b) Find the time t_2 at which the relay switches back to position 1.



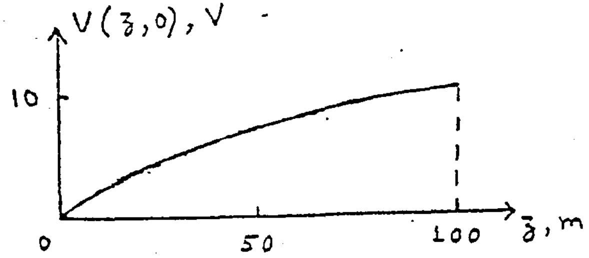
18. In the system shown, the switch S is closed at $t = 0$, with the voltage across the capacitor equal to zero. (a) Obtain the differential equation for V^- at $z = l$. (b) Find the solution for $V^-(l,t)$.



19. In Fig. (a) below, the line is short-circuited at one end $z = 0$, and open-circuited at the other end $z = 100$m. At $t = 0$, the current is zero throughout the line, and the voltage distribution is given by $V(z,0) = 10 \sin 0.005\pi z$ V as shown in Fig. (b). Find and sketch the voltage and current distributions on the line for values of t equal to 0.5 μs and 1 μs.

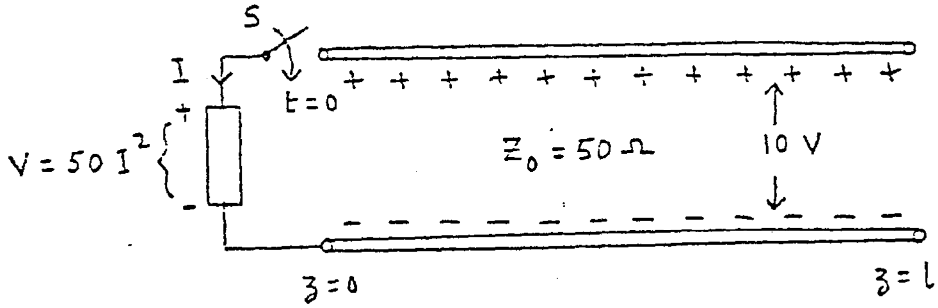


(a)

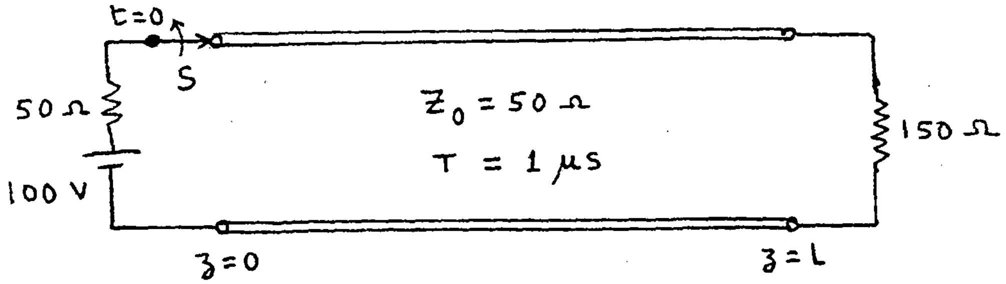


(b)

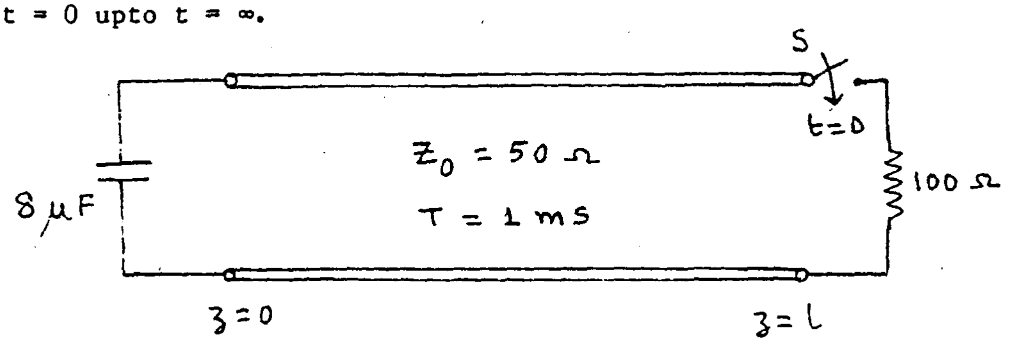
20. In the system shown, a passive nonlinear element having the indicated volt-ampere characteristic is connected to an initially charged line at $t = 0$. Find the voltage across the nonlinear element immediately after closure of the switch.



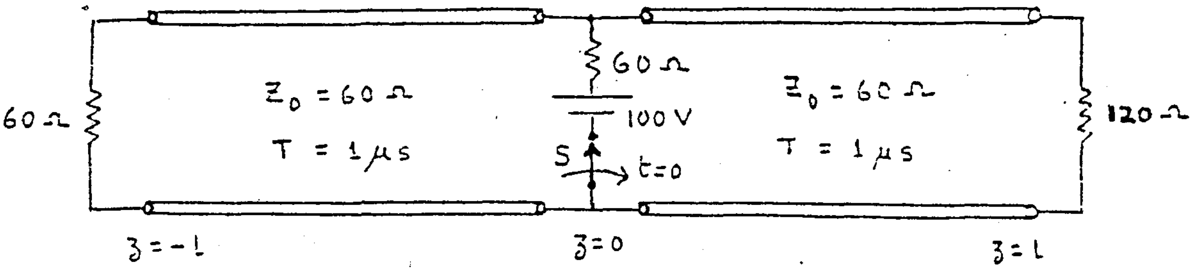
21. In the system shown, steady state conditions are established with the switch S closed. At $t = 0$, the switch is opened. (a) Find and sketch the voltage across the 150 Ω resistor for $t \geq 0$, with the aid of a bounce diagram. (b) Show that the total energy dissipated in the 150 Ω resistor after opening the switch is exactly the same as the energy stored in the line before opening the switch.



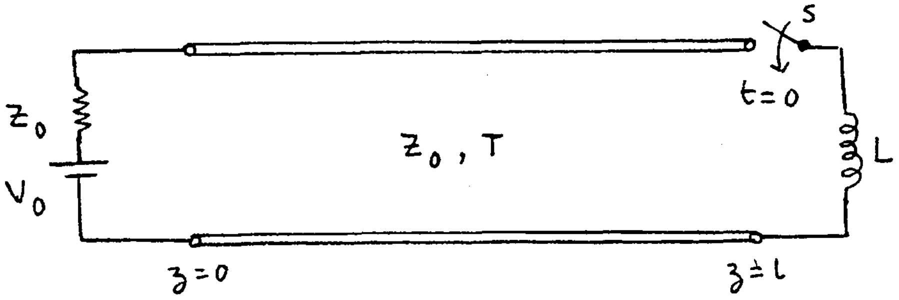
22. In the system shown, the capacitor and the line are both charged to 10 V at $t = 0-$. The switch S is closed at $t = 0$ thereby connecting the 100 Ω resistor to the line. Find the energy dissipated in the 100 Ω resistor from $t = 0$ upto $t = \infty$.



23. In the system shown, steady state conditions are established with the switch S closed. At $t = 0$, the switch is opened. (a) Sketch the voltage and current along the system for $t = 0-$. (b) Find the total energy stored in the lines for $t = 0-$. (c) Find and sketch the voltages across the two resistors for $t > 0$. (d) From your sketches of part (c), find the total energy dissipated in the resistors for $t > 0$.



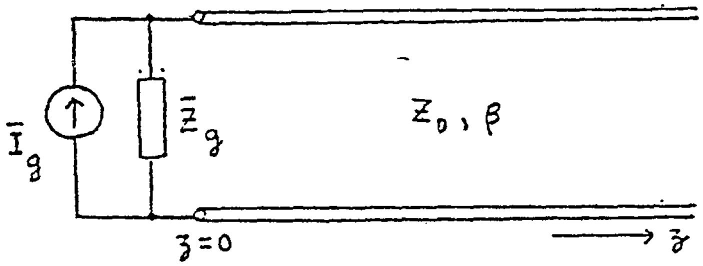
24. In the system shown, steady state conditions are established with the switch S open and no current in the inductor. At $t = 0$, the switch is closed. (a) Obtain the expression for the line voltage and current versus t at $z = l$. (b) Sketch the line voltage and current versus z for $t = T/2$.



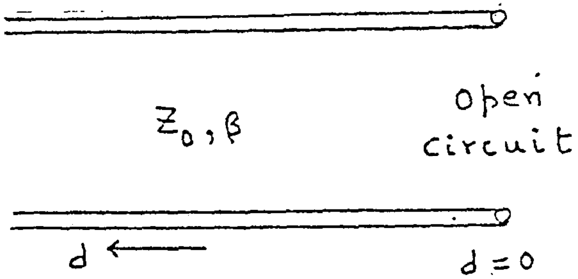
PROBLEMS

FREQUENCY DOMAIN ANALYSIS

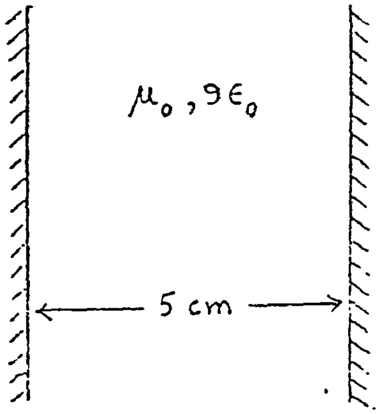
1. In the system shown, $\overline{I}_g = 10\underline{/0^\circ}$ A, $\overline{Z}_g = (12 - j24)\ \Omega$, and $Z_0 = 60\ \Omega$. Find the following quantities: (a) the phasor line voltage $\overline{V}(z)$, (b) the phasor line current $\overline{I}(z)$, and (c) the time average power flow down the line.



2. For a line open-circuited at the far end, as shown in the figure, obtain the solutions for the complex line voltage and current, and sketch the voltage and current standing wave patterns.



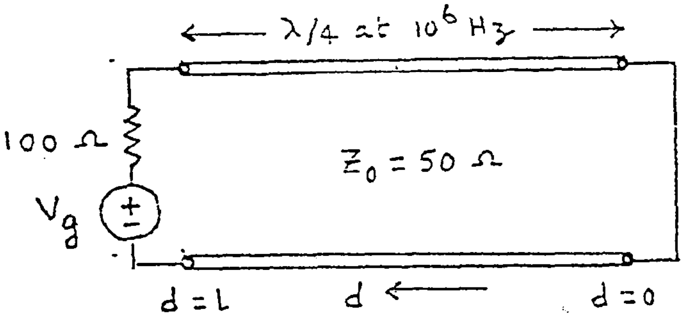
3. The arrangement shown is that of a parallel plate resonator made up of a dielectric slab sandwiched between perfect conductors, and in which uniform plane waves bounce back and forth normal to the conductors. Find the natural frequencies of oscillation of the system.



4. In the system shown, the source voltage is given by

$$V_g(t) = 10 \cos 2\pi \times 10^6 t + 5 \cos 4\pi \times 10^6 t \text{ V}$$

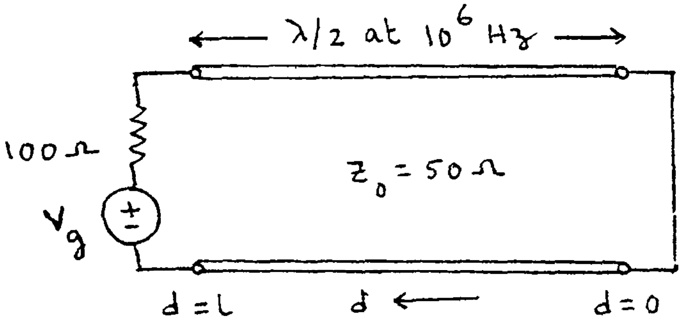
Find the root mean square values of the line voltage and current at values of d equal to 0, $l/2$, and l.



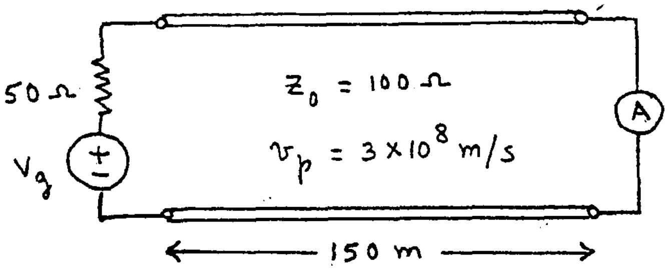
5. In the system shown, the source voltage is

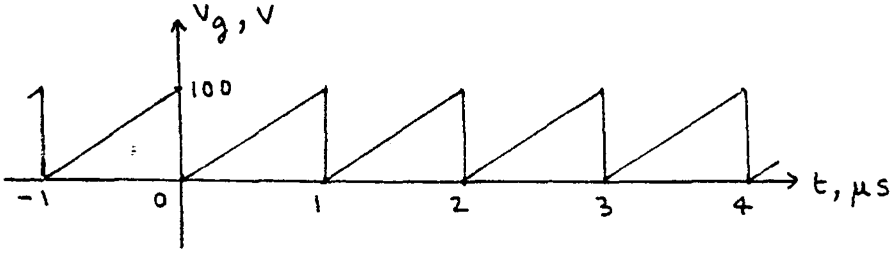
$$V_g(t) = 100 \cos^3 2\pi \times 10^6 t \text{ V}$$

Find the root mean square values of the line voltage and line current at values of d equal to 0, $l/2$, and l.



6. In the system shown, the source voltage is periodic. Find the reading of the rms reading ammeter A.





7. Find the three lowest frequencies, not including zero, for which the input reactance of a short-circuited line of length l and phase velocity v_p is equivalent to that of the total inductance Ll of the line.

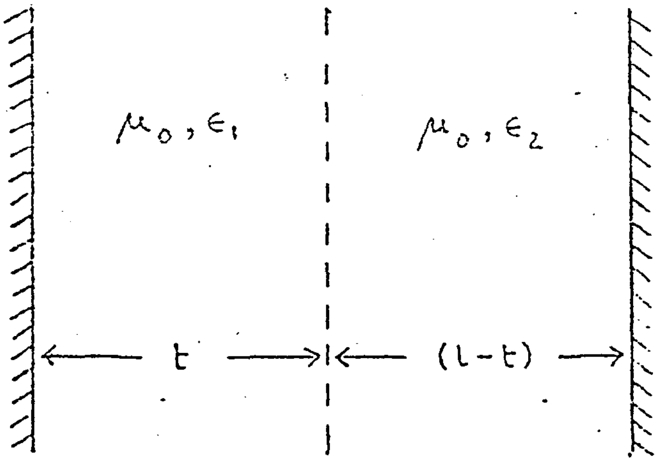
8. The arrangement shown is that of a parallel plate resonator made up of two dielectric slabs, sandwiched between perfect conductors, and in which uniform plane waves bounce back and forth normal to the conductors.

(a) Show that the resonant frequencies of the system are given by the roots of the characteristic equation

$$\tan \omega\sqrt{\mu_0 \epsilon_1}\, t + \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tan \omega\sqrt{\mu_0 \epsilon_2}\,(l - t) = 0$$

$$\frac{\sin\ \omega\sqrt{\mu_0\epsilon_1}}{\cos\ \omega\sqrt{\mu_0\epsilon_1}} + \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

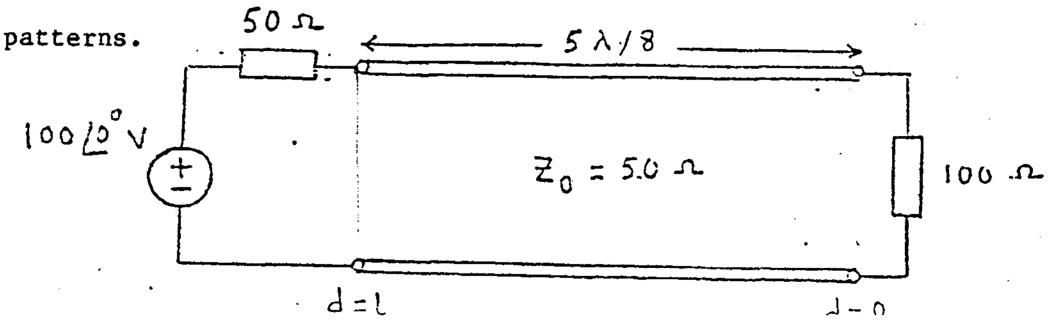
(b) Find the three lowest resonant frequencies if $t = l/2$, $l = 5.0$ cm, $\varepsilon_1 = 4\varepsilon_0$, and $\varepsilon_2 = 16\ \varepsilon_0$.



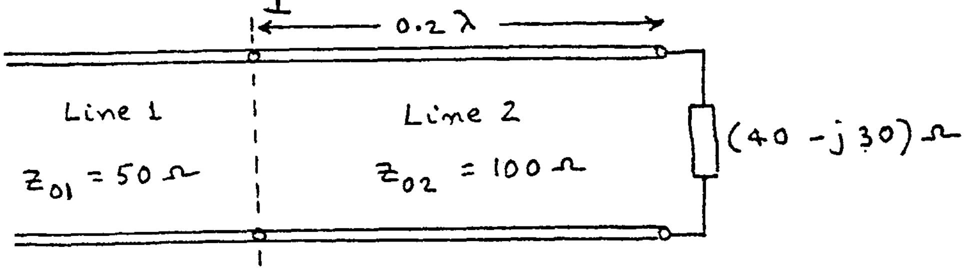
9. A transmission line of characteristic impedance $Z_0 = 100\ \Omega$ is terminated by a load impedance $\bar{Z}_R = (80 + j200)\ \Omega$. Find the values of $\bar{\Gamma}_R$, SWR, and d_{min}.

10. A slotted coaxial line of characteristic impedance 50 Ω was used to measure an unknown load impedance. First, the receiving end of the line was short-circuited. The voltage minima were found to be 0.6 m apart. One of the minima was marked as the reference point. Next, the unknown impedance was connected to the receiving end of the line. The SWR was found to be 5.0 and a voltage minimum was found to be 0.1 m from the reference point toward the load. Find the value of the unknown load impedance.

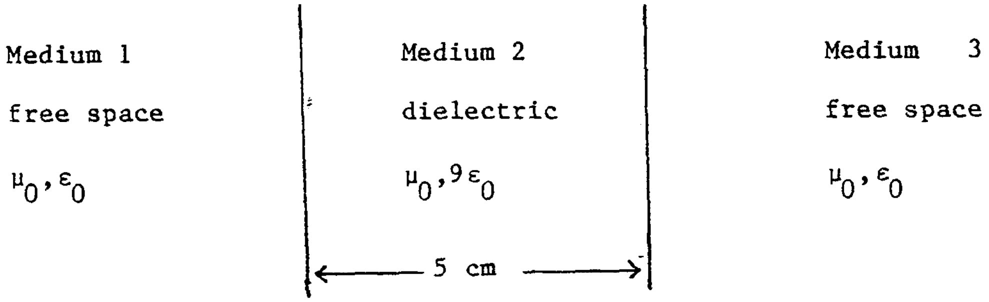
11. For the system shown, find the input impedance of the line, the time average power flow down the line, the value of $|\bar{V}^+|$, and the various voltage and current values in the standing wave patterns, and sketch the standing wave patterns.



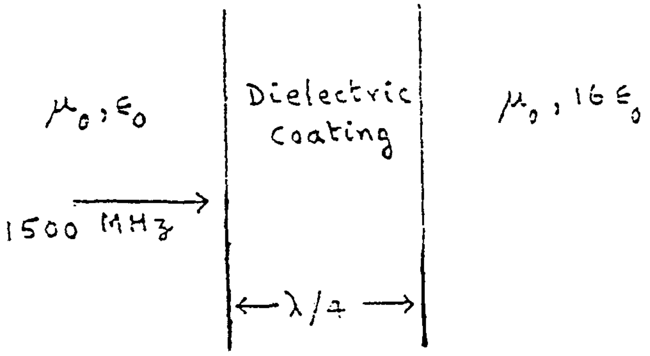
12. In the system shown, find (a) the SWR in line 2, (b) the input impedance of line 2, and (c) the SWR in line 1.



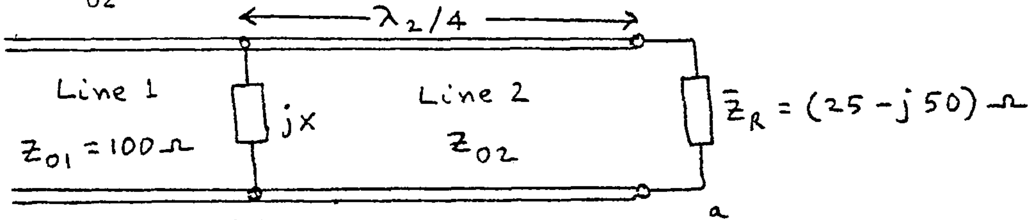
13. For the system shown, find the values of the three lowest frequencies for which complete transmission occurs from medium 1 to medium 3 for normally incident uniform plane waves.



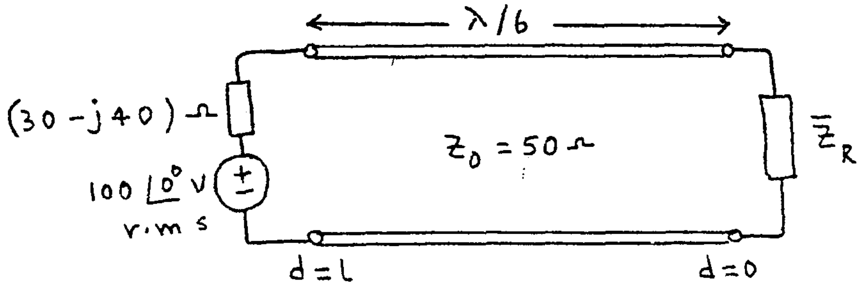
14. In the arrangement shown, a quarter-wave dielectric coating is employed to eliminate reflections of uniform plane waves of frequency 1500 MHz incident normally from free space on to a dielectric of permittivity 16 ε_0. (a) Assuming $\mu = \mu_0$, find the thickness in cm and the permittivity of the dielectric coating. (b) For the values obtained in (a), find the two frequencies on either side of 1500 MHz at which the SWR in the free space is 1.5.



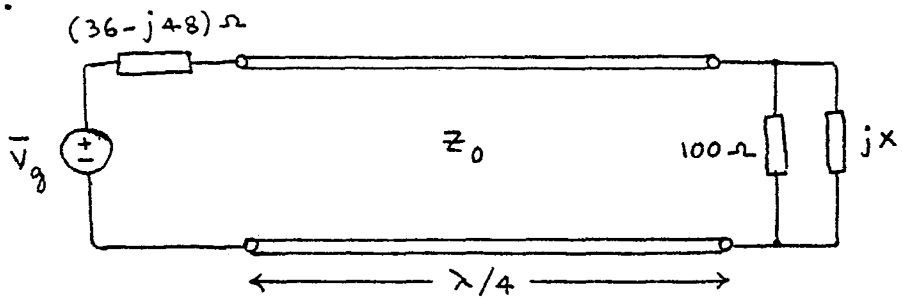
15. In the system shown, find the value of the reactance X and the characteristic impedance Z_{02} required to eliminate standing waves in line 1.



16. In the system shown, find (a) the value of the load impedance that enables maximum power transfer from the generator to the load, and (b) the power transferred to the load for the value found in (a).



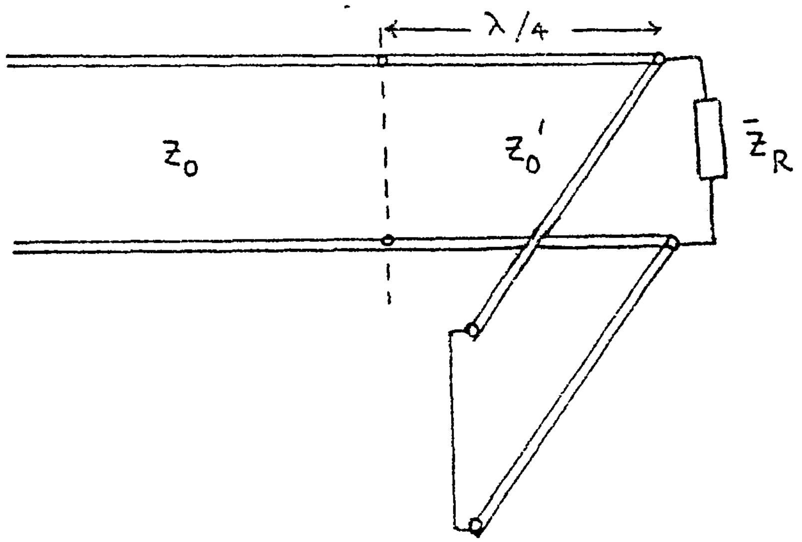
17. In the system shown, find the values of the characteristic impedance Z_0 and the reactance jX for which the power delivered to the load is a maximum.



reactive elements do not consume power

18. A transmission line of characteristic impedance 50 Ω is terminated by a certain load impedance. It is found that the SWR on the line is equal to 5.0 and that the first voltage minimum of the standing wave pattern is located to be at 0.1λ from the load. Determine the location and the length of a short-circuited stub of characteristic impedance 50 Ω connected in parallel with the line required to achieve a match between the line and the load.

19. In the system shown, find the characteristic impedance Z_0' of the quarter-wave section, and the length l of a short-circuited stub of characteristic impedance 100 Ω, required to achieve a match between the line of characteristic impedance Z_0 = 100 Ω and the load impedance $\bar{Z}_R$ = (16 - j12) Ω.



20. A transmission line of characteristic impedance 50 Ω is terminated by a certain load impedance. It is found that the SWR on the line is equal to 3.0. The first and second voltage minima of the standing wave pattern are located at 5.80 cm and 25.80 cm, respectively, from the load. (a) Find the value of the minimum SWR that can be achieved on the line by placing a stub in parallel with the line at the load. (b) Find the length of the stub required to achieve the minimum SWR, assuming that the stub is short-circuited and its characteristic impedance is 50 Ω.

EE350
(NNR)

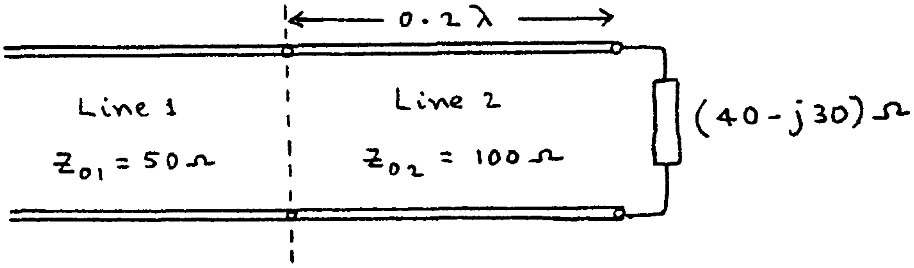
PROBLEMS

SMITH CHART

1. For a transmission line of characteristic impedance 100 Ω, terminated by a load impedance (80 + j200) Ω, find the following quantities: (a) Reflection coefficient at the load. (b) SWR on the line. (c) The distance of the first voltage minimum of the standing wave pattern from the load.

2. For a transmission line of characteristic impedance 100 Ω, terminated by a load impedance (80 + j200) Ω at d = 0, find the following quantities: (a) The line impedance at d = 0.1 λ. (b) The line admittance at d = 0.1 λ. (c) The location nearest to the load at which the real part of the line admittance is equal to the line characteristic admittance.

3. In the system shown, find (a) the SWR in line 2, (b) the input impedance of line 2, and (c) the SWR in line 1, using the Smith Chart.



4. A transmission line of characteristic impedance 50 Ω is terminated by a certain load impedance. It is found that the SWR on the line is equal to 5.0 and that the first voltage minimum of the standing wave pattern is located to be at 0.1 λ from the load. Determine the location and the length of a short-circuited stub of characteristic impedance 50 Ω connected in parallel with the line required to achieve a match between the line and the load.

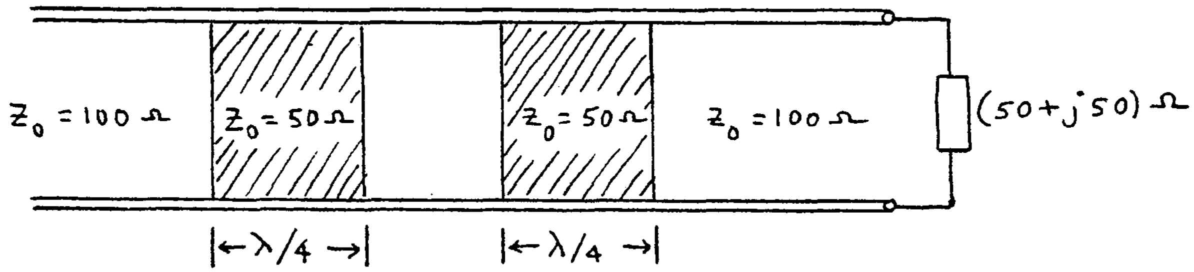
5. A transmission line of characteristic impedance 50 Ω is terminated by a certain load impedance. It is found that the SWR on the line is equal to 3.0. The first and second voltage minima of the standing wave pattern are located at 5.80 cm and 25.80 cm, respectively, from the load. (a) Find the value of the minimum SWR that can be achieved on the line by placing a stub in parallel with the line at the load. (b) Find the length of the stub required to achieve the minimum SWR, assuming that the stub is short-circuited and its characteristic impedance is 50 Ω.

6. Standing wave measurements on a line of characteristic impedance 50 Ω indicate SWR on the line to be 3.0 and the location of the first voltage minimum of the standing wave pattern to be 0.16 λ from the load. Assuming $d_1 = 0.1\ \lambda$, $d_{12} = 0.625\ \lambda$ $\simeq 5/8\,\lambda$, find the lengths of the two short-circuited stubs of characteristic impedance 50 Ω required to achieve a match between the line and the load.

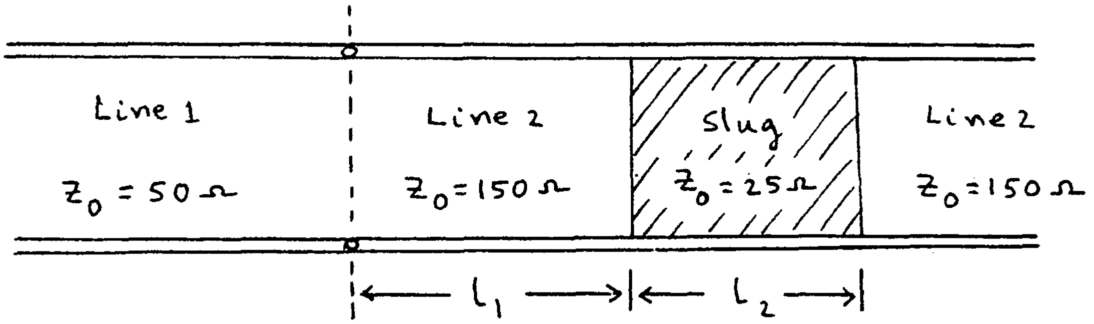
7. It is proposed to match a transmission line of characteristic impedance 100 Ω to a load impedance (7.5 - j30) Ω by using a double stub arrangement with spacing between stubs, d_{12}, equal to $3\lambda/8$. Determine the forbidden range of values of d_1 within the first half wavelength in order to achieve the match.

8. A transmission line of characteristic impedance 100 Ω is terminated by a load impedance (30 - j40) Ω. A movable element of fixed reactance 100 Ω is connected in parallel with the line. Find the minimum value of the SWR that can be achieved on the line and the location of the reactance that will achieve the minimum SWR.

9. In the system shown, two movable slugs each of length λ/4 and characteristic impedance 50 Ω are employed. Find the locations of the two slugs in order to achieve a match between the line and the load.



10. In the system shown, it is proposed to achieve a match between line 1 of characteristic impedance 50 Ω and line 2 of characteristic impedance 150 Ω by inserting a slug of characteristic impedance 25 Ω in line 2. Find the values of l_1 and l_2 to achieve the match. Assume line 2 to be infinitely long.



WAVEGUIDES

1. By expanding in Cartesian coordinates, show that

$$\nabla^2 \underset{\sim}{A} = \underset{\sim}{\nabla}(\underset{\sim}{\nabla} \cdot \underset{\sim}{A}) - \underset{\sim}{\nabla} \times \underset{\sim}{\nabla} \times \underset{\sim}{A}$$

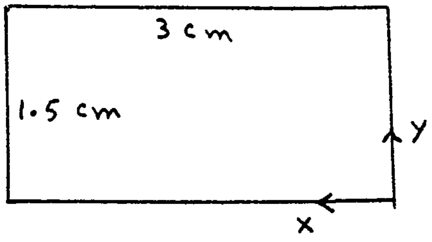
$$= (\nabla^2 A_x)\,\hat{x} + (\nabla^2 A_y)\,\hat{y} + (\nabla^2 A_z)\,\hat{z}$$

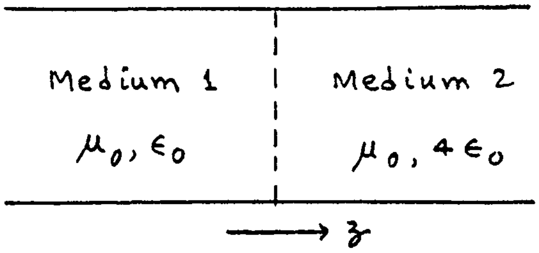
2. For a rectangular waveguide of dimensions $a = 5$ cm and $b = 5/3$ cm, and having a dielectric of $\varepsilon = 9\ \varepsilon_0$ and $\mu = \mu_0$, find all propagating modes for $f = 2500$ MHz.

3. An air dielectric rectangular waveguide of dimensions $a = 3$ cm and $b = 1$ cm is excited by a source of frequency $f = 4000$ MHz and rich in harmonics. Find all the frequencies that propagate in TE mode(s) only, and specify the mode(s) of propagation for each of these frequencies.

4. Find the values in cm of the dimensions a and b of an air dielectric rectangular waveguide such that $TE_{1,0}$ mode of frequency $f = 3000$ MHz propagates with a 30% safety factor ($f \geq 1.30\ f_c$) but also such that f is at least 30% below the cutoff frequency of the next higher order mode.

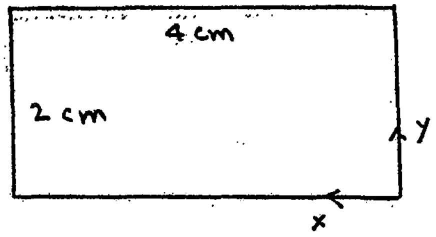
5. A rectangular waveguide of dimensions $a = 3$ cm and $b = 1.5$ cm has a dielectric discontinuity, as shown in the figure. A $TE_{1,0}$ wave of frequency 6000 MHz is incident on the discontinuity from the free space side. (a) Find the SWR in the free space section. (b) Find the length and the permittivity of a quarter-wave section required to achieve a match between the two media.

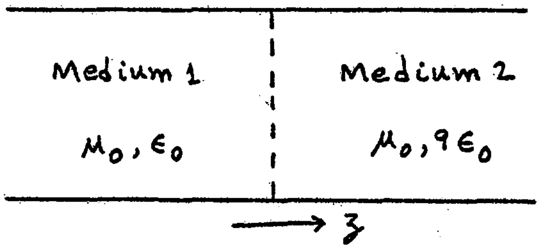




6. A rectangular waveguide of dimensions a = 4 cm and b = 2 cm has a dielectric discontinuity, as shown in the accompanying figure. A $TM_{1,1}$ wave of frequency 10,000 MHz is incident from the free space side.

(a) Find the SWR in the free space section. (b) Find the length and the permittivity of a quarter-wave section required to achieve a match between the two media.

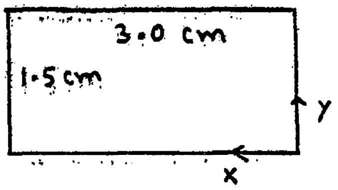


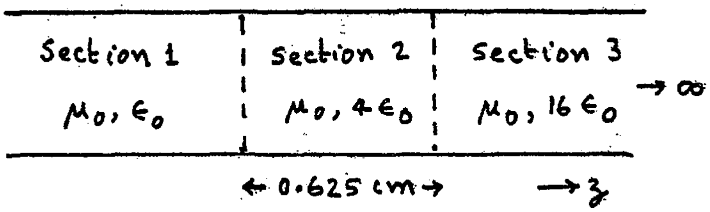


7. A rectangular waveguide of dimensions a = 3 cm and b = 1.5 cm has dielectric discontinuities, as shown in the figure. Note that Section 3 extends to infinity. A $TE_{1,0}$ wave of frequency f = 6000 MHz is incident on Section 2 from Section 1.

(a) Obtain the transmission line equivalent of the system.

(b) Using the Smith Chart, find the SWR in Section 1.

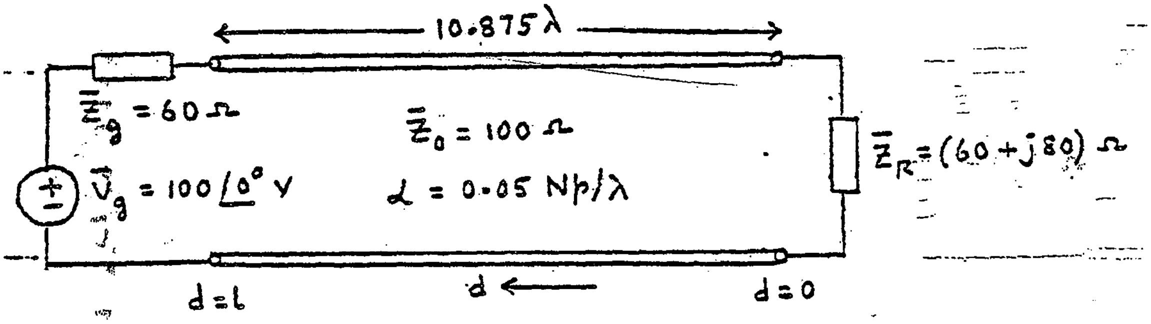




8. For an air dielectric rectangular waveguide having the dimensions a = 3 cm and b = 1.5 cm, find the group velocity for (a) a signal composed of the two frequencies f_1 = 6000 MHz and f_2 = 9000 MHz, and (b) a narrow-band signal having the center frequency 6000 MHz.

LOSSY LINES

31. For the lossy transmission line system shown ~~in figure~~ find the SWR at the load end, the SWR at the input end, the input impedance of the line, the time-average power flow at the input end of the line, the time-average power delivered to the load, and the time-average power dissipated in the line.



$\Gamma_R = j\,5$

$[SWR]_{d=0} = 3$

$\bar\Gamma(l) = .5e^{j\pi/2}\, e^{-2\,0.05 \times 10.875}\, e^{-j\frac{4\pi}{\lambda}\cdot 10.875\lambda}$

$= .1685\, e^{-j43\pi}$

$= -.1685$

$[SWR]_{d=l} = 1.405$

$Z_{in} = 100 \cdot \frac{1 - .1685}{1 + .1685} = 71.11\ \Omega$

$\bar I(l) = .762\ A$

$\bar V(l) = 54.224\ V$

$\langle P(l)\rangle = 20.659\ W$

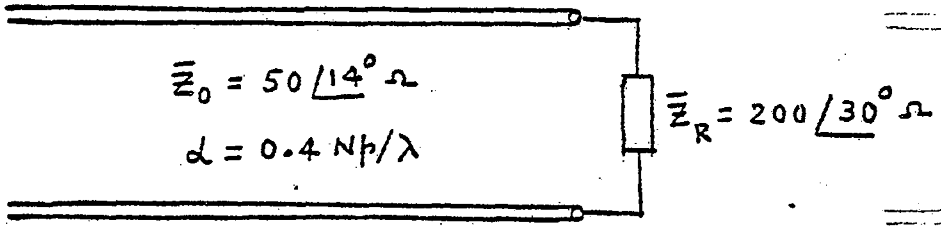
$|V^+| = 37.86\ V$

$\langle P(0)\rangle = 5.355\ W$

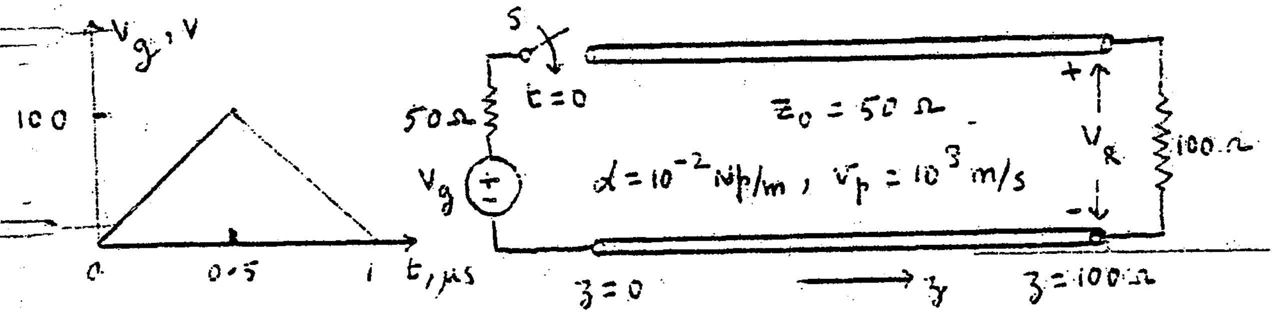
$\langle P(l)\rangle = 20.659 - 5.375 = 15.284\ W$

32. The input impedance of a lossy line of length 30 m is measured at a frequency of 100 MHz for two cases: with the output short-circuited, it is $(44 + j90)\ \Omega$, and with the output open-circuited, it is $(44 - j90)\ \Omega$. Find (a) the characteristic impedance of the line, (b) the attenuation constant of the line, and (c) the phase velocity in the line, assuming its approximate value to be 2×10^8 m/s.

33. For the system shown ~~[illegible]~~, find the location nearest to the load and the length of a lossless short-circuited stub of characteristic impedance 50 Ω required to achieve a match between the line and the load. Repeat for the second nearest location to the load.



34. For the distortionless line system shown, the source voltage V_g is a triangular pulse of duration 1 μs. Find and sketch (a) the voltage V_R across the load resistor versus t, (b) the line voltage versus z for t = 0.5 μs, and (c) the line voltage versus z for t = 1.5 μs.



ANTENNAS

35. Find the value of r at which the amplitude of the radiation field term [illegible] is equal to the resultant amplitude of the remaining two terms in the θ-component in the expression for $\underset{\sim}{E}$ for the Hertzian dipole.

36. A straight wire of length 1 m in free space carries a uniform current $10 \cos 4\pi \times 10^6 t$ amp. (a) Calculate the amplitude of the electric field intensity at a distance of 10 km in a direction at right angle to the wire. (b) Calculate the radiation resistance and the time-average power radiated by the wire.

37. The radiation pattern for the power density of an antenna located at the origin is dependent on θ in the manner $\sin^2 \theta \cos^2 \theta$. Find the directivity of the antenna.

38. Find the time-average power required to be radiated by a half-wave dipole in order to produce an electric field intensity of peak amplitude 0.01 V/m at a distance of 1 km broadside to the dipole.

39. For the array of two antennas carrying equal currents and [illegible], having $d = \lambda/4$, find the value of α for which the maxima of the group pattern are directed along $\psi = \pm 60°$, and then sketch the group pattern.

40. For a linear array of three isotropic antennas, spaced $\lambda/2$ apart, carrying unequal currents in the ratio 1:2:1, and fed in phase, obtain the resultant pattern.